MATHEMATICS

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9610EPN10MTEN2500901

# 1 Real Numbers

**Multiple choice questions** **(1 mark)**

**1.** What is the HCF of the smallest prime number and the smallest composite number?

(1) 2 (2) 4 (3) 1 (4) 3

**Sol.** **Option (1)**

The smallest prime number = 2

The smallest composite number = 4

Prime factorisation of 2 = 2

Prime factorisation of 4 = 2 × 2 = $2^2$

HCF(2, 4) = 2

Therefore, the HCF of the smallest prime number and the smallest composite number is 2.

**2.** Two positive integers p and q can be expressed as $p = ab^2$ and $q = a^2b$, a and b are prime numbers. What is the LCM of p and q?

(1) $a^2b$ (2) $a^2b^2$ (3) $ab$ (4) $ab^2$

**Sol.** **Option (2)**

$a^2b^2$

**3.** For any positive integer a and b, there exist unique integers q and r such that a = 3q + r, where r must satisfy a condition, which is

(1) $0 \le r > 3$ (2) $0 \ge r > 3$ (3) $0 \le r < 3$ (4) $0 \ge r < 3$

**Sol.** **Option (3)**

According to Euclid's Division Lemma,

a = 3q + r

$\mathbf{0 \le r < 3}$

**4.** The decimal expansion of the rational number $\frac{45}{2^4 5^3}$ will terminate after how many places of decimals?

(1) 5 (2) 3 (3) 4 (4) 1

**Sol.** **Option (3)**

$$\frac{45}{2^4 \times 5^3} = \frac{9 \times 5}{2^4 \times 5^3} = \frac{9}{2^4 \times 5^2} \times \frac{5^2}{5^2} = \frac{225}{10^4}$$

After 4 decimal places.

**5.** Given that HCF (306, 657) = 9, LCM (306, 657) will be

(1) 23328 (2) 22338 (3) 22833 (4) 222338

**Sol. Option (2)**

HCF × LCM = Product of the two numbers

9 × LCM = 306 × 657

$$\text{LCM} = \frac{(306 \times 657)}{9} = 22338$$

Therefore, LCM(306, 657) = 22338

**6.** The largest possible positive integer that will divide 398 and 436 leaving remainders 7 and 11 respectively is

(1) 15 (2) 3 (3) 17 (4) 5

**Sol. Option (3)**

Required Number = HCF(398 – 7, 436 – 11)

= HCF(391, 425) = 17

**7.** The least number of soldiers in a regiment such that they stand in rows of 15, 20, 25 and form a perfect square are

(1) 300 (2) 400 (3) 1600 (4) 900

**Sol. Option (4)**

Least number of soldiers such that they can stand in rows of 15, 20, 25 is LCM (15, 20, 25)

| | |
|---|---|
| 2 | 15, 20, 25 |
| 2 | 15, 10, 25 |
| 3 | 15, 5, 25 |
| 5 | 5, 5, 25 |
| 5 | 1, 1, 5 |
| | 1, 1, 1 |

LCM = 2 × 2 × 3 × 5 × 5 = 300

Least multiple of 300 which is a perfect square is 900.

**8.** HCF of 196 and 38220 will be

(1) 196 (2) 144 (3) 169 (4) 195

**Sol. Option (1)**

Using Euclid's division algorithm,

38220 = 196 × 195 + 0

Here the remainder is 0.

Therefore, the HCF of 196 and 38220 is 196.

### Assertion reason questions (1 mark)

**9. Assertion:** The H.C.F. of two numbers is 16 and their product is 3072. Then their L.C.M. = 162.

**Reason:** If a and b are two positive integers, then H.C.F. × L.C.M. = a × b.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol. Option (4)**

Assertion (A) is false but Reason (R) is true.

Since HCF × LCM = a × b $\Rightarrow$ 3072 = 16 × 162 $\Rightarrow$ 3072 $\neq$ 2592

**10. Assertion:** Denominator of 34.12345 when expressed in the form p/q, q ≠ 0, is of the form $2^m \times 5^n$, where m and n are non-negative integers.

**Reason:** 34.12345 is a terminating decimal fraction.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol. Option (1)**

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Here, $34.12345 = \dfrac{3412345}{100000} = \dfrac{682469}{20000} = \dfrac{682469}{2^5 \times 5^4}$

Its denominator is of the form $2^m \times 5^n$.

### Very short answer type questions (2 mark)

**11.** Find the HCF of 735 and 85 by using Euclid's algorithm.

**Sol.** HCF of 735 & 85

735 = 85 ×8 + 55

85 = 55 × 1 + 30

55 = 30 × 1 + 25

30 = 25 × 1 + 5

25 = 5 × 5 + 0

Therefore HCF = 5

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**12.** Has the rational number $\frac{441}{2^2.5^7.7^2}$ a terminating or a non-terminating decimal representation ?

**Sol.** Since, given rational number = $\frac{441}{2^2.5^7.7^2} = \frac{7^2 \times 3^2}{2^2 \times 5^7 \times 7^2} = \frac{3^2}{2^2 \times 5^7}$

Since, denominator is of form $2^n \times 5^m$.

Hence, rational number has a terminating decimal expansion.

**13.** Write whether $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ on simplification gives a rational or an irrational number.

**Sol.** $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}} = \frac{2\times3\sqrt{5}+3\times2\sqrt{5}}{2\sqrt{5}}$

$= \frac{6\sqrt{5}+6\sqrt{5}}{2\sqrt{5}}$

$= \frac{12\sqrt{5}}{2\sqrt{5}} = 6$

Which is a rational number.

Hence, $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ is a rational number.

**14.** The decimal expansion of the rational number $\frac{43}{2^4.5^3}$, will terminate after how many places of decimals?

**Sol.** We have, $\frac{43}{2^4.5^3} = \frac{43}{2\times2\times2\times2\times5\times5\times5\times5} \times 5$

$= \frac{215}{(2\times5)^4} = \frac{215}{10^4}$

$= \frac{215}{10000} = 0.0215$

The decimal expansion will terminate after four places of decimals.

**15.** Write whether the rational number $\frac{51}{1500}$ will have a terminating decimal expansion or a non-terminating repeating decimal expansion.

**Sol.** $\frac{51}{1500} = \frac{3\times17}{2\times2\times3\times5\times5\times5} = \frac{17}{2^2\times5^3}$

Since denominator is of form $2^m \times 5^n$.

Hence, $\frac{17}{2^2\times5^3}$ has a terminating decimal expansion.

**16.** Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

**Sol.** $7 \times 11 \times 13 + 13 = 13(7 \times 11 + 1) = 13 \times 78$,
which is not a prime number because it has more than two factors. So, it is a composite number.
And, $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5(7 \times 6 \times 4 \times 3 \times 2 + 1) = 5 \times 1009$,
which is not a prime number because it has more than two factors. So, it is also a composite number.

**Short answer type questions** **(3 marks)**

**17.** Can the number $4^n$, n being a natural number, end with the digit 0? Give reason.

**Sol.** If $4^n$ ends with 0, then it must have 5 as a factor. But, $(4)^n = (2^2)^n = 2^{2n}$ i.e., the only prime factor of $4^n$ is 2. Also, we know from the fundamental theorem of arithmetic that the prime factorisation of each number is unique.
$\therefore$ $4^n$ can never end with digit 0.

**18.** Use the Euclid's division algorithm to find the HCF of
(i) 650 and 1170
(ii) 870 and 225

**Sol.** (i) On applying Euclid's division lemma for 650 and 1170, we get
$1170 = 650 \times 1 + 520$
Here, remainder $= 520 \neq 0$
So, take new dividend as 650 and divisior as 520.
Then, we get $650 = 520 \times 1 + 130$
Here, remainder $= 130 \neq 0$
So, take new dividend as 520 and divisior as 130.
Then, we get $520 = 130 \times 4 + 0$
Here, remainder is 0 and last divisor is 130.
Hence, HCF of 1170 and 650 is 130.

(ii) On applying Euclid's division lemma for 870 and 225, we get
$870 = 225 \times 3 + 195$
Here, remainder $= 195 \neq 0$
So, take new dividend as 225 and divisor as 195.
Then, we get
$225 = 195 \times 1 + 30$
Here, remainder $= 30 \neq 0$
So, take new dividend as 195 and divisor as 30
$195 = 30 \times 6 + 15$
Here, remainder $= 15 \neq 0$
So, take new dividend as 30 and divisor as 15
$30 = 2 \times 15 + 0$
Here, remainder 0 and last divisor is 15
Hence, HCF of 870 and 225 is 15.

**19.** Write the denominator of the rational number $\frac{257}{500}$ in the form $2^m \times 5^n$, where m and n are non-negative integers. Hence write its decimal expansion without actual division.

**Sol.** Denominator = 500 = $2^2 \times 5^3$

Decimal expansion = $\frac{257}{500} = \frac{257 \times 2}{2 \times 2^2 \times 5^3} = \frac{514}{10^3}$ = 0.514

**20.** The HCF of two numbers is 113 and their LCM is 56952. If one number is 904, find the other number.

**Sol.** Given, HCF = 113, LCM = 56952 and one number = 904

Let another number = x

HCF × LCM = Product of two numbers

$\Rightarrow$ HCF × LCM = one number × other number

$\therefore$ HCF = $\frac{x \times 904}{LCM}$

$\Rightarrow$ 113 = $\frac{x \times 904}{56952}$

$\Rightarrow$ x = $\frac{113 \times 56952}{904}$ =7119

**21.** If HCF of 210 and 55 is expressible in the form 210 × 5 – 55x, find the value of x.

**Sol.**

$$\begin{array}{c|l} 5 & 210 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array} \qquad \begin{array}{c|l} 5 & 55 \\ \hline 11 & 11 \\ \hline & 1 \end{array}$$

210 = 2 × 3 × 5 × 7

55 = 5 × 11

HCF (55, 210) = 5

Given, HCF (210, 55) = 210 × 5 – 55x

$\therefore$ 5 = 210 × 5 – 55x

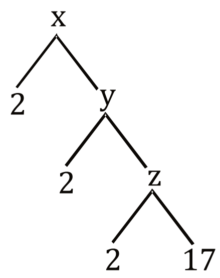
$\Rightarrow$ 55x = 1050 – 5

$\Rightarrow$ 55x = 1045

$\Rightarrow$ x = 19

**22.** Find the value of x, y and z in the given factor tree. Can the value of 'x' be found without finding the value of 'y' and 'z' ? If yes, explain.

**Sol.** $z = 2 \times 17 = 34$ ; $y = 34 \times 2 = 68$ and $x = 2 \times 68 = 136$




Yes, value of x can be found without finding value of y or z as $x = 2 \times 2 \times 2 \times 17$ which are prime factors of x.

**23.** An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

**Sol.** For the maximum number of columns, we have to find the HCF of 616 and 32.

Now, since 616 > 32, we apply division lemma to 616 and 32.

We have, $616 = 32 \times 19 + 8$

Here, remainder $= 8 \neq 0$. So we again apply division lemma to 32 and 8.

We have, $32 = 8 \times 4 + 0$

Here, remainder is zero. So, HCF (616, 32) = 8

Hence, maximum number of columns in which they can march is 8.

**24.** The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then find the other number.

**Sol.** Let HCF of the numbers be x then according to question LCM of the number will be 14x.

And $x + 14x = 600$

$\Rightarrow$ $15x = 600$

$\Rightarrow$ $x = 40$

Then HCF = 40 and LCM $= 14 \times 40 = 560$

$\because$ LCM × HCF = Product of the numbers

$560 \times 40 = 280 \times$ Second number

$\Rightarrow$ Second number $= \dfrac{560 \times 40}{280} = 80$

Therefore, other number is 80.

**25.** Prove that $2-3\sqrt{5}$ is an irrational number.

**Sol.** Let $2-3\sqrt{5}$ be rational number.

$\therefore$ We can find two integers a, b ($b \neq 0$) such that $2-3\sqrt{5}=\frac{a}{b}$

(where a and b are co-prime numbers)

$\Rightarrow$ $3\sqrt{5}=2-\frac{a}{b} \Rightarrow \sqrt{5}=\frac{1}{3}\left[2-\frac{a}{b}\right]$

$\Rightarrow$ $\sqrt{5}$ is rational number.

$\left[\because a,b \text{ are integers}, \frac{1}{3}\left[2-\frac{a}{b}\right] \text{is rational}\right]$

But this contradict to the fact that $\sqrt{5}$ is an irrational number.

Hence, our assumption is wrong.

$\therefore$ $2-3\sqrt{5}$ is an irrational number.

**26.** Prove that $\sqrt{5}$ is an irrational number.

**Sol.** Let us assume $\sqrt{5}$ is a rational number.

$\Rightarrow$ $\sqrt{5}=\frac{a}{b}$ ; where a and b are co-prime positive integers.

Squaring both sides, we get

$5=\frac{a^2}{b^2} \Rightarrow a^2=5b^2 \Rightarrow 5 \text{ divides } a^2$

$\Rightarrow$ 5 divides a ... (1)

$\Rightarrow$ a = 5 c, where c is an integer

Again, squaring both sides, we get

$a^2 = 25c^2$

$\Rightarrow$ $5b^2 = 25c^2$

$\Rightarrow$ $b^2 = 5c^2$

$\Rightarrow$ 5 divides $b^2$

$\Rightarrow$ 5 divides b ... (2)

From (1) and (2), we get 5 divides both a and b

$\Rightarrow$ a and b are not co-prime integers.

This contradicts our fact that a and b are co-prime. This is arisen due to our incorrect assumptions.

Hence, $\sqrt{5}$ is a irrational number.

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**27.** There are 156, 208 and 260 students in groups A, B and C, respectively. Buses are to be hired to take them for a field trip. Find the minimum number of buses to be hired, if the same number of students should be accommodated in each bus.

**Sol.** Given numbers are 156, 208 and 260.

Here, 260 > 208 > 156

Let us first find the HCF of 260 and 208.

By using Euclid's division lemma for 260 and 208,

we get $260 = (208 \times 1) + 52$

Here, remainder $= 52 \neq 0$

On taking 208 as new dividend and 52 as new divisor and then apply Euclid's division lemma, we get

$208 = (52 \times 4) + 0$

Here, the remainder is zero and the divisor is 52.

So, 52 is the HCF of 208 and 260.

Now, 156 > 52

Let us find the HCF of 52 and 156. By using Euclid's division lemma, we get

$156 = (52 \times 3) + 0$

Here, the remainder is zero and the divisor is 52.

So, 52 is the HCF of 52 and 156.

Thus, HCF of 156, 208 and 260 is 52.

Hence, the minimum number of buses $= \frac{156}{52} + \frac{208}{52} + \frac{260}{52}$

$= 3 + 4 + 5 = 12$

**28.** Find the largest number that will divide 398, 436 and 542 leaving remainder 7, 11 and 15 respectively.

**Sol.** It is given that the required number when divides 398, 436 and 542 the remainder are 7, 11 and 15 respectively.

i.e. $398 - 7 = 391$, $436 - 11 = 425$ and $542 - 15 = 527$ are completely divisible by the required number.

$\Rightarrow$ Required number

= H.C.F. (391, 425, 527)

= H.C.F. [H.C.F. (425, 527), 391]

Now applying Euclid's division Lemma for 425, 527

$527 = 425 \times 1 + 102$

$425 = 102 \times 4 + 17$

$102 = 17 \times 6 + 0$

$\Rightarrow$ H.C.F. (425, 527) = 17

Applying Euclid's division Lemma for 391 and 17

$391 = 17 \times 23 + 0$

H.C.F. (391, 17) = 17

H.C.F. (391, 425, 527) = 17

$\Rightarrow$ Required number is 17

**29.** The length, breadth and height of a room are 8 m 25 cm, 6 m 75 cm and 4 m 50 cm respectively. Determine the longest rod which can measure the three dimensions of the room exactly.

**Sol.** Length = 8 m 25 cm = 825 cm

Breadth = 6 m 75 cm = 675 cm

Height = 4 m 50 cm = 450 cm

To determine the length of longest rod, that can measure the length of all dimension of the room exactly, length of rod should be H.C.F. of all three dimension of a room.

Length of rod = H.C.F. (825, 675, 450)

= H.C.F. [H.C.F. (675, 825), 450]

applying the Euclid's division lemma for 675 and 825

$825 = 675 \times 1 + 150$

$675 = 150 \times 4 + 75$

$150 = 75 \times 2 + 0$

$\Rightarrow$ H.C.F. (675, 825) = 75

Applying Euclid's Division Lemma for 75 and 450

$450 = 75 \times 6 + 0$

H.C.F. (75, 450) = 75

$\Rightarrow$ length of rod = H.C.F. (75, 450) = 75 cm

**30.** Show that the square of any positive odd integer is of the form $8m + 1$, for some integer m.

**Sol.** Let a be any positive integer and let $b = 8$.

Then, $a = 8n + r$, where $0 \le r < 8$ and n is an integer.

$\therefore$ $a = 8n$ or $8n + 1$ or $8n + 2$ or $8n + 3$ or $8n + 4$

or $8n + 5$ or $8n + 6$ or $8n + 7$

So, every odd positive integer is of the form $8n + 1$ or $8n + 3$ or $8n + 5$ or $8n + 7$.

Now, consider these four cases.

**Case 1 :** When $a = 8n + 1$

$a^2 = (8n + 1)^2 = 64n^2 + 16n + 1 = 8(8n^2 + 2n) + 1$

$= 8m + 1$ [where $m = (8n^2 + 2n)$ is an integer.]

**Case 2 :** When $a = 8n + 3$

$a^2 = (8n + 3)^2 = 64n^2 + 2 \times 8n \times 3 + 9 = 8(8n^2 + 6n + 1) + 1$

$= 8m + 1$ [where $m = 8n^2 + 6n + 1$ is an integer.]

**Case 3 :** When $a = 8n + 5$

$a^2 = (8n + 5)^2 = 64n^2 + 2(8n)(5) + 25 = 8(8n^2 + 10n + 3) + 1$

$= 8m + 1$ [where $m = 8n^2 + 10n + 3$ is an integer.]

**Case 4 :** When $a = 8n + 7$

$a^2 = (8n + 7)^2 = 64n^2 + 2(8n)(7) + 49$

$= 8(8n^2 + 14n + 6) + 1 = 8m + 1$ [where $m = 8n^2 + 14n + 6$ is an integer.]

Thus, it can be seen that square of any positive odd integer is of the form $8m + 1$ for some integer m.

### Case study type questions **(4 marks)**

**1.** To enhance the reading skills of grade X students, the school nominates you and two of your friends to set up a class library. There are two sections- section A and section B of grade X. There are 32 students in section A and 36 students in section B.

(i) What is the minimum number of books you will acquire for the class library, so that they can be distributed equally among students of Section A or Section B?

(ii) If the product of two positive integers is equal to the product of their HCF and LCM is true then find the HCF (32, 36).

(iii) Express 36 as a product of its primes.

(iv) Find the type of given number $7 \times 11 \times 13 \times 15 + 15$.

**[OR]**

(iv) If p and q are positive integers such that $p = ab^2$ and $q = a^2b$, where a, b are prime numbers, then find the LCM (p, q).

**Sol.** (i)

$$\begin{array}{r|l} 2 & 32, 36 \\ \hline 2 & 16, 18 \\ \hline 2 & 8, 9 \\ \hline 2 & 4, 9 \\ \hline 2 & 2, 9 \\ \hline 3 & 1, 9 \\ \hline 3 & 1, 3 \\ \hline & 1, 1 \end{array}$$

Minimum number of books required = LCM (32, 36) = $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 288$

(ii) HCF × LCM = Product of two number

$\Rightarrow$ HCF (32, 36) = $\dfrac{32 \times 36}{288} = 4$

(iii) $36 = 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3$

(iv) $7 \times 11 \times 13 \times 15 + 15 = 15(7 \times 11 \times 13 + 1)$

$= 15 \times 1002$ = Composite number

**[OR]**

(iv) $p = a \times b \times b$

$q = a \times a \times b$

LCM (p, q) = $a \times a \times b \times b = a^2b^2$

**2.** A seminar is being conducted by an Educational Organisation, where the participants will be educators of different subjects. The number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively.

(i) In each room the same number of participants are to be seated and all of them being in the same subject, find the maximum number of participants that can be accommodated in each room.

(ii) What is the minimum number of rooms required during the event?

(iii) Find the LCM of 60, 84 and 108.

(iv) Find the product of HCF and LCM of 60,84 and 108.

**[OR]**

(iv) Express 108 as a product of its primes.

**Sol.** (i) Maximum number of participates required = HCF (60, 84, 108)

$60 = 2^2 \times 3 \times 5$

$84 = 2^2 \times 3 \times 7$

$108 = 2^2 \times 3^3$

HCF (60, 84, 108) = $2^2 \times 3 = 12$

(ii) Maximum number of participants that can be accommodated in each room = 12

Total number of rooms required = $\frac{60}{12} + \frac{84}{12} + \frac{108}{12} = 5 + 7 + 9 = 21$

(iii)

| | |
|---|---|
| 2 | 60, 84, 108 |
| 2 | 30, 42, 54 |
| 3 | 15, 21, 27 |
| 3 | 5, 7, 9 |
| 3 | 5, 7, 3 |
| 5 | 5, 7, 1 |
| 7 | 1, 7, 1 |
| | 1, 1, 1 |

LCM (60, 84, 108) = $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 7 = 3780$

(iv) Product of HCF and LCM of 60, 84 and 108

$= 12 \times 3780 = 45360$

**[OR]**

(iv) $108 = 2 \times 2 \times 3 \times 3 \times 3$

$= 2^2 \times 3^3$

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# 2 Polynomials

**Multiple choice questions** **(1 mark)**

**1.** For what value of k, $x = -2$ is the zero of polynomial $2x^4 + 3x^3 + 2kx^2 + 3x + 6$?

(1) 1 (2) –1 (3) 2 (4) –2

**Sol. Option (2)**

$P(x) = 2x^4 + 3x^3 + 2kx^2 + 3x + 6$

As $x = -2$ is zero of P(x)

$\therefore$ $P(-2) = 0$

$32 - 24 + 8k - 6 + 6 = 0$

$8k = -8$

$k = -1$

**2.** The quadratic polynomial p(x) with –81 and 3 as product and one of the zero respectively is

(1) $x^2 + 24x - 81$ (2) $x^2 - 24x - 81$ (3) $x^2 - 24x + 81$ (4) $x^2 + 24x + 81$

**Sol. Option (1)**

Let $\alpha$ & $\beta$ are zeroes of poly. p(x)

$\therefore$ $\alpha\beta = -81$ & $\alpha = 3$

$\therefore$ $\beta = -27$

$\therefore$ $\alpha + \beta = -24$

$x^2 + 24x - 81$

**3.** If $\alpha, \beta$ are roots of the equation $2x^2 + 6x - 4 = 0$, then $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is equal to

(1) 8 (2) $\frac{-13}{2}$ (3) 3 (4) $\frac{-9}{2}$

**Sol. Option (2)**

As $\alpha, \beta$ are roots of $2x^2 + 6x - 4 = 0$

$\alpha + \beta = -\frac{6}{2} = -3$ and $\alpha\beta = \frac{-4}{2} = -2$

Therefore,

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{9+4}{-2} = \frac{-13}{2}$$

**4.** If $p(x) = x^2 - 2\sqrt{2}x + 1$, then $p(2\sqrt{2})$ is equal to

(1) 0 (2) 1 (3) $4\sqrt{2}$ (4) $8\sqrt{2}+1$

**Sol. Option (2)**

$p(x) = x^2 - 2\sqrt{2}x + 1$

$p(2\sqrt{2}) = (2\sqrt{2})^2 - 2\sqrt{2} \times 2\sqrt{2} + 1$

$p(2\sqrt{2}) = 8 - 8 + 1$

$p(2\sqrt{2}) = 1$

**5.** If the quadratic polynomial $ax^2 + bx + c$ has equal zeros, then the value of c is –

(1) $\frac{-b}{2a}$ (2) $\frac{b}{2a}$ (3) $\frac{-b^2}{4a}$ (4) $\frac{b^2}{4a}$

**Sol. Option (4)**

$P(x) = ax^2 + bx + c$

zeros are $\alpha, \alpha$

Product of zeros $= \alpha^2 = \frac{c}{a}$; Sum of zeros $= 2\alpha = \frac{-b}{a}$

$\alpha = \frac{-b}{2a}$

$\therefore \left(\frac{-b}{2a}\right)^2 = \frac{c}{a}$

$\frac{b^{\ 2}}{4a^2} = \frac{c}{a}$

$\Rightarrow \frac{b^2}{4a} = c$

**6.** Degree of the polynomial $4x^4 + 5x + 7$ is

(1) 4 (2) 5 (3) 2 (4) 7

**Sol. Option (1)**

Given polynomial is $4x^4 + 5x + 7$

The highest power of x having a nonzero coefficient is 4.

Hence, the degree is 4.

**7.** The value of the polynomial $5x^3 - 4x^2 + 3$ when $x = -1$ is

(1) –6 (2) 6 (3) 1 (4) –5

**Sol. Option (1)**

$5x^3 - 4x^2 + 3$

If $x = -1$, then replace x with $-1$, we get

$5 \times (-1)^3 - 4(-1)^2 + 3$

$= -5 - 4 + 3 = -6$

**8.** If 2 and –3 are the zeros of the quadratic polynomial $x^2 + (a + 1)x + b$, then find the values of a and b.

(1) 4, –1 (2) 6, 3 (3) 5, –5 (4) 0, –6

**Sol. Option (4)**

As 2 & – 3 are zeros of given quad. poly.

$\therefore$ Sum of zeros = $2 + (-3) = -(a + 1)$

$-a - 1 = -1 \Rightarrow a = 0$

Product of zeros = $2(-3) = b$

$b = -6$

$\therefore$ $a = 0, b = -6$

## Assertion reason questions (1 marks)

**9. Assertion:** If the sum of the zeroes of the quadratic polynomial $x^2 - 2kx + 8$ is 2 then value of k is 1.

**Reason:** Sum of zeroes of a quadratic polynomial $ax^2 + bx + c$ is $\frac{-b}{a}$

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol. Option (1)**

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Sum of zeroes = $\frac{-b}{a}$

$\Rightarrow \frac{-(2k)}{1} = 2$

$\Rightarrow k = 1$

**10. Assertion:** Degree of a zero polynomial is not defined.

**Reason:** Degree of a non-zero constant polynomial is 0.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol. Option (2)**

Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

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**Very short answer type questions** **(2 mark)**

**11.** For what value of k, is (–4) a zero of the polynomial $x^2 - x - (2k + 2)$ ?

**Sol.** Let $p(x) = x^2 - x - (2k + 2)$

If (–4) is a zero of p(x), then $p(-4) = 0$

$\Rightarrow$ $(-4)^2 - (-4) - 2k - 2 = 0$

$\Rightarrow$ $16 + 4 - 2k - 2 = 0$

$\Rightarrow$ $2k = 18$

$\Rightarrow$ $k = 9$

**12.** If zeroes $\alpha$ and $\beta$ of a polynomial $x^2 - 7x + k$ are such that $\alpha - \beta = 1$, then find the value of k.

**Sol.** Sum of zeroes = $\dfrac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

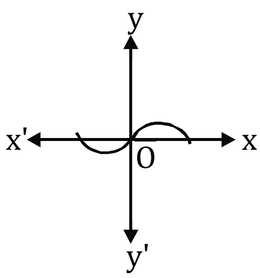
$\alpha + \beta = 7$ ... (i)

$\alpha - \beta = 1$ ... (ii)

On solving eq.(i) and (ii), we get $\alpha = 4$ and $\beta = 3$

Now, $k = \alpha\beta = 12$

**13.** Write the number of zeroes of the polynomial $y = f(x)$ whose graph is given in figure.




**Sol.** The graph of the polynomial $y = f(x)$ intersects x-axis at three points. Thus, the polynomial $y = f(x)$ has three zeroes.

**14.** If $\alpha$, $\beta$ are the zeroes of a polynomial, such that $\alpha + \beta = 6$ and $\alpha\beta = 6$, then write the polynomial.

**Sol.** Given $\alpha$ and $\beta$ are the zeroes of required polynomial p(x)

Given, $\alpha + \beta = 6$ and $\alpha\beta = 6$

$\therefore$ $p(x) = k[x^2 - (6)x + 6]$

For $k = 1$, $p(x) = x^2 - 6x + 6$

Thus, one of the polynomial which satisfy the given condition is $x^2 - 6x + 6$.

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**15.** If sum of the zeroes of the quadratic polynomial $3x^2 - kx + 6$ is 3, then find the value of k.

**Sol.** $p(x) = 3x^2 - kx + 6$

Sum of the zeroes = 3 = $-\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$\Rightarrow \quad 3 = -\dfrac{(-k)}{3}$

$\therefore \quad k = 9$

**16.** If $\alpha$, $\beta$ are the zeroes of the polynomial $f(x) = x^2 - 3x + 2$, then find $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$.

**Sol.** Given polynomial $f(x) = x^2 - 3x + 2$

Since $\alpha$ and $\beta$ are the zeroes of f(x)

$\alpha + \beta = 3 \qquad \left[\because \text{Sum of zeroes} = \dfrac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}\right]$

$\alpha\beta = 2 \qquad \left[\text{Q Product of zeroes} = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}\right]$

Now, $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\alpha + \beta}{\alpha\beta} = \dfrac{3}{2}$

**Short answer type questions** **(3 marks)**

**17.** What number should be added to the polynomial $x^2 + 7x - 35$ so that 3 is the zero of the polynomial?

**Sol.** Let k should be added to the polynomial $x^2 + 7x - 35$ so that 3 is the zero of the polynomial.

Thus, $x - 3$ is factor of $x^2 + 7x - 35 + k$

$$\begin{array}{r|l} & x + 10 \\ \hline x - 3 & x^2 + 7x - 35 + k \\ & \underline{-x^2 \mp 3x} \\ & \quad 10x - 35 + k \\ & \quad \underline{-10x \mp 30} \\ & \qquad\quad -5 + k \end{array}$$

Since, $x - 3$ is factor of $x^2 + 7x - 35 + k$

Hence, $-5 + k = 0$

$k = 5$

Thus, 5 should be added to polynomial $x^2 + 7x - 35$ so that 3 is the zero of the polynomial.

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**18.** If p, q are zeroes of polynomial $f(x) = 2x^2 - 7x + 3$, find the value of $p^2 + q^2$.

**Sol.** $f(x) = 2x^2 - 7x + 3$

Sum of zeroes = $p + q = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\left(\frac{-7}{2}\right) = \frac{7}{2}$

Product of zeroes = $pq = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{3}{2}$

We know that

$(p + q)^2 = p^2 + q^2 + 2pq$

$\Rightarrow \quad p^2 + q^2 = (p + q)^2 - 2pq$

$= \left(\frac{7}{2}\right)^2 - 2\left(\frac{3}{2}\right) = \frac{49}{4} - \frac{3}{1} = \frac{49-12}{4} = \frac{37}{4}$

**19.** If $\alpha$ and $\beta$ are the zeroes of the polynomial $f(x) = x^2 - 6x + k$, find the value of k, such that $\alpha^2 + \beta^2 = 40$

**Sol.** $\alpha + \beta = -\frac{b}{a}$

$= \frac{-(-6)}{1} = 6$

and $\alpha\beta = \frac{c}{a} = \frac{k}{1} = k$

$\therefore \quad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 40$

$\Rightarrow \quad (6)^2 - 2k = 40$

$\Rightarrow \quad 36 - 2k = 40$

$\Rightarrow \quad -2k = 4$

$\therefore \quad k = -2$

**20.** If one of the zeroes of the quadratic polynomial $f(x) = 14x^2 - 42k^2x - 9$ is negative of the other, find the value of 'k'.

**Sol.** Given, $f(x) = 14x^2 - 42k^2x - 9$

Let one zero be $\alpha$

$\therefore \quad$ The other = $-\alpha$,

$\therefore \quad$ Sum of zeroes = $\alpha + (-\alpha) = 0$

Sum of zeroes = $-\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

According to the question,

Sum of zeroes = $\frac{42k^2}{14} = 3k^2$

$\therefore \quad 3k^2 = 0$

$\Rightarrow \quad k = 0.$

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**21.** If one zero of a polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

**Sol.** Let $\alpha$ and $\beta$ be the zeroes of the polynomial, then as per the question $\beta = 7\alpha$

$\therefore$ Sum of zeroes $= \alpha + 7\alpha = 8\alpha = -\left(-\frac{8}{3}\right)$

$\Rightarrow \alpha = \frac{1}{3}$

and product of zeroes $= \alpha \times 7\alpha = \frac{2k+1}{3}$

$\Rightarrow 7\alpha^2 = \frac{2k+1}{3}$

$\Rightarrow 7\left(\frac{1}{3}\right)^2 = \frac{2k+1}{3}$

$\Rightarrow 7 \times \frac{1}{9} = \frac{2k+1}{3}$

$\Rightarrow \frac{7}{3} - 1 = 2k$

$\Rightarrow k = \frac{2}{3}$

**22.** If two zeroes of the polynomial $x^3 - 4x^2 - 3x + 12$ are $\sqrt{3}$ and $-\sqrt{3}$, then find its third zero.

**Sol.** Given, $\sqrt{3}$ and $-\sqrt{3}$ are zeroes of $x^3 - 4x^2 - 3x + 12$

$\therefore (x-\sqrt{3})(x+\sqrt{3}) = x^2 - 3$

Thus, $x^2 - 3$ is a factor of $x^3 - 4x^2 - 3x + 12$

$$
\begin{array}{r|l}
x^2 - 3 \;) & x^3 - 4x^2 - 3x + 12 \;(\; x - 4 \\
 & \underline{\overset{-}{x^3} \;\overset{+}{-}\; 3x} \\
 & -4x^2 + 12 \\
 & \underline{\overset{+}{-}4x^2 \;\overset{-}{+}\; 12} \\
 & 0
\end{array}
$$

$\Rightarrow$ $x - 4$ is also the factor of $x^3 - 4x^2 - 3x + 12$

Hence, the third zero of the given polynomial is 4.

**23.** If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out to be $(ax + b)$, find a and b.

**Sol.** We divide the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ by $3x^2 + 4x + 1$.

$$\begin{array}{r|l} & 2x^2 + 5 \\ \hline 3x^2 + 4x + 1 & 6x^4 + 8x^3 + 17x^2 + 21x + 7 \\ & \underline{\overset{-}{6x^4} + \overset{-}{8x^3} + \overset{-}{2x^2}} \\ & \qquad 15x^2 + 21x + 7 \\ & \qquad \underline{\overset{-}{15x^2} + \overset{-}{20x} + \overset{-}{5}} \\ & \qquad\qquad\quad x + 2 \end{array}$$

According to equation, $x + 2 = ax + b$

$\therefore$ On comparing, we get $a = 1$ and $b = 2$

**24.** If the sum and product of the zeroes of the polynomial $ax^2 - 5x + c$ is equal to 10 each, find the value of 'a' and 'c'.

**Sol.** Given, polynomial, $f(x) = ax^2 - 5x + c$

Let the zeroes of f(x) are $\alpha$ and $\beta$, then according to the question

Sum of zeroes = Product of zeroes = 10

$\alpha + \beta = \alpha\beta = 10$

Now $\quad \alpha + \beta = -\dfrac{\text{Coeff.of } x}{\text{Coeff.of } x^2} = -\dfrac{-5}{a}$

$\Rightarrow \quad 10 = \dfrac{+5}{a}$

$\therefore \quad a = \dfrac{1}{2}$ and $\alpha\beta = \dfrac{\text{Constant term}}{\text{Coeff.of } x^2} = \dfrac{c}{a}$

$\Rightarrow \quad 10 = 2c$

$\therefore \quad c = 5$

Hence $a = \dfrac{1}{2}$ and $c = 5$

**25.** Show that $\dfrac{1}{2}$ and $\dfrac{-3}{2}$ are the zeroes of the polynomial $4x^2 + 4x - 3$ and verify the relationship between zeroes and coefficients of the polynomial.

**Sol.** $f(x) = 4x^2 + 4x - 3$

$\Rightarrow f\left(\dfrac{1}{2}\right) = 4\left(\dfrac{1}{4}\right) + 4\left(\dfrac{1}{2}\right) - 3 = 1 + 2 - 3 = 0$

and $f\left(-\dfrac{3}{2}\right) = 4\left(\dfrac{9}{4}\right) + 4\left(-\dfrac{3}{2}\right) - 3 = 9 - 6 - 3 = 0$

$\therefore \quad \dfrac{1}{2}, -\dfrac{3}{2}$ are zeroes of polynomial $4x^2 + 4x - 3$.

Sum of zeroes $= \dfrac{1}{2} - \dfrac{3}{2} = -1 = \dfrac{-4}{4} = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of zeroes $= \left(\dfrac{1}{2}\right)\left(-\dfrac{3}{2}\right) = \dfrac{-3}{4} = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\therefore$ Relationship between zeroes and coefficients of polynomial is verified.

**26.** Find the zeroes of the quadratic polynomial $5x^2 + 8x - 4$ and verify the relationship between the zeroes and the coefficients of the polynomial.

**Sol.** $p(x) = 5x^2 + 8x - 4$

$= 5x^2 + 10x - 2x - 4$

$= 5x(x + 2) - 2(x + 2)$

$= (x + 2)(5x - 2)$

Hence, zeroes of the quadratic polynomial $5x^2 + 8x - 4$ are $-2$ and $\frac{2}{5}$.

**Verification :**

Sum of zeroes $= -2 + \frac{2}{5} = \frac{-8}{5} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of zeroes $= (-2) \times \left(\frac{2}{5}\right) = \frac{-4}{5} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

Thus, the relationship is verified.

**27.** If $\alpha$ and $\beta$ are the zeroes of a quadratic polynomial such that $\alpha + \beta = 24$ and $\alpha - \beta = 8$. Find the quadratic polynomial having $\alpha$ and $\beta$ as its zeroes. Verify the relationship between the zeroes and coefficients of the polynomial.

**Sol.** Given,

$\alpha + \beta = 24$ ... (i)

$\alpha - \beta = 8$ ... (ii)

Adding equations (i) and (ii),

$2\alpha = 32$

$\Rightarrow \alpha = 16$

Put the value of $\alpha$ in equation (i),

$16 + \beta = 24$

$\Rightarrow \beta = 24 - 16 = 8$

Hence, the quadratic polynomial

$= x^2 - (\text{Sum of zeroes})\, x + (\text{Product of zeroes})$

$= x^2 - (\alpha + \beta)\, x + \alpha\beta$

$= x^2 - (16 + 8)\, x + (16)(8)$

$= x^2 - 24x + 128$

**Verification :** $\alpha + \beta = 24 = -\frac{\text{Cofficient of } x}{\text{Cofficient of } x^2}$

and $\alpha\beta = 128 = \frac{\text{Constant term}}{\text{Cofficient of } x^2}$

Hence, the relationship is verified.

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**28.** On dividing $x^3 - 5x^2 + 6x + 4$ by a polynomial g(x), the quotient and the remainder were $x - 3$ and 4 respectively. Find g(x).

**Sol.** $x^3 - 5x^2 + 6x + 4 = g(x)\,(x - 3) + 4$

$x^3 - 5x^2 + 6x = g(x) \times (x - 3)$

$$\Rightarrow \quad g(x) = \frac{x^3 - 5x^2 + 6x}{x - 3}$$

$$\begin{array}{r|l}
x - 3 & x^3 - 5x^2 + 6x \;(\; x^2 - 2x \\
 & \underset{-}{x^3} \underset{+}{- 3x^2} \\
\hline
 & -2x^2 + 6x \\
 & +2x^2 \underset{-}{+} 6x \\
\hline
 & 0 \\
\hline
\end{array}$$

Hence $g(x) = x^2 - 2x$

**29.** What should be added to the polynomial $x^3 + 2x^2 - 9x + 1$ so that it is completely divisible by $x + 4$ ?

**Sol.** On dividing $x^3 + 2x^2 - 9x + 1$ by $x + 4$, remainder should be zero.

$$\begin{array}{r|l}
x + 4 & x^3 + 2x^2 - 9x + 1 \;(\; x^2 - 2x - 1 \\
 & \underset{-}{x^3} + \underset{-}{4x^2} \\
\hline
 & -2x^2 - 9x + 1 \\
 & \underset{+}{-} 2x^2 \underset{+}{-} 8x \\
\hline
 & -x + 1 \\
 & \underset{+}{-} x \underset{+}{-} 4 \\
\hline
 & 5 \\
\hline
\end{array}$$

Hence $-5$ should be added to $x^3 + 2x^2 - 9x + 1$ to make it is completely divisible by $x + 4$.

**30.** On dividing a polynomial $3x^3 + 4x^2 + 5x - 13$ by a polynomial g(x), the quotient and the remainder were $(3x + 10)$ and $(16x - 43)$ respectively. Find g(x).

**Sol.** $3x^3 + 4x^2 + 5x - 13 = (3x + 10)\, g(x) + (16x - 43)$

$3x^3 + 4x^2 + 5x - 16x - 13 + 43 = (3x + 10) \times g(x)$

$$\Rightarrow \quad \frac{3x^3 + 4x^2 - 11x + 30}{3x + 10} = g(x)$$

$$\begin{array}{r|l}
3x + 10 & 3x^3 + 4x^2 - 11x + 30 \;(\; x^2 - 2x + 3 \\
 & \underset{-}{3x^3} + \underset{-}{10x^2} \\
\hline
 & -6x^2 - 11x + 30 \\
 & \underset{+}{-} 6x^2 \underset{+}{-} 20x \\
\hline
 & 9x + 30 \\
 & \underset{-}{9x} \underset{-}{+} 30 \\
\hline
 & 0 \\
\hline
\end{array}$$

Hence, $g(x) = x^2 - 2x + 3$

**31.** If $\alpha$, $\beta$ are zeroes of the polynomial $x^2 - 2x - 15$, then form a quadratic polynomial whose zeroes are $2\alpha$ and $2\beta$.

**Sol.** Given polynomial $x^2 - 2x - 15$

Then, $\alpha + \beta = 2, \alpha\beta = -15$

Now, sum of zeroes $= 2\alpha + 2\beta = 2(\alpha + \beta)$

$= 2 \times 2 = 4$

and product of zeroes $= 2\alpha.2\beta = 4\alpha\beta = 4(-15) = -60$

Thus, required polynomial is

$x^2 -$ (sum of zeroes) x + product by zeroes

$= x^2 - 4x - 60$

**Long answer type questions** **(5 marks)**

**32.** Polynomial $x^4 + 7x^3 + 7x^2 + px + q$ is exactly divisible by $x^2 + 7x + 12$, then find the values of p and q.

**Sol.** Factors of $x^2 + 7x + 12$

$x^2 + 7x + 12 = 0$

$\Rightarrow x^2 + 4x + 3x + 12 = 0$

$\Rightarrow x(x + 4) + 3(x + 4) = 0$

$\Rightarrow (x + 4)(x + 3) = 0$

$\Rightarrow x = -4, -3$ .... (i)

Let $p(x) = x^4 + 7x^3 + 7x^2 + px + q$

If p(x) is exactly divisible by $x^2 + 7x + 12$, then $x = -4$ and $x = -3$ are zeroes of p(x) from eq.(i)

$p(x) = x^4 + 7x^3 + 7x^2 + px + q$

$p(-4) = (-4)^4 + 7(-4)^3 + 7(-4)^2 + p(-4) + q$

but $p(-4) = 0$

$\therefore 0 = 256 - 448 + 112 - 4p + q$

$\Rightarrow 0 = -4p + q - 80$

$\Rightarrow 4p - q = -80$ ... (ii)

and $p(-3) = (-3)^4 + 7(-3)^3 + 7(-3)^2 + p(-3) + q$

but $p(-3) = 0$

$\therefore 0 = 81 - 189 + 63 - 3p + q$

$\Rightarrow 0 = -3p + q - 45$

$\Rightarrow 3p - q = -45$ ... (iii)

On solving eq. (ii) and eq. (iii) by elimination method, we get.

$$\begin{array}{r} 4p - q = -80 \\ \underset{-}{3p} \underset{+}{-} \ q = \underset{+}{-45} \\ \hline p = -35 \end{array}$$

On putting the value of p in eq.(i),

$4(-35) - q = -80$

$\Rightarrow -140 - q = -80$

$\Rightarrow -q = 140 - 80$

$\Rightarrow -q = 60$

$\therefore q = -60$

Hence, $p = -35, q = -60$

**33.** If $\alpha$ and $\beta$ are the zeroes of the polynomial $2x^2 - 4x + 5$, find the value of :

(i) $\alpha^2 + \beta^2$ (ii) $\frac{1}{\alpha} + \frac{1}{\beta}$ (iii) $(\alpha - \beta)^2$

(iv) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ (v) $\alpha^3 + \beta^3$

**Sol.** Given $\alpha$ and $\beta$ are the zeroes of $2x^2 - 4x + 5$

$\Rightarrow \alpha + \beta = \frac{-(-4)}{2} = 2$ and $\alpha\beta = \frac{5}{2}$

Now,

(i) $(\alpha^2 + \beta^2) = (\alpha + \beta)^2 - 2\alpha\beta = 4 - 5 = -1$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{2\times2}{5} = \frac{4}{5}$

(iii) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = 2^2 - \frac{4\times5}{2} = 4 - 10 = -6$

(iv) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$

From (i)

$= \frac{-1}{\left(\frac{5}{2}\right)^2} = \frac{-4}{25}$

(v) $(\alpha^3 + \beta^3) = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = 2^3 - 3 \times \frac{5}{2} \times 2 = 8 - 15 = -7$

**34.** If two zeroes of the polynomial $p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$. Find the other zeroes.

**Sol.** As $x = 2 \pm \sqrt{3}$ are the zeroes of p(x), so $x - 2 \pm \sqrt{3}$ are the factors of p(x).

Now, $\{x-(2+\sqrt{3})\}\{x-(2-\sqrt{3})\}$

$\{(x-2)-\sqrt{3})\}\{(x-2)+\sqrt{3})\}$

$= (x-2)^2 - (\sqrt{3})^2$

$= x^2 - 4x + 1$

Dividing p(x) by $x^2 - 4x + 1$

$$\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & + \quad - \quad + \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\
\hline
 & 0 \\
\hline
\end{array}$$

$\therefore \quad p(x) = \{x - (2 + \sqrt{3})\} \{x - (2 - \sqrt{3})\} (x^2 - 2x - 35)$

As, $x^2 - 2x - 35 = x^2 + 5x - 7x - 35$

$= x(x + 5) - 7(x + 5) = (x + 5)(x - 7)$

Hence, other two zeroes of p(x) are $-5$ and 7.

**35.** Find the values of a and b so that $8x^4 + 14x^3 - 2x^2 + ax + b$ is exactly divisible by $4x^2 + 3x - 2$.

**Sol.**

$$
\begin{array}{r|l}
 & 2x^2 + 2x - 1 \\
4x^2 + 3x - 2 & 8x^4 + 14x^3 - 2x^2 + ax + b \\
 & 8x^4 + 6x^3 - 4x^2 \\
 & - \quad - \quad + \\
\hline
 & 8x^3 + 2x^2 + ax + b \\
 & 8x^3 + 6x^2 - 4x \\
 & - \quad - \quad + \\
\hline
 & -4x^2 + (a + 4)x + b \\
 & -4x^2 - 3x + 2 \\
 & + \quad + \quad - \\
\hline
 & (a + 7)x + b - 2
\end{array}
$$

For exact divisible, remainder is zero, then

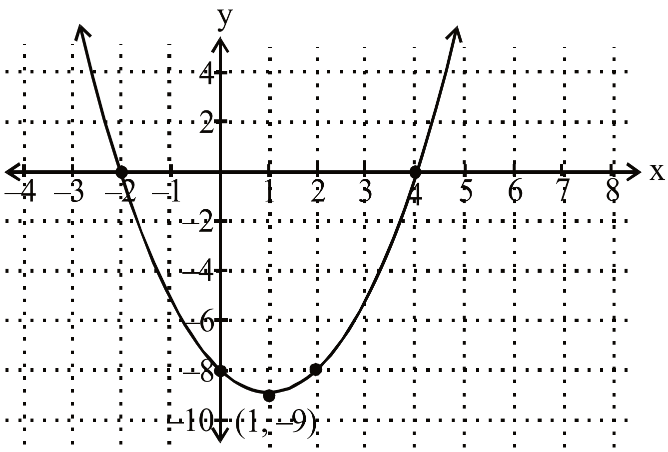
$(a + 7)x + b - 2 = 0$

$\Rightarrow \quad a + 7 = 0, b - 2 = 0$

$\Rightarrow \quad a = -7, b = 2$

**Case Study type questions** **(4 marks)**

**1.** A straight necklace, when worn around the neck, follows a mathematical shape of "Parabola", See the image and answer the following question.




(i) Find the zeroes of the polynomial by reading the graph.

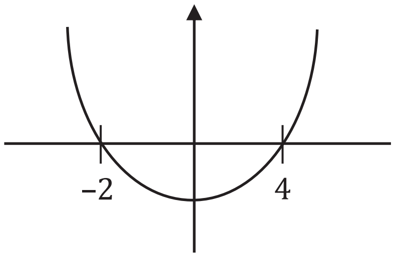
(ii) Write the expression of the polynomial.

(iii) Find the value of polynomial at x = 1.

**[OR]**

(iii) If we move the parabola by one unit to the right side, then what will be the expression of polynomial?

**Sol.**




(i) From the graph zeroes are $x = -2$ & $x = 4$

(ii) Polynomial is

$P(x) = x^2 - [-2 + 4]x + (-2 \times 4)$

$P(x) = x^2 - 2x - 8$

(iii) at $x = 1$

$P(1) = (1)^2 - 2(1) - 8$

$P(1) = -9$

**[OR]**

(iii) According to question, new zeroes are –1 & 5 then new polynomial expression is

$P(x) = x^2 - (-1 + 5)\,x + (-1 \times 5)$

$\Rightarrow$ $P(x) = x^2 - 4x - 5$

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# 3 Pair of Linear Equation in Two Variables

**Multiple choice questions** **(1 mark)**

**1.** Which of the following is not a solution of the pair of equations $3x - 2y = 4$ and $6x - 4y = 8$?

(1) $x = 2, y = 1$ (2) $x = 4, y = 4$ (3) $x = 6, y = 7$ (4) $x = 5, y = 3$

**Sol.** **Option (4)**

$3x - 2y = 4$ ; $6x - 4y = 8$

Put all the options in the given equations and check.

Values of x and y satisfies option (1), (2) and (3) but not option (4)

$\Rightarrow 3(5) - 2(3) = 15 - 6 = 9 \neq 4$

$\Rightarrow 6(5) - 4(3) = 30 - 12 = 18 \neq 8$

**2.** Path of two birds are represented by given equation $3x + 2y - 11 = 0$ and $2x - 3y + 10 = 0$. Find at which point they will cross each other.

(1) (–2, 2) (2) (–1, 7) (3) (1, 4) (4) (3, 1)

**Sol.** **Option (3)**

$3x + 2y = 11$ ... (i)

$2x - 3y = -10$ ... (ii)

Multiply equation (i) by 3 and equation (ii) by 2 and add

$$\begin{array}{l} 9x + \cancel{6y} = 33 \\ 4x - \cancel{6y} = -20 \\ \hline 13x = 13 \\ \quad x = 1 \end{array}$$

from eq. (i) $y = \dfrac{11-3}{2} = 4$

$\therefore \quad (x, y) = (1, 4)$

**3.** Find the solution of the pair of simultaneous equations.

$3x + 4y = -6$

$4x + 3y = -1$

(1) $x = 3, y = -2$ (2) $x = 2, y = -3$ (3) $x = -3, y = 2$ (4) $x = -2, y = -3$

**Sol. Option (2)**

$3x + 4y = -6$

$\Rightarrow \quad 3x = -6 - 4y \Rightarrow \; x = \dfrac{-6-4y}{3}$

$4x + 3y = -1$

$\Rightarrow \quad 4\left(\dfrac{-6-4y}{3}\right) + 3y = -1 \Rightarrow \; -24 - 16y + 9y = -3$

$\Rightarrow \quad -7y = 21 \Rightarrow y = -3$

$x = \dfrac{-6-4(-3)}{3} = \dfrac{-6+12}{3} = 2$

$x = 2, y = -3$

**4.** What is the value of k if (k, 5) is the solution of the simultaneous equations $4x + 3y = 19$ and $4x - 3y = -11$?

(1) 4 (2) $\dfrac{-1}{3}$ (3) 5 (4) 1

**Sol. Option (4)**

Since, (k, 5) is the solution of simultaneous equation.

Therefore, it should satisfy the given equations.

$\Rightarrow 4(k) + 3(5) = 19$

$\Rightarrow 4k = 4$

$k = 1$

Also,

$\Rightarrow 4k - 3(5) = -11$

$\Rightarrow 4k = 4$

$k = 1$

Therefore, the value of $k = 1$

**5.** The graphs of $2x + 3y - 6 = 0$, $4x - 3y - 6 = 0$, $x = 2$ and $y = \dfrac{2}{3}$ intersect at:

(1) 6 points (2) 1 point (3) 2 points (4) no points

**Sol. Option (2)**

We are required to solve the system of equations

$2x + 3y = 6$ ... (1)

$4x - 3y = 6$ ... (2)

$2x + 3y = 6$

| | | | |
|---|---|---|---|
| x | 0 | 2 | 3 |
| y | 2 | 2/3 | 0 |

$4x - 3y = 6$

| | | | |
|---|---|---|---|
| x | 0 | 2 | 3/2 |
| y | −2 | 2/3 | 0 |



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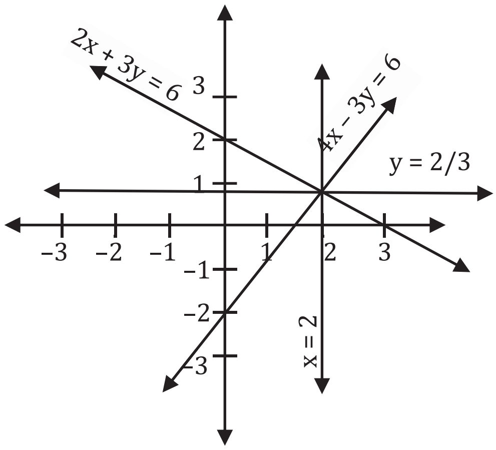



So, both of these lines intersect at one point x = 2, y = $\frac{2}{3}$

Solve the first two equations simultaneously, obtaining x = 2, y = $\frac{2}{3}$. Since these results are consistent with the last two equations, the solution is $\left(2, \frac{2}{3}\right)$.

So, all the four equations intersect at one point only.

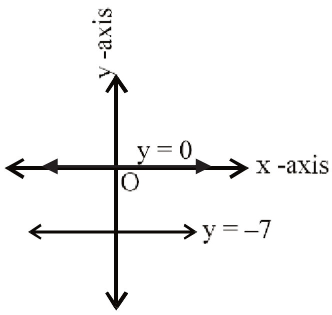
**6.** The pair of equations y = 0 and y = –7 has

(1) One solution (2) two solution (3) infinitely many solutions (4) no solution

**Sol. Option (4)**

y = 0 $\Rightarrow$ x-axis

y = –7 $\Rightarrow$ parallel to x-axis




So, no solution or parallel lines

**7.** The graphical representation of the pair of equations 2x + 2y – 4 = 0 and 2x + 4y –12 = 0 is

(1) Intersecting lines (2) Parallel lines

(3) Coincident lines (4) all of the above

**Sol. Option (1)**

2x + 2y = 4

| | | | |
|---|---|---|---|
| x | 0 | 2 | –2 |
| y | 2 | 0 | 4 |

2x + 4y = 12

| | | | |
|---|---|---|---|
| x | 0 | 6 | –2 |
| y | 3 | 0 | 4 |

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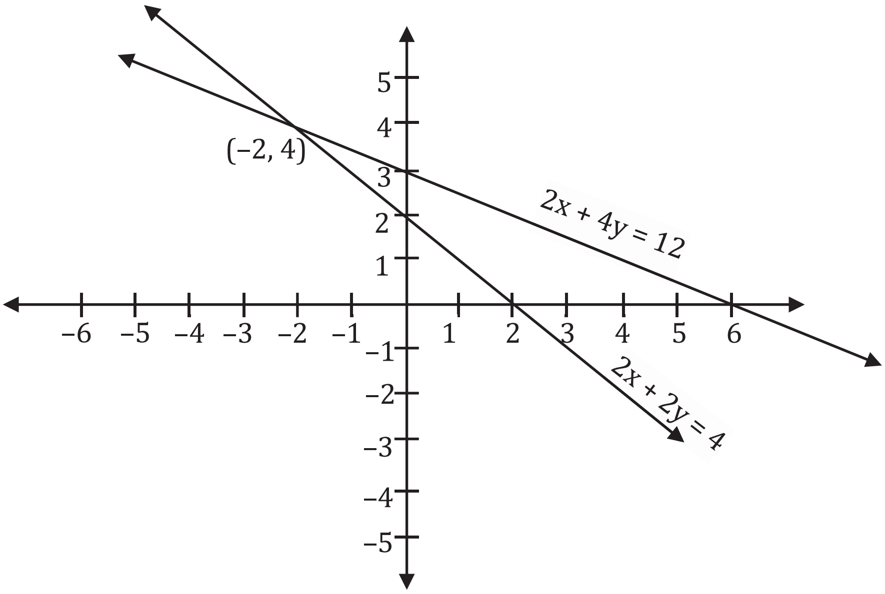


The graphical representation of the pair of equations $2x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ is intersecting lines

**8.** Which of the following pair of equations represent inconsistent system?

(1) $3x - 2y = 8$ ; $2x + 3y = 1$ (2) $3x - y = -8$; $3x - y = 24$

(3) $6x - y = 4$ ; $x + 2y = l$ (4) $5x - y = 10$; $10x - 2y = 20$

**Sol. Option (2)**

Inconsistent system has no solution

(1) $\frac{3}{2} \neq \frac{-2}{3}$ Unique solution

(2) $\frac{3}{3} = \frac{-1}{-1} \neq \frac{-1}{3}$ Inconsistent system

(3) $\frac{6}{1} \neq \frac{-1}{2}$ Unique solution

(4) $\frac{5}{10} = \frac{(-1)}{(-2)} = \frac{-10}{-20} \Rightarrow \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$ Infinitely many solutions

## Assertion reason questions (1 marks)

**9. Assertion:** If the pair of lines are coincident, then we say that pair of lines is consistent and it has a unique solution.

**Reason:** If the pair of lines are parallel, then the pairs has no solution and is called inconsistent pair of equations.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol. Option (4)**

Assertion (A) is false but Reason (R) is true.

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**10.** **Assertion:** The linear equations $x - 2y - 3 = 0$ and $3x + 4y - 20 = 0$ have exactly one solution

**Reason:** The linear equation $2x + 3y - 9 = 0$ and $4x + 6y - 18 = 0$ have a unique solution.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol.** **Option (3)**

**Assertion:** $\frac{1}{3} \neq \frac{-2}{4}$

**Reason :** $\frac{2}{4} = \frac{3}{6} = \frac{-9}{-18} \Rightarrow \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$

It has infinite many solutions.

Assertion (A) is true but Reason (R) is false.

**Very short answer type questions** **(2 mark)**

**11.** On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pairs of linear equations are consistent or inconsistent ?

$4x - y = 4$ and $3x + 2y = 14$.

**Sol.** On comparing the given equations with standard form, we get

$$\frac{a_1}{a_2} = \frac{4}{3}, \frac{b_1}{b_2} = \frac{-1}{2}, \frac{c_1}{c_2} = \frac{-4}{-14} = \frac{2}{7}$$

$$\therefore \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence, given equations are consistent.

**12.** On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines represent the following pairs of linear equations intersect at a point or parallel or coincide?

$6x - 3y - 12 = 0$ & $2x - y = 4$

**Sol.** On comparing the given equations with standard form, we get

$$\frac{a_1}{a_2} = \frac{6}{2} = \frac{3}{1}, \frac{b_1}{b_2} = \frac{-3}{-1} = \frac{3}{1}, \frac{c_1}{c_2} = \frac{-12}{-4} = \frac{3}{1}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, lines represented by given equations are coincident.

**13.** One equation of a pair of dependent linear equations is $-5x + 7y = 2$, find the second equation.

**Sol.** As one of the equation of pair of dependent linear equation is – 5x + 7y = 2, then the second equation is – 10x + 14y = 4 $\left(\because \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}\right)$

**14.** Find the number of solutions of the pair of linear equations x + 2y – 8 = 0 and 2x + 4y = 16

**Sol.** As given pair of linear equation are

x + 2y – 8 = 0

2x + 4y – 16 = 0

$$\therefore \quad \frac{a_1}{a_2}=\frac{1}{2}, \frac{b_1}{b_2}=\frac{2}{4}=\frac{1}{2}, \frac{c_1}{c_2}=\frac{-8}{-16}=\frac{1}{2}$$

As $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Hence, given pair of equations has infinite number of solutions.

**Short answer type questions** **(3 marks)**

**15.** Find the value of k for which the following pair of linear equations have infinitely many solutions.

2x + 3y = 7; (k – 1) x + (k + 2)y = 3k

**Sol.** The given pair of linear equations are

2x + 3y – 7 = 0 ;

(k – 1)x + (k + 2)y – 3k = 0

Since, given equations have infinitely many solutions.

$$\therefore \quad \frac{2}{k-1}=\frac{3}{k+2}=\frac{-7}{-3k}$$

Now, $\frac{2}{k-1}=\frac{7}{3k}$

$\Rightarrow$ 6k = 7k – 7

$\Rightarrow$ k = 7

**16.** For what value of k will the pair of linear equations 2x + 3y = 9 & 6x + (k – 2) y = 3k – 2 have no solution?

**Sol.** We have, 2x + 3y = 9

and 6x + (k – 2)y = 3k – 2

For no solution, we have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$$\frac{2}{6}=\frac{3}{k-2}\neq\frac{-9}{-(3k-2)}$$

Now, $\frac{2}{6}=\frac{3}{k-2}$

$\Rightarrow$ 2k – 4 = 18

$\Rightarrow$ 2k = 22

$\Rightarrow$ k = 11

Verification : $\frac{c_1}{c_2}=\frac{9}{33-2}=\frac{9}{31}$

$\therefore \quad \frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

**17.** On comparing the ratios $\frac{a_1}{a_2},\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations is consistent or inconsistent. $\frac{3}{2}x+\frac{5}{3}y=7; 9x-10y=-14$

**Sol.** We have,

$\frac{3}{2}x+\frac{5}{3}y=7$ ... (i)

$9x - 10y = -14$ ... (ii)

Here, $a_1=\frac{3}{2}, b_1=\frac{5}{3}, c_1=-7$

$a_2 = 9$, $b_2 = -10$, $c_2 = 14$

Thus, $\frac{a_1}{a_2}=\frac{3}{2\times 9}=\frac{1}{6}, \frac{b_1}{b_2}=\frac{5}{3\times(-10)}=-\frac{1}{6}$

Hence, $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$. So, it has a unique solution and it is consistent.

**18.** If x = a, y = b is the solution of the pair of equations x – y = 2 and x + y = 4 then find the value of a and b.

**Sol.** x – y = 2 ... (i)

x + y = 4 ... (ii)

On adding (i) and (ii), we get 2x = 6 or x = 3

From (i),

3 – y = 2

$\Rightarrow$ y = 1

$\therefore$ a = 3, b = 1

**19.** In $\Delta ABC$, $\angle A = x$, $\angle B = 3x$, and $\angle C = y$ if $3y - 5x = 30°$, show that triangle is right angled.

**Sol.** $\angle A + \angle B + \angle C = 180°$ (Sum of interior angle of $\Delta ABC$)

$x + 3x + y = 180°$

$\Rightarrow \quad 4x + y = 180° \quad \text{... (i)}$

and $3y - 5x = 30°$ (Given) ... (ii)

Multiply equation (i) by 3 and subtracting from eq.(ii), we get

$$-17x = -510° \Rightarrow x = \frac{510°}{17} = 30°$$

then $\angle A = x = 30°$ and $\angle B = 3x = 3 \times 30° = 90°$

$\angle C = y = 180° - (\angle A + \angle B) = 180° - 120° = 60°$

$\therefore \quad \angle A = 30°, \angle B = 90°, \angle C = 60°$

Hence, $\Delta ABC$ is right angle triangle at B.

**20.** For which value of p does the pair of equations given below has unique solution.

$4x + py + 8 = 0$; $2x + 2y + 2 = 0$

**Sol.** Given system of equation are

$4x + py + 8 = 0$, $2x + 2y + 2 = 0$

for unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{4}{2} \neq \frac{p}{2} \Rightarrow p \neq 4$$

$\therefore$ For all real vales of p except 4 has unique solution.

**21.** Solve the following pair of equations by using the method of substitution.

$x + 2y - 3 = 0$ and $3x - 2y + 7 = 0$

**Sol.** Given, pair of linear equations are

$x + 2y - 3 = 0$ ... (i)

and $3x - 2y + 7 = 0$ ... (ii)

From Eq. (i), $x = 3 - 2y$ ... (iii)

Substituting the value of x in eq.(ii), we get

$3(3 - 2y) - 2y + 7 = 0$

$\Rightarrow \quad 9 - 6y - 2y + 7 = 0$

$\Rightarrow \quad -8y + 16 = 0$

$\Rightarrow \quad -8y = -16$

$\Rightarrow \quad y = 2$

On putting $y = 2$ in Eq. (iii), we get

$x = 3 - 4 = -1$

Hence, the solution of the given system of equations is $x = -1$ and $y = 2$

**22.** Solve for x and y:

$37x + 43y = 123$

$43x + 37y = 117$

**Sol.** The given equations are

$37x + 43y = 123$ ... (i)

$43x + 37y = 117$ ... (ii)

Adding (i) and (ii), we get

$80(x + y) = 240$

$\Rightarrow \quad x+y=\frac{240}{80}=3$

$\Rightarrow \quad y = 3 - x$ ... (iii)

Substituting the value of y in (i), we get

$37x + 43(3 - x) = 123$

$\Rightarrow \quad 37x + 129 - 43x = 123$

$\Rightarrow \quad -6x = -6$

$\Rightarrow \quad x = 1$

Substituting the value of x in (iii), we get

$y = 3 - 1 = 2$

Hence, solution of given system of equations is

$x = 1 ; y = 2$

**23.** **Solve:** $\frac{bx}{a}+\frac{ay}{b}=a^2+b^2; x+y=2ab$

**Sol.** $\frac{bx}{a}+\frac{ay}{b}=a^2+b^2$

$\Rightarrow \quad b^2x + a^2y = a^3b + b^3a$ ... (1)

$x + y = 2ab$ ... (2)

Multiply eq.(2) by $b^2$ and subtract from eq.(1)

$$\begin{array}{l} b^2x+a^2y=a^3b+b^3a \\ b^2x+b^2y=2ab^3 \\ - \quad - \qquad - \\ \hline (a^2-b^2)y=a^3b-ab^3 \end{array}$$

$(a^2 - b^2)y = ab(a^2 - b^2)$

$y = ab$ ... (3)

Substitute the value of y in eq. (2)

$x + ab = 2ab$

$x = ab$

$\therefore \quad x = ab, y = ab$

**24.** Solve the following pair of equations:

$$\frac{5}{x-1}+\frac{1}{y-2}=2\,;\ \frac{6}{x-1}+\frac{3}{y-2}=1$$

**Sol.** The given pair of linear equations is

$\Rightarrow \quad \dfrac{5}{x-1}+\dfrac{1}{y-2}=2$ ... (i)

$\Rightarrow \quad \dfrac{6}{x-1}-\dfrac{3}{y-2}=1$ ... (ii)

Let $\dfrac{1}{x-1}=u$ and $\dfrac{1}{y-2}=v$

Then equation (1) and (2) become

$5u + v = 2$ ... (iii)

$6u - 3v = 1$ ... (iv)

On multiplying equation (iii) by 3 and adding it to equation (iv), we get $u=\dfrac{1}{3}$

On substituting the value of u in equation (iii), we get $\dfrac{5}{3}+v=2 \Rightarrow v=2-\dfrac{5}{3}=\dfrac{1}{3}$

Now, $\dfrac{1}{x-1}=\dfrac{1}{3}$ and $\dfrac{1}{y-2}=\dfrac{1}{3}$

$\Rightarrow \quad x - 1 = 3$ and $y - 2 = 3$

$\Rightarrow \quad x = 4$ and $y = 5$

**25.** Solve the following pair of equations:

$$\frac{10}{x+y}+\frac{2}{x-y}=4\,;\ \frac{15}{x+y}-\frac{5}{x-y}=-2$$

**Sol.** The given pair of linear equations is

$\Rightarrow \quad \dfrac{10}{x+y}+\dfrac{2}{x-y}=4$ ... (i)

$\Rightarrow \quad \dfrac{15}{x+y}-\dfrac{5}{x-y}=-2$ ... (ii)

Let $\dfrac{1}{x+y}=u$ and $\dfrac{1}{x-y}=v$

Then equations (i) and (ii) become

$10u + 2v = 4$ ... (iii)

$15u - 5v = -2$ ... (iv)

On multiplying equation (iii) with 5 and equation (iv) with 2, and then adding, we obtain

$80u = 16 \Rightarrow u = \frac{1}{5}$

Now substitute the value of u in eq.(iii)

$\frac{10}{5} + 2v = 4 \Rightarrow v = 1$

Now, $\frac{1}{x+y} = \frac{1}{5}$ and $\frac{1}{x-y} = 1$

$x + y = 5$ $\qquad$ $x - y = 1$

On solving, we get

$2x = 6$

$\Rightarrow$ $x = 3$ and $y = 2$

**26.** The sum of numerator and denominator of a fraction is 3 less than twice the denominator. If each of the numerator and denominator is decreased by 1, the fraction becomes $\frac{1}{2}$. Find the fraction.

**Sol.** Let the fraction be $= \frac{x}{y}$.

According to question

$x + y = 2y - 3$

$\Rightarrow$ $x - y = - 3$ ... (i)

Also, $\frac{x-1}{y-1} = \frac{1}{2}$

$\Rightarrow$ $2x - 2 = y - 1$

$\Rightarrow$ $2x - y = 1$ ... (ii)

On subtracting (i) from (ii), we get $x = 4$

Substituting the value of x in (i), we get $y = 7$

Thus, the required fraction is $\frac{4}{7}$.

**27.** A number consists of two digits. When the number is divided by the sum of its digits, the quotient is 7. If 27 is subtracted from the number, the digits interchange their places. Find the number.

**Sol.** Let the ten's digit of a number be a and ones digit be b.

$\therefore$ The number be $10a + b$

According to question,

$\frac{10a+b}{a+b} = 7 \Rightarrow 10a + b = 7a + 7b$

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$\Rightarrow$ $3a - 6b = 0$

$\Rightarrow$ $a - 2b = 0$ ... (i)

and $10a + b - 27 = 10b + a$

$\Rightarrow$ $10a - a + b - 10b = 27$

$\Rightarrow$ $9a - 9b = 27$

$\Rightarrow$ $a - b = 3$ ... (ii)

Subtracting (i) from (ii), we get $b = 3$

On substituting value of b in (ii), we get

$a - 3 = 3 \Rightarrow a = 6$

$\therefore$ Required number is 63.

**Long answer type questions** **(5 marks)**

**28.** Form the pair of linear equations in this problem and find its solution graphically: 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

**Sol.** Let x be the number of girls and y be the number of boys.

According to question, we have

$x = y + 4$

$\Rightarrow$ $x - y = 4$

Again, total number of students = 10

$\therefore$ $x + y = 10$

Hence, we have following system of equations

$x - y = 4$ ... (i)

and $x + y = 10$ ... (ii)

From equation (i), we have the following table:

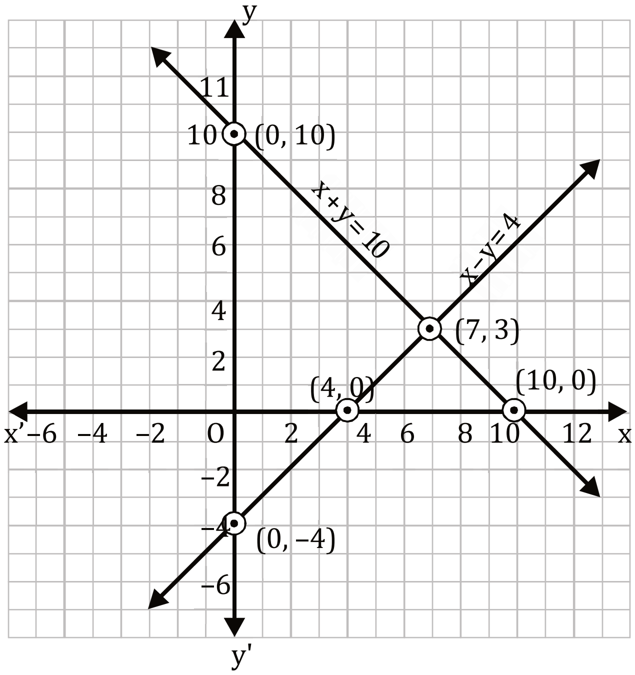
Plotting these we have

| x | 0 | 4 | 7 |
|---|---|---|---|
| y | –4 | 0 | 3 |

From equation (ii), we have the following table:

Plotting these we have

| x | 0 | 10 | 7 |
|---|---|---|---|
| y | 10 | 0 | 3 |




Here, the two lines intersect at point (7, 3) i.e., x = 7, y = 3.

So, the number of girls = 7

and number of boys = 3.

**29.** A man travels 600 km partly by train and partly by car. It takes 8 hr and 40 min, if he travels 320 km by train and the rest by car. It would take 30 min more, if he travels 200 km by train and the rest by car. Find the speed of the train and the car separately.

**Sol.** Let the speed of train be x km/h and that of car be y km/hr, then

$$\Rightarrow \frac{320}{x}+\frac{280}{y}=\frac{26}{3} \qquad \text{... (i)}$$

$$\Rightarrow \frac{200}{x}+\frac{400}{y}=\frac{55}{6} \qquad \text{... (ii)}$$

Let $\frac{1}{x}=u$ and $\frac{1}{y}=v$

$\therefore$ eq.(i) and eq. (ii) becomes

$$320u + 280v = \frac{26}{3}$$

$$200u + 400v = \frac{55}{6}$$

or, $$8u + 7v = \frac{13}{60} \qquad \text{... (iii)}$$

$$4u + 8v = \frac{11}{60} \qquad \text{... (iv)}$$

Now, multiply eq.(iv) by 2 and subtract (iii)

$$-9v = \frac{-3}{20}$$

$$v = \frac{1}{60}, u = \frac{1}{80}$$

$\therefore \quad \frac{1}{y} = \frac{1}{60}, \frac{1}{x} = \frac{1}{80}$

$\therefore \quad x = 80, y = 60$

Speed of train = 80 km/hr

Speed of car = 60 km/hr

**30.** A boat goes 30 km upstream and 44 km downstream in 10 hrs. In 13 hrs., it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

**Sol.** Let speed of the boat in still water be x km/hr and speed of the stream be y km/hr.

Then, speed of the boat downstream = (x + y) km/hr

and speed of the boat upstream = (x – y) km/hr

**For condition I**

When boat goes 30 km upstream, let the time taken be $t_1$ and

When boat goes 44 km downstream, let the time taken be $t_2$.

But, total time taken = 10 hrs. [i.e. $t_1 + t_2 = 10h$]

$\therefore \quad \frac{30}{x-y} + \frac{44}{x+y} = 10 \quad \text{... (i)}$

**For condition II**

When boat goes 40 km upstream, let the time taken be $T_1$ and when boat goes 55 km downstream, let the time taken be $T_2$.

But, total time taken = 13 hrs. [i.e. $T_1 + T_2 = 13$ hrs.]

$\therefore \quad \frac{40}{x-y} + \frac{55}{x+y} = 13 \quad \text{... (ii)}$

Here, Eqs. (i) and (ii) are not in linear form, so we reduce them in linear form by putting.

$$\frac{1}{x-y} = u \text{ and } \frac{1}{x+y} = v$$

$\therefore$ Eq. (i) becomes

$30u + 44v = 10$

$\Rightarrow \quad 30u + 44v - 10 = 0 \quad \text{... (iii)}$

and Eq. (ii) becomes $40u + 55v = 13$

$\Rightarrow \quad 40u + 55v - 13 = 0 \quad \text{... (iv)}$

These equations are in linear form.

Now, by cross-multiplication method, we get

$$\frac{u}{(44)(-13)-(55)(-10)}=\frac{v}{40(-10)-30(-13)}=\frac{1}{30(55)-44(40)}$$

$$\left[\because \frac{u}{b_1c_2-b_2c_1}=\frac{v}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}\right]$$

$$\Rightarrow \quad \frac{u}{-572+550}=\frac{v}{-400+390}=\frac{1}{1650-1760}$$

$$\Rightarrow \quad \underset{\text{I}}{\frac{u}{-22}}=\underset{\text{II}}{\frac{v}{-10}}=\underset{\text{III}}{\frac{1}{-110}}$$

On taking I and III terms, we get

$$\frac{u}{-22}=\frac{1}{-110}\Rightarrow u=\frac{22}{110}=\frac{1}{5}$$

On taking II and III terms, we get

$$\frac{v}{-10}=\frac{1}{-110}\Rightarrow v=\frac{10}{110}=\frac{1}{11}$$

Now, on putting the values of u and v, we get

$$\frac{1}{x-y}=\frac{1}{5}\Rightarrow x-y=5 \quad \text{... (v)}$$

and $\frac{1}{x+y}=\frac{1}{11}\Rightarrow x+y=11$ ... (vi)

On adding Eqs. (v) and (vi), we get

$$2x=16\Rightarrow x=\frac{16}{2}=8$$

On putting x = 8 in Eq. (v), we get

$8 - y = 5 \Rightarrow y = 8 - 5 = 3$

Hence, the speed of the boat in still water is 8 km/hr and speed of the stream is 3 km/hr.

**31.** 4 men and 6 boys can finish a piece of work in 5 days, while 3 men and 4 boys can finish it in 7 days. Find the time taken by 1 man alone or that by 1 boy alone.

**Sol.** Let the man finishes the work in x days and that the boy finishes in y days.

One day work of a man $=\frac{1}{x}$

and one day work of a boy $=\frac{1}{y}$

Since, 4 men and 6 boys finish a piece of work in 5 days.

$\Rightarrow \quad \frac{4}{x}+\frac{6}{y}=\frac{1}{5}$ .... (i)

Similarly, in second case,

3 men and 4 boys can finish it in 7 days

and $\frac{3}{x}+\frac{4}{y}=\frac{1}{7}$ ... (ii)

Here, Eqs. (i) and (ii) are not in linear form, so we reduce them in linear form by putting $\frac{1}{x}$ = u and $=\frac{1}{y}$ v.

The, Eq. (i) becomes $4u+6v=\frac{1}{5}$ ... (iii)

and Eq. (ii) becomes $3u+4v=\frac{1}{7}$ ... (iv)

On multiplying Eq. (iii) by 3 and Eq. (iv) by 4 and then on subtracting we get

$\Rightarrow \quad 18v-16v=\frac{3}{5}-\frac{4}{7}$

$\Rightarrow \quad 2v=\frac{21-20}{35} \Rightarrow 2v=\frac{1}{35} \Rightarrow v=\frac{1}{70}$

Put $v=\frac{1}{70}$ in Eq. (iv), we get

$3u+\frac{4}{70}=\frac{1}{7}$

$\Rightarrow \quad 3u=\frac{1}{7}-\frac{4}{70}$

$\Rightarrow \quad 3u=\frac{6}{70}$

$\Rightarrow \quad u=\frac{1}{35}$

Thus, $u=\frac{1}{35}$ and $v=\frac{1}{70}$

On substituting the value of u & v we get

$\Rightarrow \quad \frac{1}{x}=\frac{1}{35}$ and $\frac{1}{y}=\frac{1}{70}$

$\Rightarrow$ x = 35 and y = 70

hence, 1 man alone and a boy alone finishes the work in 35 and 70 days, respectively.

**32.** Solve the following pair of linear equations by cross-multiplication method.

$3x - 5y + 1 = 0, 2x + 3y - 12 = 0$

**Sol.** $3x - 5y + 1 = 0$

$2x + 3y - 12 = 0$

$$\frac{x}{\begin{matrix}-5 & 1\\ 3 & -12\end{matrix}} = \frac{y}{\begin{matrix}1 & 3\\ -12 & 2\end{matrix}} = \frac{1}{\begin{matrix}3 & -5\\ 2 & 3\end{matrix}}$$

$$\Rightarrow \frac{x}{60-3} = \frac{y}{2+36} = \frac{1}{9+10}$$

$$\Rightarrow \frac{x}{57} = \frac{y}{38} = \frac{1}{19}$$

$$\Rightarrow \frac{x}{57} = \frac{1}{19} \Rightarrow x = 3$$

$$\Rightarrow \frac{y}{38} = \frac{1}{19} \Rightarrow y = 2$$

$\therefore$ value of x & y are 3 & 2 respectively.

**33.** Five years ago, A was thrice as old as B and ten years later, A shall be twice as old as B. What are the present ages of A and B ?

**Sol.** Let the present ages of B and A be x years and y years respectively. Then

B's age 5 years ago = $(x - 5)$ years

and A's age 5 years ago = $(y - 5)$ years

$\therefore \quad (y - 5) = 3(x - 5)$

$\Rightarrow \quad 3x - y = 10 \qquad$ .... (i)

B's age 10 years hence = $(x + 10)$ years

A's age 10 years hence = $(y + 10)$ years

$\therefore \quad y + 10 = 2(x + 10) \Rightarrow 2x - y = -10 \qquad$ .... (ii)

On subtracting (ii) from (i) we get $x = 20$

Putting $x = 20$ in (i) we get

$(3 \times 20) - y = 10 \Rightarrow y = 50$

$\therefore \quad x = 20$ and $y = 50$

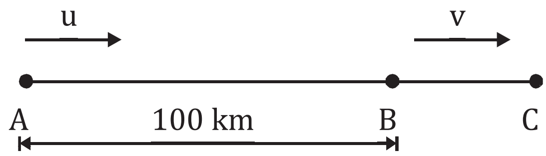
Hence B's present age = 20 years and

A's present age = 50 years.

**34.** Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hours. What are the speeds of the two cars?

**Sol.** Let the speed of car at A be u km/hr and car at B be v km/hr.

**Case 1:** When both cars travel in the same direction




Let both the cars meet at point C in 5 hours.

Car at A travels distance AC, whereas car at B travels distance BC.

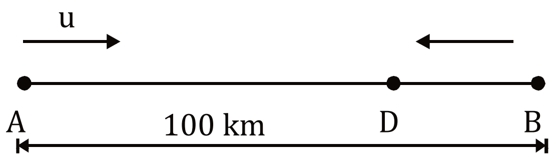
$\therefore$ AC = 5 × u and BC = 5 × v

Now, AC – BC = 100

$\Rightarrow$ 5u – 5v = 100

$\Rightarrow$ u – v = 20 ... (i)

**Case 2 :** When both cars travel in opposite directions. Let both cars meet at point D.




Car at A will travel distance AD, whereas car at B will travel distance BD.

$\therefore$ AD = 1 × u and BD = 1 × v

Now, AD + BD = 100

$\Rightarrow$ u + v = 100 ... (ii)

On adding equations (i) and (ii), we get

2u = 120

$\Rightarrow$ u = 60

From equation (ii), we get

60 + v = 100 $\Rightarrow$ v = 40

$\therefore$ Speed of car at A is 60 km/hr and car at B is 40 km/hr

**35.** Draw the graph of the equations x – y + 1 = 0 and 3x + 2y – 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis and shade the triangular region. Also find area.

**Sol.** We have,

x – y + 1 = 0 and 3x + 2y – 12 = 0

Thus,

x – y = – 1 $\Rightarrow$ x = y – 1 ... (i)

3x + 2y = 12 $\Rightarrow$ $x = \frac{12-2y}{3}$ ... (ii)

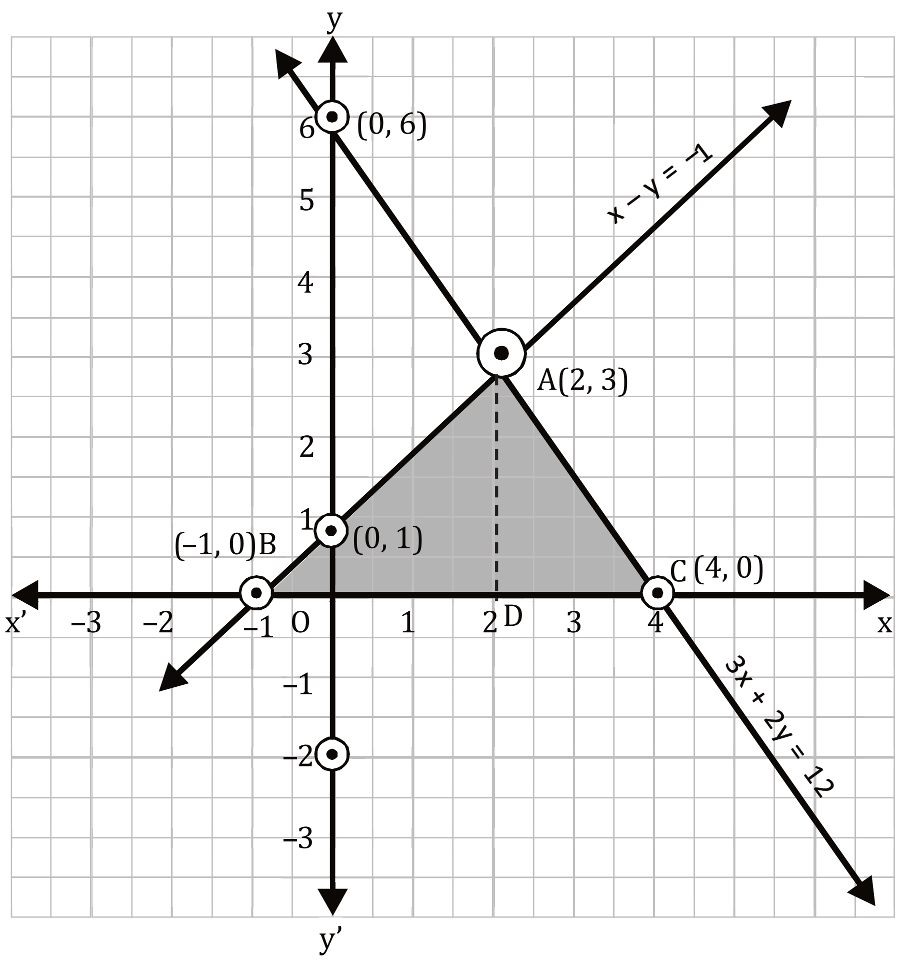
From equation (i), we have

| x | –1 | 0 | 2 |
|---|---|---|---|
| y | 0 | 1 | 3 |

From equation (ii), we have

| x | 0 | 4 | 2 |
|---|---|---|---|
| y | 6 | 0 | 3 |

Plotting these points, we have




ABC is the required (shaded) region and point of intersection is (2, 3).

$\therefore$ The vertices of the triangle are

A (2, 3), B (–1, 0) and C (4, 0)

Area of $\Delta ABC = \frac{1}{2} BC \times AD$

$= \frac{1}{2}(5 \times 3) = \frac{15}{2} = 7.5$ sq. unit

**Case Study type questions** **(4 marks)**

**1.** ATC saw a fighter plane of enemy coming down. It directs own country fighter plane from ground to move towards enemy fighter plane. The path traced by enemy fighter plane and country's fighter plane given by equations x + y = 10 and 3x – 2y = 0, where 'y' is time in minutes and x is height about ground in Km.

(i) By what condition, we can check that two planes will collide or not ?

(ii) At what height above the ground the planes will collide?

(iii) If the enemy plane was not intercepted by opponent (country's) plane and continued its path, after how many minutes would it touch the ground?

**[OR]**

(iii) At what height is the country plane after 2 minutes ?

**Sol.** (i) Equation of path of country fighter plane and enemy fighter plane

$x + y = 10$ and $3x - 2y = 0$

$$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{1}{-2}$$

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, there will be a point of intersection so, planes will collide.

(ii) let us find point of intersection

$x + y = 10$ ... (1)

multiply equation (1) by 2

$$\begin{array}{rl} 2x + 2y = 20 & \quad ...(3) \\ 3x - 2y = 0 & \quad ...(2) \\ \hline 5x = 20 & \end{array}$$

$x = 4$

Put $x = 4$ in (1)

$4 + y = 10$

$y = 6$

Height from the ground where planes will collide = 4 km

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(iii) Touching ground means height = 0

$\Rightarrow$ x = 0

x + y = 10

0 + y = 10

y = 10 min

**[OR]**

(iii) 3x – 2y = 0

If y = 2

3x – 2(2) = 0

3x – 4 = 0

3x = 4

$x = \frac{4}{3}$ km.

**2.** A part of monthly hostel charges in a college is fixed and the remaining, depends on the number of days one has taken food in the mess. When a student Anu takes food for 25 days, she has to pay Rs. 4500 as hostel charges, where as another student Bindu, who takes food for 30 days, has to pay Rs. 5200 as hostel charges.

Considering the fixed charges per month as Rs. x and the cost of food per day as Rs. y, answer the following questions

(i) Represent the situation faced by Anu and Bindu algebraically

(ii) Determine whether the pair of linear equations are consistent or inconsistent?

(iii) If Bindu takes food for 20 days, then what amount she has to pay?

**[OR]**

(iii) Find the cost of food per day.

**Sol.** Hostel monthly charges

(i) For student Anu

x + 25y = 4500 ... (1)

For student Bindu

x + 30y = 5200 ... (2)

(ii) $\frac{a_1}{a_2}=\frac{1}{1}, \frac{b_1}{b_2}=\frac{25}{30}=\frac{5}{6}, \frac{c_1}{c_2}=\frac{-4500}{-5200}=\frac{45}{52}$

$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}\Rightarrow$ system of linear equation will have unique solution.

(iii) Solving equation (1) & (2)

$$\begin{array}{r} x + 30y = 5200 \\ x + 25y = 4500 \\ - \quad - \quad\quad - \\ \hline 5y = 700 \end{array}$$

$y = \frac{700}{5} = 140$

Put y = 140 in equation (2)

$x + 30(140) = 5200$

$x + 4200 = 5200$

$x = 1000$

For 20 days, charges will be-

$x + 20y$

$\Rightarrow$ $1000 + 20 \times 140$

$\Rightarrow$ $1000 + 2800$

$\Rightarrow$ Rs. 3800

**[OR]**

(iii) Solving equation (1) & (2)

$$\begin{array}{r} x + 30y = 5200 \\ x + 25y = 4500 \\ - \quad - \quad\quad - \\ \hline 5y = 700 \end{array}$$

$y = \frac{700}{5} = 140$

Cost of food per day = Rs. 140

**3.** Puneet sells a saree at 8% profit and a sweater at 10% discount, thereby getting a sum of Rs. 1008. If he had sold the saree at 10% profit and the sweater at 8% discount, he would have got Rs.1028.

Denote the cost price of the saree and the list price (price before discount) of the sweater by x and y respectively and answer the following questions.

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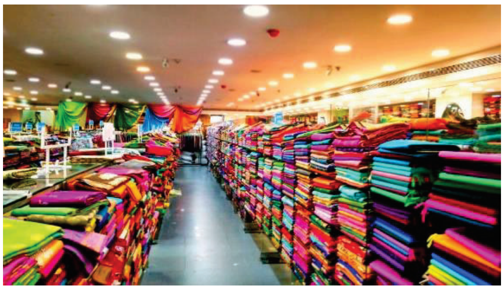

(i) Represent the above situations algebraically.

(ii) Find the common solutions for pair of linear equations in 2 variables.

(iii) Find the co-ordinates of point where equation representing I situation intersecting x-axis.

**[OR]**

(iii) Find the co-ordinates of point where equation representing II situation intersecting y -axis.

**Sol.** (i) **Case-I**

Saree at 8% profit + sweater at 10% discount = Rs. 1008

$\Rightarrow$ (100 + 8)% of x + (100 – 10)% of y = 1008

$$\Rightarrow \frac{108}{100}x + \frac{90}{100}y = 1008$$

$$\Rightarrow 1.08x + 0.9y = 1008 \quad \text{... (1)}$$

**Case-II**

Saree at 10% profit + sweater at 8% discount = ₹ 1028

$\Rightarrow$ (100 + 10)% of x + (100 – 8)% of y = 1028

$\Rightarrow$ 110% of x + 92% of y = 1028

$$\Rightarrow 1.1x + 0.92y = 1028 \quad \text{... (2)}$$

(ii) Value of x from equation (1)

$$x = \frac{1008 - 0.9y}{1.08} \quad \text{... (3)}$$

Put value of x in (2)

$$\Rightarrow 1.1\left(\frac{1008 - 0.9y}{1.08}\right) + 0.92y = 1028$$

$$\Rightarrow 1108.8 - 0.99y + 0.9936y = 1110.24$$

$$\Rightarrow 0.0036y = 1.44$$

$$\Rightarrow y = \frac{1.44}{0.0036} = 400$$

put y = 400 in (3)

$$\Rightarrow x = \frac{1008 - 0.9 \times 400}{1.08} = \frac{648}{1.08} = 600$$

unique solution $\Rightarrow$ x = Rs. 600, y = Rs. 400

(iii) To find coordinates of point, put y = 0 in equation (1)

$\Rightarrow$ 1.08x + 0.9y = 1008

$\Rightarrow$ 1.08x + 0.9(0) = 1008

$$x = \frac{1008}{1.08}$$

Coordinate (933.33, 0)

**[OR]**

(iii) To find coordinates of point, put x = 0 in equation (2)

1.1(0) + 0.92y = 1028

$$y = \frac{1028}{0.92} = 1117.39$$

Coordinates = (0, 1117.39)

**4.** The scissors is so common in our daily life use. Its blades represent the graph of linear equations : x + 3y = 6 and 2x – 3y = 12.

(i) Find the pivot point (point of intersection) of the blades represented by the linear equations x + 3y = 6 and 2x – 3y = 12 of the scissors.

(ii) Find the number of solutions of the system of liner equations x + 2y – 8 = 0 and 2x + 4y =

(iii) If (1, 2) is the solution of linear equations ax + y = 3 and 2x + by = 12, then find the values of a and b.

**[OR]**

(iii) Find the points at which linear equations x + 3y = 6 and 2x – 3y = 12 intersect y-axis.

**Sol.** (i) Pivot point = Part of intersection of lines = Common solution

(x + 3y = 6) × 2 ... (1)

$$\begin{array}{rl} 2x + 6y = 12 & \quad \ldots (2) \\ 2x - 3y = 12 & \quad \ldots (3) \\ - \quad + \quad\quad - & \\ \hline 9y = 0 & \end{array}$$

$y = 0$

Put $y = 0$ in equation $x + 3y = 6$

$x + 3(0) = 6$

$x = 6$

Pivot Point = (6, 0)

(ii) Given equations are

$x + 2y - 8 = 0$

$2x + 4y - 16 = 0$

$$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, the given pair of linear equations will have infinite solutions

(iii) Put $x = 1$ & $y = 2$ in $ax + y = 3$

$\Rightarrow a(1) + 2 = 3$

$a = 3 - 2 = 1$

Put $x = 1$ & $y = 2$ in $2x + by = 12$

$\Rightarrow 2(1) + b(2) = 12$

$\Rightarrow 2b = 10$

$b = 5$

So, $a = 1$, $b = 5$

**[OR]**

(iii) To find point of intersection with y-axis we have to put $x = 0$ in both equations.

$0 + 3y = 6$

$y = 2$

Point of intersection with y-axis is (0, 2)

$2(0) - 3y = 12$

$-3y = 12$

$y = -4$

Point of intersection with y-axis is (0, –4)

**5.** Deepesh bought 3 notebooks and 2 pens for Rs. 80. His friend Ramesh said that price of each notebook could be Rs. 25, then three notebooks would cost Rs. 75, the two pens would cost Rs. 5 then each pen could be for Rs. 2.50. Another friend Amar felt that Rs. 2.50 for one pen was too much less. It should be at least Rs. 16. Then the price of each notebook would also be Rs. 16. Lokesh also bought the same types of notebooks and pens as Deepesh. He paid Rs. 110 for 4 notebooks and 3 pens.

(i) Let the cost of one notebook be x and that of pen be y. Write the equations describing the given problem.

(ii) What is exact cost of the notebook?

(iii) What is the total cost if they will purchase the same type of 15 notebooks and 12 pens?

**[OR]**

(iii) Write an equation of line parallel to the line $3x + 2y = 80$.

**Sol.** (i) According to the statement we have

$3x + 2y = 80$ and $4x + 3y = 110$

(ii) Solving $3x + 2y = 80$ and $4x + 3y = 110$

we get $x = 20$ and $y = 10$.

Thus, cost of 1 notebook is Rs. 20 and cost of 1 pen is Rs. 10

(iii) Total cost $= 15 \times 20 + 12 \times 10 =$ Rs. 420

**[OR]**

(iii) There can be infinite such lines. One such line can be $6x + 4y = 160$

# 4 Quadratic Equations

**Multiple choice questions** **(1 marks)**

**1.** If discriminant of quadratic equation $mx^2 - 4x + 7 = 0$ is $-68$, then value of 'm' is

(1) 1 (2) 2 (3) $-1$ (4) 3

**Sol.** **Option (4)**

$D = 16 - 4 \times 7 \times m = -68$

$\Rightarrow 16 + 68 = 28 \times m$

$\Rightarrow \frac{84}{28} = m$

$\Rightarrow m = 3$

**2.** The two roots of quadratic equation $abx^2 + (b^2 - ac)x - bc = 0$ are

(1) $\left(-\frac{b}{a}, \frac{c}{b}\right)$ (2) $\left(1, -\frac{b}{a}\right)$ (3) $\left(\frac{b}{c}, -\frac{b}{a}\right)$ (4) $\left(-\frac{a}{b}, -\frac{c}{b}\right)$

**Sol.** **Option (1)**

$\Rightarrow abx^2 + (b^2 - ac)x - bc = 0$

$\Rightarrow abx^2 + b^2x - acx - bc = 0$

$\Rightarrow bx(ax + b) - c(ax + b) = 0$

$\Rightarrow (bx - c)(ax + b) = 0$

$x = \frac{c}{b}, -\frac{b}{a}$

**3.** For what values of k, the quadratic equation $kx^2 - 6x - 2 = 0$ has real roots?

(1) $k \le -\frac{9}{2}$ (2) $k \ge -\frac{9}{2}$ (3) $k \le -2$ (4) None of these

**Sol.** **Option (2)**

For real Roots

$D \ge 0$

$\Rightarrow 36 - 4(-2)(k) > 0$

$\Rightarrow 36 + 8k \ge 0$

$\Rightarrow 8k \ge -36$

$\Rightarrow k \ge -\frac{36}{8}$

$k \ge -\frac{9}{2}$

**4.** The value of 'k' so that equation $4x^2 - 8kx - 9 = 0$ has one root as negative of other is

(1) 0 (2) 2 (3) –2 (4) $\frac{1}{2}$

**Sol. Option (1)**

Sum of roots = 0

$\alpha + (-\alpha) = \frac{8k}{4} = 0$

k = 0

**5.** If $\alpha$, $\beta$ are roots of equation $x^2 - 5x + k = 0$, then value of k such that $\alpha - \beta = 1$ is

(1) 1 (2) 5 (3) 6 (4) 4

**Sol. Option (3)**

$\alpha + \beta = 5, \alpha\beta = k$

$\Rightarrow (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\Rightarrow (1)^2 = (5)^2 - 4k$

$\Rightarrow 1 = 25 - 4k$

$\Rightarrow 4k = 24$

$\Rightarrow k = 6$

**6.** If –4 is root of equation $x^2 + px - 4 = 0$ and equation $x^2 + px + q = 0$ has equal roots, then q is

(1) $\frac{4}{9}$ (2) $\frac{9}{4}$ (3) $-\frac{2}{3}$ (4) None of these

**Sol. Option (2)**

$\Rightarrow (-4)^2 + p(-4) - 4 = 0$

$\Rightarrow 16 - 4p - 4 = 0$

$\Rightarrow 12 = 4p \Rightarrow p = 3$

and $x^2 + 3x + q = 0$

For equal roots

$\Rightarrow (3)^2 - 4q = 0$

$\Rightarrow \frac{9}{4} = q$

**7.** Find the values of k so that the quadratic equation $(4 - k)x^2 + 2(k + 2)x + (8k + 1) = 0$ has equal roots.

(1) 2 (2) 4 (3) 3 (4) 5

**Sol. Option (3)**

$(4 - k)x^2 + 2(k + 2)x + (8k + 1) = 0$ for equal roots, D = 0

$a = (4 - k), b = 2k + 4, c = 8k + 1, D = b^2 - 4ac$

$D = (2k + 4)^2 - 4(8k + 1)(4 - k)$

$0 = 4k^2 + 16 + 16k - 4(32k - 8k^2 + 4 - k)$

$0 = 4k^2 + 16k + 16 - 128k + 32k^2 - 16 + 4k$

$0 = 36k^2 - 108k$

$0 = k(36k - 108)$

$\therefore$ k = 0, k = 3

36k = 108

k = 3

**8.** If one root of $px^2 - 14x + 8 = 0$ is 6 times the other, then p =

(1) 3 (2) –3 (3) 2 (4) 1

**Sol. Option (1)**

$\Rightarrow$ The given quadratic equation is $px^2 - 14x + 8 = 0$

$\Rightarrow$ Let the root be $\alpha$ then other root will be $6\alpha$.

$\Rightarrow$ Sum of the roots = $\frac{-b}{a}$

$\Rightarrow$ $\alpha + 6\alpha = \frac{-(-14)}{p}$

$\Rightarrow$ $7\alpha = \frac{14}{p}$

$\therefore$ $\alpha = \frac{2}{p}$ ... (1)

$\Rightarrow$ Product of the roots = $\frac{c}{a}$

$\Rightarrow$ $\alpha.6\alpha = \frac{8}{p}$

$\Rightarrow$ $6\alpha^2 = \frac{8}{p}$

$\Rightarrow$ $6\left(\frac{2}{p}\right)^2 = \frac{8}{p}$ [From (1)]

$\Rightarrow$ $6 \times \frac{4}{p^2} = \frac{8}{p}$

$\therefore$ $p = 3$

**Assertion reason questions** **(1 marks)**

**9.** **Assertion:** The roots of the quadratic equation $x^2 + 2x + 2 = 0$ are imaginary.

**Reason:** If discriminant $D = b^2 - 4ac < 0$ then the roots of quadratic equation $ax^2 + bx + c = 0$ are imaginary.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol.** **Option (1)**

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

$x^2 + 2x + 2 = 0$

Discriminant, $D = b^2 - 4ac$

$= (2)^2 - 4 \times 1 \times 2$

$= 4 - 8 = -4 < 0$

As we know roots are imaginary when $D < 0$.

**10.** **Assertion:** $3x^2 - 6x + 3 = 0$ has repeated roots.

**Reason:** The quadratic equation $ax^2 + bx + c = 0$ have repeated roots if discriminant $D > 0$.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol.** **Option (3)**

Assertion (A) is true but Reason (R) is false.

$3x^2 - 6x + 3 = 0$

$D = b^2 - 4ac$

$= (-6)^2 - 4(3)(3)$

$= 36 - 36 = 0$

Roots are repeated as $D = 0$.

## Very short answer type questions (2 mark)

**11.** If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then find the value of k.

**Sol.** $\because$ $\frac{1}{2}$ is a root of quadratic equation.

$\therefore$ It must satisfy the quadratic equation.

$\Rightarrow$ $x^2 + kx - \frac{5}{4} = 0$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0 \Rightarrow \frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0$$

$$\Rightarrow \frac{1 + 2k - 5}{4} = 0$$

$\Rightarrow$ $2k - 4 = 0$

$\Rightarrow$ $k = 2$

**12.** Find the discriminant of the quadratic equation $4\sqrt{2}x^2 + 8x + 2\sqrt{2} = 0$.

**Sol.** $D = b^2 - 4ac = (8)^2 - 4(4\sqrt{2})(2\sqrt{2}) = 64 - 64 = 0$

**13.** If a and b are the roots of the equation $x^2 + ax - b = 0$, then find a and b.

**Sol.** Sum of the roots = $a + b = -\dfrac{\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of the roots = $ab = -\dfrac{\text{coefficient of } x}{\text{coefficient of } x^2}$

$\Rightarrow\ a + b = -a$ and $ab = -b$

$\Rightarrow\ 2a = -b$ and $a = -1$

So, $-b = 2(-1)$

$\Rightarrow\ b = 2$

**14.** If the discriminant of the equation $6x^2 - bx + 2 = 0$ is 1, then find the value of b.

**Sol.** Given, $6x^2 - bx + 2 = 0$

We know that,

$D = b^2 - 4ac$

$\therefore\ 1 = (-b)^2 - 4(6)(2)$ [$\because$ given, $D = 1$]

$\Rightarrow\ 1 = b^2 - 48$

$\Rightarrow\ b^2 = 49$

$\Rightarrow\ b = \pm 7$

Hence, the value of b is 7 or – 7.

**15.** If $ax^2 + bx + c = 0$ has equal roots, find the value of c.

**Sol.** For equal roots $D = 0$

i.e., $b^2 - 4ac = 0$

$\Rightarrow\ b^2 = 4ac$

$\Rightarrow\ c = \dfrac{b^2}{4a}$

**16.** If one root of the quadratic equation $x^2 - 5x + 6k = 0$ is reciprocal of other, find the value of k.

**Sol.** One root of the equation is reciprocal of other root Let roots are $\alpha, \dfrac{1}{\alpha}$

So, product of roots = $\dfrac{6k}{1}$

$\alpha. = 6k$ ; $6k = 1$

$k = \dfrac{1}{6}$

**Short answer type questions** **(3 marks)**

**17.** If $-5$ is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the value of k.

**Sol.** Since $-5$ is a root of the equation $2x^2 + px - 15 = 0$

$\therefore$ $2(-5)^2 + p(-5) - 15 = 0$

$\Rightarrow$ $50 - 5p - 15 = 0$ or $5p = 35$ or $p = 7$

Again $p(x^2 + x) + k = 0$ or $7x^2 + 7x + k = 0$ has equal roots

$\therefore$ $D = 0$

i.e., $b^2 - 4ac = 0$ or $49 - 4 \times 7k = 0$

$\Rightarrow$ $k = \dfrac{49}{28} = \dfrac{7}{4}$

**18.** A two digit number is four times the sum of the digits. It is also equal to 3 times the product of digits. Find the number.

**Sol.** Let the ten's digit be x and unit's digit = y

Number = $10x + y$

$\therefore$ $10x + y = 4(x + y)$

$\Rightarrow$ $6x = 3y$

$\Rightarrow$ $2x = y$

Again $10x + y = 3xy$

$10x + 2x = 3x(2x)$

$\Rightarrow$ $12x = 6x^2$

$\Rightarrow$ $x = 2$ (rejecting $x = 0$)

$2x = y$

$\Rightarrow$ $y = 4$

$\therefore$ The required number is 24

**19.** Solve the following quadratic equation for x :

$4x^2 - 4a^2x + (a^4 - b^4) = 0.$

**Sol.** We have, $4x^2 - 4a^2x + (a^4 - b^4) = 0$

$\Rightarrow$ $(2x)^2 - 2(2x)a^2 + (a^2)^2 - (b^2)^2 = 0$

$\Rightarrow$ $(2x - a^2)^2 - (b^2)^2 = 0$

$\Rightarrow$ $(2x - a^2 + b^2)(2x - a^2 - b^2) = 0$

$\Rightarrow$ $2x - a^2 + b^2 = 0$ or $2x - a^2 - b^2 = 0$

$\Rightarrow$ $2x = a^2 - b^2$ or $2x = a^2 + b^2$

$\Rightarrow$ $x = \dfrac{a^2 - b^2}{2}$ or $x = \dfrac{a^2 + b^2}{2}$

**20.** Solve for $x : \sqrt{2x+9}+x=13$

**Sol.** We have, $\sqrt{2x+9}+x=13$

Or $\sqrt{2x+9}=13-x$

Squaring both sides, we have

$2x + 9 = (13 - x)^2$

$\Rightarrow$ $2x + 9 = 169 + x^2 - 26x$

$\Rightarrow$ $x^2 - 28x + 160 = 0$

$\Rightarrow$ $(x - 20)(x - 8) = 0$

$\therefore$ $x = 20$ or $8$

Since $x = 20$ is not satisfying given equation.

Hence, $x = 8$

**21.** For what value of k the equation $4x^2 - 2(k + 1) = 0$ has real and equal roots ?

**Sol.** $4x^2 - 2(k + 1) = 0$ has real and equal roots.

$D = 0$

$b^2 - 4ac = 0$

$(0)^2 - 4 \times 4 \times [-2(k + 1)] = 0$

$32(k + 1) = 0$

$\Rightarrow$ $k + 1 = 0$

$\Rightarrow$ $k = -1$

**22.** Find the roots of the quadratic equation

$3x^2-2\sqrt{6}x+2=0$

**Sol.** $3x^2-2\sqrt{6}x+2=0$

$\Rightarrow 3x^2-\sqrt{6}x-\sqrt{6}x+2=0$

$\Rightarrow \sqrt{3}x\ \ \sqrt{3}x-\sqrt{2}\ \ -\sqrt{2}\ \ \sqrt{3}x-\sqrt{2}\ \ =0$

$\Rightarrow \ \sqrt{3}x-\sqrt{2}\ \ \ \sqrt{3}x-\sqrt{2}\ \ =0$

$\Rightarrow \ \sqrt{3}x-\sqrt{2}=0$ or $\sqrt{3}x-\sqrt{2}=0$

$\Rightarrow \ x=\dfrac{\sqrt{2}}{\sqrt{3}},\dfrac{\sqrt{2}}{\sqrt{3}}$

$\Rightarrow \ x=\dfrac{\sqrt{2}}{\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}},\ \dfrac{\sqrt{2}}{\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow \ x=\dfrac{\sqrt{6}}{3},\ \dfrac{\sqrt{6}}{3}$

**23.** The side of a square exceeds the side of the another square by 4 cm and the sum of the areas of the two squares is 400 sq.cm. Find the dimensions of the squares.

**Sol.** Let $S_1$ and $S_2$ be two squares. Let the side of the square $S_2$ be x cm in length. Then, the side of square $S_1$ is (x + 4) cm.

$\therefore$ Area of square $S_1 = (x + 4)^2$

and Area of square $S_2 = x^2$

It is given that

Area of square $S_1$ + Area of square $S_2$ = 400 $cm^2$

$\Rightarrow (x + 4)^2 + x^2 = 400$

$\Rightarrow (x^2 + 8x + 16) + x^2 = 400$

$\Rightarrow 2x^2 + 8x - 384 = 0$

$\Rightarrow x^2 + 4x - 192 = 0$

$\Rightarrow x^2 + 16x - 12x - 192 = 0$

$\Rightarrow x(x + 16) - 12(x + 16) = 0$

$\Rightarrow (x + 16)(x - 12) = 0$

$\Rightarrow x = 12$ or, $x = -16$

As the length of the side of a square cannot be negative. Therefore, x = 12.

$\therefore$ Side of square $S_1 = x + 4 = 12 + 4 = 16$ cm and, Side of square $S_2$ = 12 cm.

**24.** Solve for x

$$\frac{2x-3}{x-1} - \frac{4(x-1)}{2x-3} = 3;\ x \neq 1, \frac{3}{2}$$

**Sol.** $\dfrac{2x-3}{x-1} - \dfrac{4(x-1)}{2x-3} = 3;\ x \neq 1, \dfrac{3}{2}$

$\Rightarrow (2x - 3)^2 - 4(x - 1)^2 = 3(x - 1)(2x - 3)$

$\Rightarrow 4x^2 + 9 - 12x - 4x^2 - 4 + 8x = 3(2x^2 - 5x + 3)$

$\Rightarrow 4x^2 - 4x^2 - 12x + 8x + 9 - 4 = 6x^2 - 15x + 9$

$\Rightarrow -4x + 5 = 6x^2 - 15x + 9$

$\Rightarrow 6x^2 - 11x + 4 = 0$

$\Rightarrow 6x^2 - 8x - 3x + 4 = 0$

$\Rightarrow 2x(3x - 4) - 1(3x - 4) = 0$

$\Rightarrow (2x - 1)(3x - 4) = 0$

$\Rightarrow 2x = 1$ or $3x = 4$

$x = \dfrac{1}{2}$ or $x = \dfrac{4}{3}$

**25.** Solve the equation :

$$\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}, x \neq -4,7$$

**Sol.** $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}$

$\Rightarrow \frac{(x-7)-(x+4)}{(x+4)(x-7)}=\frac{11}{30}$

$\Rightarrow \frac{-11}{(x+4)(x-7)}=\frac{11}{30}$

$\Rightarrow \frac{-1}{(x+4)(x-7)}=\frac{1}{30}$

$\Rightarrow (x + 4)(x - 7) + 30 = 0$

$\Rightarrow x^2 - 3x - 28 + 30 = 0$

$\Rightarrow x^2 - 2x - x + 2 = 0$

$\Rightarrow x(x - 2) - 1(x - 2) = 0$

$\Rightarrow (x - 1)(x - 2) = 0$

$x = 1$ or $x = 2$

**26.** Solve for x by factorization method :

$x^2 + 5x - (a^2 + a - 6) = 0$

**Sol.** $x^2 + 5x - (a^2 + a - 6) = 0$

$\Rightarrow x^2 + 5x - (a^2 + 3a - 2a - 6) = 0$

$\Rightarrow x^2 + 5x - [a(a + 3) - 2(a + 3)] = 0$

$\Rightarrow x^2 + 5x - (a - 2)(a + 3) = 0$

$\Rightarrow x^2 + x[(a + 3) - (a - 2)] - (a - 2)(a + 3) = 0$

$\Rightarrow x^2 + (a + 3)x - (a - 2)x - (a - 2)(a + 3) = 0$

$\Rightarrow x[x + (a + 3)] - (a - 2)[x + (a + 3)] = 0$

$\Rightarrow [x + (a + 3)].[x - (a - 2)] = 0$

$\therefore x = -(a + 3)$ or $x = (a - 2)$

**27.** Find the roots of the quadratic equation :

$a^2b^2x^2 + b^2x - a^2x - 1 = 0$

**Sol.** $a^2b^2x^2 + (b^2 - a^2)x - 1 = 0$

$D = B^2 - 4AC$

$= (b^2 - a^2)^2 - 4 \times (a^2b^2)(-1)$

$\Rightarrow (b^2 - a^2)^2 + 4a^2b^2 = (b^4 + a^4 - 2b^2a^2 + 4b^2a^2)$

$D = (b^2 + a^2)^2$

$x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$x = \dfrac{-(b^2 - a^2) \pm \sqrt{(b^2 + a^2)^2}}{2.a^2b^2}$

$x = \dfrac{a^2 - b^2 \pm (a^2 + b^2)}{2a^2b^2}$

Either $x = \dfrac{a^2 - b^2 + a^2 + b^2}{2a^2b^2}$

or $x = \dfrac{a^2 - b^2 - a^2 - b^2}{2a^2b^2}$

$\Rightarrow \quad x = \dfrac{2a^2}{2a^2b^2} = \dfrac{1}{b^2}$ or $x = \dfrac{-2b^2}{2a^2b^2} = \dfrac{-1}{a^2}$

$\Rightarrow \quad x = \dfrac{1}{b^2}$ or $\dfrac{-1}{a^2}$

**28.** Solve for x :

$\dfrac{1}{x+1} + \dfrac{2}{x+2} = \dfrac{4}{x+4}; x \neq -1, -2, -4$

**Sol.** $\dfrac{1}{x+1} + \dfrac{2}{x+2} = \dfrac{4}{x+4}; x \neq -1, -2, -4$

$\Rightarrow \quad \dfrac{(x+2)+2(x+1)}{(x+1)(x+2)} = \dfrac{4}{(x+4)}$

$\Rightarrow \quad \dfrac{3x+4}{(x+1)(x+2)} = \dfrac{4}{(x+4)}$

$\Rightarrow \quad (3x + 4)(x + 4) = 4(x^2 + 3x + 2)$

$\Rightarrow \quad 3x^2 + 12x + 4x + 16 = 4x^2 + 12x + 8$

$\Rightarrow \quad x^2 - 4x - 8 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-4) \pm \sqrt{16+32}}{2} = \dfrac{4 \pm \sqrt{48}}{2}$

$= \dfrac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}$

**29.** Solve for x : $9x^2 - 3(a + b)x + ab = 0$

**Sol.** $9x^2 - 3(a + b)x + ab = 0$

$D = B^2 - 4AC$

$= 9(a + b)^2 - 4 \times 9 \times ab$

$= 9a^2 + 9b^2 + 18ab - 36ab$

$= 9a^2 + 9b^2 - 18ab$

$= 9(a - b)^2$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$x = \frac{3(a+b) \pm \sqrt{9(a-b)^2}}{2 \times 9}$$

$$x = \frac{3(a+b) \pm 3(a-b)}{18}$$

Either $x = \frac{3(a+b) + 3(a-b)}{18}$

or $x = \frac{3(a+b) - 3(a-b)}{18}$

$$x = \frac{6a}{18} = \frac{a}{3} \text{ or } x = \frac{6b}{18} = \frac{b}{3}$$

**30.** Solve the following equation for x :

$$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$$

**Sol.** $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

$\Rightarrow \frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

$\Rightarrow \frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

$\Rightarrow \frac{-(a+b)}{(a+b+x).x} = \frac{1}{a} + \frac{1}{b}$

$\Rightarrow \frac{-(a+b)}{(a+b+x).x} = \frac{a+b}{ab} \Rightarrow \frac{-1}{(a+b+x)x} = \frac{1}{ab}$

$\Rightarrow (a + b + x)\, x = -ab$

$\Rightarrow (a + b + x).x + ab = 0$

$\Rightarrow ax + bx + x^2 + ab = 0$

$\Rightarrow x^2 + ax + bx + ab = 0$

$\Rightarrow x(x + a) + b\,(x + a) = 0$

$\Rightarrow (x + a)\,(x + b) = 0$

$\Rightarrow x = -a \text{ or } x = -b$

**31.** Two numbers differ by 3 and their product is 504. Find the numbers.

**Sol.** $x - y = 3$

$\Rightarrow$ $x = 3 + y$ ... (1)

$\Rightarrow$ $xy = 504$

$\Rightarrow$ $(3 + y)y = 504$ (from (1))

$\Rightarrow$ $3y + y^2 = 504$

$\Rightarrow$ $y^2 + 3y - 504 = 0$

$\Rightarrow$ $y^2 + 24y - 21y - 504 = 0$

$\Rightarrow$ $(y + 24)(y - 21) = 0$

$\Rightarrow$ $y = 21, y = -24$

$\Rightarrow$ $x = 24, x = -21$

Thus, numbers are 21 and 24 or – 24 and – 21.

**32.** If the roots of the quadratic equation

$(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal, then prove that $b + c = 2a$.

**Sol.** $(a - b)x^2 + (b - c)x + (c - a) = 0$

Since, roots are equal D = 0

$\Rightarrow$ $B^2 - 4AC = 0$

$\Rightarrow$ $(b - c)^2 - 4(a - b)(c - a) = 0$

$\Rightarrow$ $b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ba) = 0$

$\Rightarrow$ $b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab = 0$

$\Rightarrow$ $b^2 + c^2 + 4a^2 + 2bc - 4ac - 4ab = 0$

$\Rightarrow$ $(b + c - 2a)^2 = 0$

$\Rightarrow$ $b + c = 2a$

**33.** If the equation $(1 + m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2(1 + m^2)$.

**Sol.** Since the equation has equal roots.

$\Rightarrow$ $D = 0$

$\Rightarrow$ $b^2 - 4ac = 0$

$\Rightarrow$ $(2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$

$\Rightarrow$ $4m^2c^2 - 4(1 + m^2)(c^2 - a^2) = 0$

$\Rightarrow$ $m^2c^2 - (1 + m^2)(c^2 - a^2) = 0$

$\Rightarrow$ $m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0$

$\Rightarrow$ $c^2 = a^2 + m^2a^2$

$\Rightarrow$ $c^2 = a^2(1 + m^2)$ Hence proved

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**34.** Solve the given quadratic equations by factorization method : $4x^2 - 2(a^2 + b^2)x + a^2b^2 = 0$

**Sol.** We have,

$4x^2 - 2(a^2 + b^2)x + a^2b^2 = 0$

$\therefore\quad 4x^2 - 2(a^2 + b^2)x + a^2b^2 = 0$

$\Rightarrow\quad 4x^2 - 2a^2x - 2b^2x + a^2b^2 = 0$

$\Rightarrow\quad (4x^2 - 2a^2x) - (2b^2x - a^2b^2) = 0$

$\Rightarrow\quad 2x(2x - a^2) - b^2(2x - a^2) = 0$

$\Rightarrow\quad (2x - a^2)(2x - b^2) = 0$

$\Rightarrow\quad (2x - a^2) = 0$ or $(2x - b^2) = 0$

$\Rightarrow\quad x = \frac{a^2}{2}$ or $x = \frac{b^2}{2}$

**35.** Factorise : $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

**Sol.** We have,

$9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

Here, constant term $= 2a^2 + 5ab + 2b^2$

$= 2a^2 + 4ab + ab + 2b^2$

$= 2a(a + 2b) + b(a + 2b)$

$= (2a + b)(a + 2b)$ and Coefficient of middle term

$= -9(a + b) = -3\{(2a + b) + (a + 2b)\}$

$\therefore\quad 9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

$\Rightarrow\quad 9x^2 - 3\{(2a + b) + (a + 2b)\}x + (2a + b)$
$(a + 2b) = 0$

$\Rightarrow\quad 9x^2 - 3(2a + b)x - 3(a + 2b)x + (2a + b)$
$(a + 2b) = 0$

$\Rightarrow\quad 3x\{3x - (2a + b)\} - (a + 2b)\{3x - (2a + b)\} = 0$

$\Rightarrow\quad \{3x - (2a + b)\}\{3x - (a + 2b)\} = 0$

$\Rightarrow\quad \{3x - (2a + b)\} = 0$ or, $\{3x - (a + 2b)\} = 0$

$\Rightarrow\quad x = \frac{2a + b}{3}$ or, $x = \frac{a + 2b}{3}$

### Long answer type questions (5 marks)

**36.** A train travels at a uniform speed for a distance of 63 km and then travels a distance of 72 km at an average speed of 6 km/h more than its original speed. If it takes 3 hours to complete the total journey, what is the original speed of the train ?

**Sol.** Let the original speed of the train be v km/hr.

Time taken for a distance of 63 km = $\frac{63}{v}$

Time taken for a distance of 72 km = $\frac{72}{v+6}$

Total time taken = 3 hr

$\Rightarrow \frac{63}{v}+\frac{72}{v+6}=\frac{3}{1} \Rightarrow 3\left(\frac{21}{v}+\frac{24}{v+6}\right)=3$

$\Rightarrow \frac{21}{v}+\frac{24}{v+6}=1 \Rightarrow \frac{21(v+6)+24v}{v(v+6)}=1$

$\Rightarrow$ $21(v + 6) + 24v = v(v + 6)$

$\Rightarrow$ $21v + 126 + 24v = v^2 + 6v$

$\Rightarrow$ $v^2 - 39v - 126 = 0$

$\Rightarrow$ $v^2 - 42v + 3v - 126 = 0$

$\Rightarrow$ $v(v - 42) + 3(v - 42) = 0$

$\Rightarrow$ $(v + 3)(v - 42) = 0$

$\Rightarrow$ $v = -3$ (Not possible) or $v = 42$

$\Rightarrow$ Original speed = 42 km/h

**37.** Sum of the areas of two squares is 468 $m^2$. If the difference of their perimeters is 24 m, find the sides of the two squares.

**Sol.** Let side of squares are 'a' m and 'b' m.

Then, $a^2 + b^2 = 468\ m^2$ and $4a - 4b = 24$

$a - b = 6$

Squaring both sides

$a^2 + b^2 - 2ab = 36$

$\Rightarrow$ $468 - 36 = 2ab$

$432 = 2ab$

$\Rightarrow$ $216 = ab$

And $(a + b)^2 = (a - b)^2 + 4ab$

$(a + b)^2 = 36 + 4 \times 216$

$\Rightarrow$ $(a + b)^2 = 36 + 864 = 900$

$\Rightarrow$ $(a + b) = 30$ ... (1)

and $(a - b) = 6$ ... (2)

Adding equation (1) & (2) we get

$2a = 36$

$a = 18$

From (1), $b = 12$

Thus, sides of two squares are 18 m and 12 m.

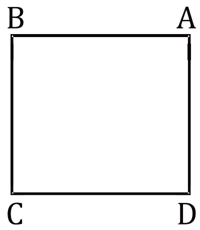
**38.** A person has a rectangular garden whose area is 100 sq m. He fences three sides of the garden with 30 m barbed wire. On the fourth side, the wall of his house is constructed; find the dimensions of the garden.

**Sol.** Let $\ell$ = length of rectangular garden ABCD

& b = breadth of rectangular garden ABCD

Given, area of garden = $\ell b = 100\ m^2$ ... (1)

Let's assume if person start fencing from point A and moves towards B, C and then D.




Thus, total fencing

$\Rightarrow$ $2(\ell) + b = 30$ ... (2)

from (1) & (2)

$\Rightarrow$ $2\ell + \dfrac{100}{\ell} = 30$

$2\ell^2 + 100 = 30\ell$

$\Rightarrow$ $2\ell^2 - 30\ell + 100 = 0$

Or $\ell^2 - 15\ell + 50 = 0$

$\Rightarrow$ $\ell^2 - 10\ell - 5\ell + 50 = 0$ [by splitting middle term]

$\Rightarrow$ $\ell(\ell - 10) - 5(\ell - 10) = 0$

$\Rightarrow$ $(\ell - 10)(\ell - 5) = 0$

$\Rightarrow$ either $\ell = 10$ m or $\ell = 5$ m

Thus, for $\ell = 10$ m

then from equation (1) $\Rightarrow$ $b = \dfrac{100}{10} = 10$ m

So, $\ell = 10$m, b = 10 m (Not possible as square is formed)

and for $\ell = 5$m

then from equation (1) $\Rightarrow$ b = $\frac{100}{5}$ = 20 m

So, $\ell$ = 5m, b = 20 m

Similarly, he could start fencing from point B and move towards C, D and A.

$2b + \ell = 30$ ... (3)

$\Rightarrow \quad 2b + \frac{100}{b} = 30$

from (1) & (3)

$\Rightarrow \quad 2b^2 - 30b + 100 = 0$

$\Rightarrow \quad b^2 - 15b + 20 = 0$

$\Rightarrow \quad b = 5 \text{ m} \ \& \ b = 10 \text{ m}$

Corresponding to b = 5m, $\ell$ = 20 m

and corresponding to b = 10m, $\ell$ = 10 m

Thus, dimensions of the garden,

$\ell$ = 20 m, b = 5 m

**39.** Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

**Sol.** Let tap of smaller diameter fill the tank in x hours then tap of larger diameter fill in (x – 10) hours.

Then $\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$

$\Rightarrow \quad \frac{x-10+x}{x(x-10)} = \frac{8}{75}$

$\Rightarrow \quad \frac{2x-10}{x^2-10x} = \frac{8}{75}$

$\Rightarrow \quad \frac{2x-10}{x^2-10x} = \frac{8}{75}$

$\Rightarrow \quad \frac{x-5}{x^2-10x} = \frac{4}{75}$

$\Rightarrow \quad -40x = 75x - 375$

$\Rightarrow \quad 4x^2 - 115x + 375 = 0$

$\Rightarrow \quad 4x^2 - 100x - 15x + 375 = 0$

$\Rightarrow \quad 4x(x - 25) - 15(x - 25) = 0$

$\Rightarrow \quad (4x - 15)(x - 25) = 0$

$x = \frac{15}{4}$ or $x = 25$

If $x = \frac{15}{4}$ then $x - 10 = \frac{15}{4} - 10 = -\frac{25}{4}$ (Not possible)

So, smaller diameter tap takes 25 hours and larger diameter tap takes = 15 hours.



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**40.** Some students went for a picnic. The budget for the food was Rs. 240. Because four students of the group failed to go, the cost of food to each student got increased by Rs. 5. How many students went for the picnic.

**Sol.** The budget for food = Rs. 240

Let total number of students who went for picnic = x

Thus, cost of food to each student = $\left(\frac{240}{x}\right)$

If 4 students failed each to go.

Rest students, went for picnic = (x – 4)

Thus, cost of food to each student = $\left(\frac{240}{x-4}\right)$

So, cost of food to each student increase by Rs. 5

According to question,

$\frac{240}{x-4}-\frac{240}{x} = 5$

$\Rightarrow \quad 240\left\{\frac{1}{x-4}-\frac{1}{x}\right\} = 5$

$\Rightarrow \quad 240\left\{\frac{x-x+4}{x(x-4)}\right\} = 5$

$\Rightarrow \quad \frac{240\times 4}{x(x-4)} = 5$

$= 192 = x(x-4)$

$\Rightarrow \quad x^2 - 4x = 192$

$\Rightarrow \quad x^2 - 4x - 192 = 0$

$\Rightarrow \quad x^2 - 16x + 12x - 192 = 0$

$\Rightarrow \quad x(x-16) + 12(x-16) = 0$

$\Rightarrow \quad (x-16)(x+12) = 0$

either x = 16 or x = –12 (Not possible)

Thus, total number of students, who went for picnic = 16

**41.** A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 250 km/h from the usual speed. Find its usual speed.

**Sol.** Let usual speed = x km/h

Distance = 1500 km

Usual time = $\frac{1500}{x}$ hour

New speed = (x + 250) km/h

New time taken = $\frac{1500}{(x+250)}$ hour

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According to question

$\Rightarrow \dfrac{1500}{x}-\dfrac{1500}{(x+250)}=\dfrac{30}{60}$

$\Rightarrow \dfrac{1500[x+250-x]}{x(x+250)}=\dfrac{1}{2}$

$\Rightarrow \dfrac{1500\times 250}{x^2+250x}=\dfrac{1}{2}$

$\Rightarrow$ $x^2 + 250x = 3000 \times 250$

$\Rightarrow$ $x^2 + 250x - 750000 = 0$

$\Rightarrow$ $x^2 + 1000x - 750x - 750000 = 0$

$\Rightarrow$ $(x + 1000)(x - 750) = 0$

$\Rightarrow$ $x = -1000$ (Not possible)

or $x = 750$

Thus, Usual speed = 750 km/h

**42.** One-fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels.

**Sol.** Let the total number of camels be x. Then,

Number of camels seen in the forest = $\dfrac{x}{4}$

Number of camels gone to mountains = $2\sqrt{x}$

Number of camels on the bank to river = 15

Total number of camels = $\dfrac{x}{4}+2\sqrt{x}+15$

By hypothesis, we have

$\dfrac{x}{4}+2\sqrt{x}+15=x$

$\Rightarrow 3x-8\sqrt{x}-60=0$

$\Rightarrow$ $3y^2 - 8y - 60 = 0$, where $x = y^2$

$\Rightarrow$ $3y^2 - 18y + 10y - 60 = 0$

$\Rightarrow$ $3y(y - 6) + 10(y - 6) = 0$

$\Rightarrow$ $(3y + 10)(y - 6) = 0$

$\Rightarrow$ $y = 6$ or, $y = \dfrac{10}{3}$

Now, $y=-\dfrac{10}{3}\Rightarrow x=\left(-\dfrac{10}{3}\right)^2=\dfrac{100}{9}$ [$\because x = y^2$]

But, the number of camels cannot be a fraction.

$\therefore$ $y = 6 \Rightarrow x = 6^2 = 36$

Hence, the number of camels = 36. [$\because x = y^2$]

**43.** Two trains leave a railway station at the same time. The first train travels due west and the second train due north. The first train travels 5 km/hr faster than the second train. If after two hours, they are 50 km apart, find the average speed of each train.

**Sol.** Let the speed of the second train be x km/hr. Then, the speed of the first train is (x + 5) km/hr.

Let O be the position of the railway station from which the two trains, leave.

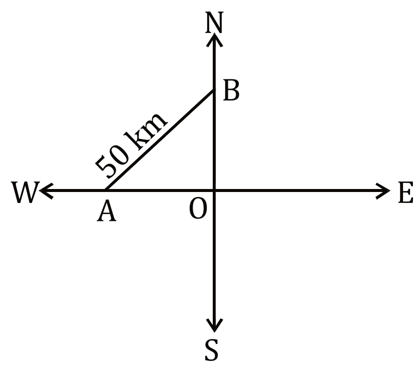
Distance travelled by the first train in 2 hours = OA = Speed × Time

= 2 (x + 5) km

Distance travelled by the second train in 2 hours = OB = Speed × Time = 2x km

By using pythagoras theorem, we have,

$AB^2 = OA^2 + OB^2$




$\Rightarrow \quad 50^2 = \{2(x + 5)\}^2 + \{2x\}^2$

$\Rightarrow \quad 2500 = 4(x + 5)^2 + 4x^2$

$\Rightarrow \quad 8x^2 + 40x - 2400 = 0$

$\Rightarrow \quad x^2 + 5x - 300 = 0$

$\Rightarrow \quad x^2 + 20x - 15x - 300 = 0$

$\Rightarrow \quad x(x + 20) - 15(x + 20) = 0$

$\Rightarrow \quad (x + 20)(x - 15) = 0$

$\Rightarrow \quad x = -20$ or $x = 15$

$\Rightarrow \quad x = 15$

Hence, the speed of the second train is 15 km/hr and the speed of the first train is 20 km/hr.

**44.** The hypotenuse of a right triangle is $3\sqrt{5}$ cm. If the smaller side is tripled and the larger side is doubled, the new hypotenuse will be 15 cm. Find the length of each side.

**Sol.** Let the smaller side of the right triangle be x cm and the larger side by y cm. Then,

$x^2 + y^2 = (3\sqrt{5})^2$ [Using pythagoras Theorem]

$\Rightarrow \quad x^2 + y^2 = 45$ ... (i)

If the smaller side is tripled and the larger side be doubled, the new hypotenuse is 15 cm.

$\therefore \quad (3x)^2 + (2y)^2 = 15^2$

$\Rightarrow \quad 9x^2 + 4y^2 = 225$ ... (ii)

From equation (i), we get $y^2 = 45 - x^2$

Putting $y^2 = 45 - x^2$ in equation (ii), we get

$9x^2 + 4(45 - x^2) = 225$

$\Rightarrow \quad 5x^2 + 180 = 225$

$\Rightarrow \quad 5x^2 = 45$

$\Rightarrow \quad x^2 = 9$

$\Rightarrow \quad x = \pm 3$

But, length of a side cannot be negative. Therefore, x = 3.

Putting x = 3 in (i), we get

$9 + y^2 = 45 \Rightarrow y^2 = 36 \Rightarrow y = 6$

Hence, the length of the smaller side is 3 cm and the length of the larger side is 6 cm.

**45.** The angry Arjun carried some arrows for fighting with Bheeshm. With half the arrows, he cut down the arrows thrown by Bheeshm on him and with six other arrows he killed the rath driver of Bheeshm. With one arrow each he knocked down respectively the rath, flag and the bow of Bheeshm. Finally, with one more than four times the square root of arrows he laid Bheeshm unconscious on an arrow bed. Find the total number of arrows Arjun had.

**Sol.** Suppose Arjun had x arrows.

Number of arrows used to cut arrows of Bheeshm = $x/2$

Number of arrows used to kill the rath driver = 6

Number of other arrows used = 3

Remaining arrows = $4\sqrt{x}+1$

By hypothesis, we have

$\therefore \quad \frac{x}{2}+6+3+4\sqrt{x}+1=x$

$\Rightarrow \quad x+20+8\sqrt{x}=2x$

$\Rightarrow \quad x=20+8\sqrt{x}$

Putting $x = y^2$, the above equation becomes

$y^2 = 20 + 8y$

$\Rightarrow \quad y^2 - 8y - 20 = 0$

$\Rightarrow \quad y^2 - 10y + 2y - 20 = 0$

$\Rightarrow \quad (y - 10)(y + 2) = 0$

$\Rightarrow \quad y = 10$ or, $y = -2$

$\Rightarrow \quad y = 10$ [$\because$ y cannot be negative]

$\Rightarrow \quad x = y^2$

$\Rightarrow \quad x = 100$

Hence, the number of arrows which Arjun had = 100

**Case Study type questions** **(4 marks)**

**1.** Rahul's father gave him some money to buy avocado from the market at the rate of $p(x) = x^2 - 24x + 128$. Let $\alpha$, $\beta$ are the zeroes of $p(x)$.

Based on above information, answer the following questions

(i) Find the value of $p(2)$.

(ii) Find the value of $\alpha + \beta$ and $\alpha\beta$.

(iii) Find the value of $\alpha$ and $\beta$ where $\alpha < \beta$

**[OR]**

(iii) If sum of zeroes of $q(x) = kx^2 + 2x + 3k$ is equal to their product, Find k.

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**Sol.** (i) Put x = 2 in p(x)

$p(2) = (2)^2 - 24(2) + 128$

$= 4 - 48 + 128$

$= 132 - 48$

$= 84$

(ii) $\alpha + \beta = \frac{-b}{a} = \frac{-(-24)}{1} = 24$

$\alpha\beta = \frac{c}{a} = \frac{128}{1} = 128$

(iii) $x^2 - 24x + 128 = 0$

$\Rightarrow x^2 - 8x - 16x + 128 = 0$

$\Rightarrow x(x - 8) - 16(x - 8) = 0$

$\Rightarrow (x - 8)(x - 16) = 0$

$x = 8, x = 16$

$\alpha = 8, \beta = 16$

**[OR]**

(iii) $q(x) = kx^2 + 2x + 3k$

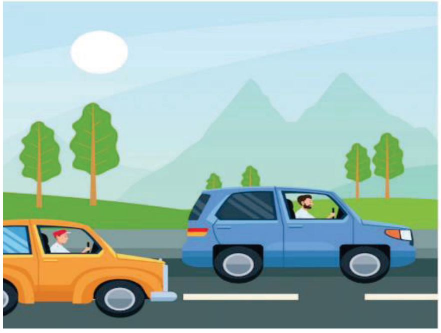
$\Rightarrow \alpha + \beta = \alpha\beta$

$\Rightarrow \frac{-2}{k} = \frac{3k}{k}$

$\Rightarrow \frac{-2}{k} = 3$

$k = -\frac{2}{3}$

**2.** Raj and Ajay are very close friends. Both the families decide to go to Ranikhet, by their own cars. Raj's car can travel at a speed of 'x' km/hr while Ajay's car travels 5 km/h faster than Raj's car. Raj took 4 hours more than Ajay to complete the journey of 400 km.



(i) What will be the distance covered by Ajay's car in two hours (in terms of x) ?

(ii) What quadratic equation describes the speed of Raj's car ?

(iii) Find the roots of the quadratic equation which describes the speed of Raj's car.

**[OR]**

(iii) Find the positive root of the equation $\sqrt{3x^2 + 6} = 9$

**Sol.** (i) Ajay's car is 5 km/hr faster than Raj's car

Speed of Ajay's car = (x + 5) km/hr

Distance covered by Ajay in 2 hours = speed × time

$\Rightarrow$ (x + 5) × 2

$\Rightarrow$ 2(x + 5) Km

(ii) Speed of Raj's car = x km/hr

Distance = 400 km

time taken = $\frac{400}{x}$ hr

Speed of Ajay's car = (x + 5) km/hr

Distance = 400 km

Time taken = $\frac{400}{x+5}$ hrs

According to question,

Raj's time = Ajay's time + 4

$\Rightarrow \frac{400}{x} = \frac{400}{x+5} + 4$

$\Rightarrow \frac{400}{x} - \frac{400}{x+5} = 4$

$\Rightarrow 400\left(\frac{x+5-x}{x(x+5)}\right) = 4$

$\Rightarrow$ 500 = x(x + 5)

$\Rightarrow$ 500 = $x^2$ + 5x

$x^2 + 5x - 500 = 0$

(iii) $x^2 + 5x - 500 = 0$

$x^2 + 25x - 20x - 500 = 0$

$x(x + 25) - 20(x + 25) = 0$

$(x + 25)(x - 20) = 0$

$x = -25, x = 20$

**[OR]**

(iii) Squaring both sides,

$\Rightarrow 3x^2 + 6 = 81$

$\Rightarrow 3x^2 = 75$

$\Rightarrow x^2 = 25$

$\Rightarrow x = \pm 5$

Positive root = 5

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# 5 Arithmetic Progressions

**Multiple choice questions** **(1 mark)**

**1.** If $\frac{2}{3}, k, \frac{5k}{8}$ are in A.P, find the value of k.

(1) $\frac{16}{11}$ (2) $\frac{16}{33}$ (3) $\frac{8}{33}$ (4) $\frac{8}{11}$

**Sol. Option (2)**

$\frac{2}{3}, k, \frac{5k}{8}$ are in AP.

$$k - \frac{2}{3} = \frac{5k}{8} - k$$

$$2k - \frac{5k}{8} = \frac{2}{3}$$

$$\Rightarrow \frac{11k}{8} = \frac{2}{3}$$

$$k = \frac{16}{33}$$

**2.** The sum of all odd integers between 2 and 100 divisible by 3 is

(1) 965 (2) 865 (3) 867 (4) 1067

**Sol. Option (3)**

The odd integer between 2 and 100 which are divisible by 3 are 3, 9, 15, 21, ....,99.

Here, a = 3 and d = 6

$a_n = 99$

$\Rightarrow a + (n - 1) \times d = 99$

$\Rightarrow 3 + (n - 1) \times 6 = 99$

$\Rightarrow 6(n - 1) = 96$

$\Rightarrow n - 1 = 16$

$\Rightarrow n = 17$

Therefore,

Required sum = $\frac{n}{2}(a + a_n) = \frac{17}{2}(3 + 99) = 867$

**3.** How many terms are there in the AP : 7, 11, 15 ............ 139 ?

(1) 31 (2) 32 (3) 33 (4) 34

**Sol. Option (4)**

$a = 7, d = 11 - 7 = 4$

Let $a_n = 139$

$a + (n - 1)d = 139$

$7 + (n - 1)4 = 139$

$n - 1 = \frac{139 - 7}{4}$

$n = 33 + 1$

$n = 34$

**4.** Which term of the AP 2, –1, –4, –7, ____ is –40?

(1) $8^{th}$ (2) $11^{th}$ (3) $15^{th}$ (4) $23^{rd}$

**Sol. Option (3)**

$a = 2$

$d = t_2 - t_1$

$= -1 - 2 = -3$

$t_n = -40$

$t_n = a + (n - 1)d$

$-40 = 2 + (n - 1) \times -3$

$-40 = 2 - 3n + 3$

$-40 = 5 - 3n$

$3n = 45$

$n = \frac{45}{3}$

$n = 15$

$\therefore$ $15^{th}$ term is –40.

**5.** Find the $11^{th}$ term from the last term of the A.P. : 10, 7, 4, .............. –69.

(1) 29 (2) –29 (3) –39 (4) –99

**Sol. Option (3)**

AP (Reverse) = –69, –66......, 4, 7, 10

$a = -69, d = 3$

required term $= a_{11} = a + 10d$

$= -69 + 10 \times 3$

$= -69 + 30 = -39$

**6.** If $S_n$ denotes the sum of the first n terms of an A.P. Then, $S_{3n} : (S_{2n} - S_n)$ is

(1) n (2) 3n (3) 3 (4) None of these

**Sol. Option (3)**

$$S_{2n} = \frac{2n}{2}[2a + (2n - 1)d]$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{3n} = \frac{3n}{2}[2a + (3n - 1)d]$$

$$S_{2n} - S_n = \frac{2n}{2}[2a + (2n - 1)d] - \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{1n}{2}[2a + (3n - 1)d]$$

$$S_{3n} : (S_{2n} - S_n) = \frac{3n}{2}[2a + (3n - 1)d] : \frac{1n}{2}[2a + (3n - 1)d] = 3$$

**7.** Which term of the AP 3, 15, 27,39, _____ will be 120 more than its 21st term?

(1) 21 (2) 18 (3) 31 (4) 36

**Sol. Option (3)**

AP : 3, 15, 27, 39, ...

$a = 3$, $d = 15 - 3 = 12$

According to question,

$a_n = 120 + a_{21}$

$a + (n - 1)d = 120 + a + 20d$

$(n - 1)12 = 120 + 20 \times 12$

$n - 1 = 30$

$n = 31$

**8.** Sum of n terms of the series $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+....$ is

(1) $\frac{n(n+1)}{2}$ (2) $2n(n + 1)$ (3) $\frac{n(n+1)}{\sqrt{2}}$ (4) 1

**Sol. Option (3)**

The series can be rewritten as $\sqrt{2}+2\sqrt{2}+3\sqrt{2}+4\sqrt{2}+.........$

So, $S_n = \sqrt{2} \times (1 + 2 + 3 + .........+ n)$

$$= \frac{\sqrt{2}\times n\times(n+1)}{2} = \frac{n(n+1)}{\sqrt{2}}$$

**Assertion reason questions** **(1 marks)**

**9.** **Assertion (A):** Sum of first 15 terms of 2 + 5 + 8… is 345.

**Reason (R):** Sum of first n terms in an A.P. is given by the formula: $S_n = \frac{n}{2} \times [2a + (n-1)d]$

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol.** **Option (1)**

$S_{15} = \frac{15}{2} \times [2 \times 2 + (15-1)3] = 345$

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of (A).

**10.** **Assertion (A):** The common difference of an A.P. 2, 4, 6, 8 is 2.

**Reason (R):** The constant difference between any two consecutive terms of an AP is commonly known as common difference.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol.** **Option (1)**

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

**Very short answer type questions** **(2 mark)**

**11.** Write the common difference of an A.P. whose nth term is 3n + 5.

**Sol.** $T_n = 3n + 5$

$\therefore \quad T_1 = 3(1) + 5 = 8$

$T_2 = 3(2) + 5 = 11$

$\Rightarrow \quad d = T_2 - T_1$

$= 11 - 8 = 3$

Thus, the common difference = 3.

**12.** For what value of k, are the numbers x, (2x + k) and (3x + 6) three consecutive terms of an A.P.?

**Sol.** Here, $T_1 = x$, $T_2 = (2x + k)$ and $T_3 = (3x + 6)$

For an A.P., we have

$T_2 - T_1 = T_3 - T_2$

i.e., $2x + k - x = 3x + 6 - (2x + k)$

$\Rightarrow$ $x + k = 3x + 6 - 2x - k$

$\Rightarrow$ $x + k = x + 6 - k$

$\Rightarrow$ $k + k = x + 6 - x$

$\Rightarrow$ $2k = 6$

$\Rightarrow$ $k = \frac{6}{2} = 3$

**13.** If $\frac{4}{5}$, a, 2 are three consecutive terms of an A.P., then find the value of a?

**Sol.** Here, $T_1 = \frac{4}{5}$

$T_2 = a$

$T_3 = 2$

$\therefore$ For an A.P.,

$T_2 - T_1 = T_3 - T_2$

$\therefore$ $a - \frac{4}{5} = 2 - a$

$\Rightarrow$ $a + a = 2 + \frac{4}{5}$

$\Rightarrow$ $2a = \frac{14}{5}$

$\Rightarrow$ $a = \frac{14}{5} \times \frac{1}{2} = \frac{7}{5}$

Thus, $a = \frac{7}{5}$

**14.** Find the sum of first 16 terms of the A.P.

10, 6, 2, .......

**Sol.** Here, $a = 10$, $d = 6 - 10 = -4$, $n = 16$

$\because$ $S = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_{16} = \frac{16}{2}[2 \times 10 + (16-1)(-4)]$

$= 8[20 + 15 \times (-4)]$

$= 8[20 - 60]$

$= 8 \times (-40)$

$= -320$

**15.** Find the next term of the A.P. $\sqrt{2}, \sqrt{8}, \sqrt{18}$ .....

**Sol.** Here, $T_1 = \sqrt{2} \Rightarrow a = \sqrt{2}$

$T_2 = \sqrt{8} = 2\sqrt{2}$

$T_3 = \sqrt{18} = 3\sqrt{2}$

Now, $d = T_2 - T_1$

$= 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

Now, using $T_n = a + (n - 1) \times d$, we have

$T_4 = a + 3d$

$= \sqrt{2} + 3(\sqrt{2})$

$= \sqrt{2}\,[1+3] = 4\sqrt{2}$

$= \sqrt{16 \times 2} = \sqrt{32}$

Thus, the next term = $\sqrt{32}$

**16.** In the A.P., if $d = -2$, $n = 5$ and $a_n = 0$, then find the value of a.

**Sol.** Given $d = -2$, $n = 5$, $a_n = 0$

Since, $a_n = a + (n - 1)d$

$\Rightarrow \quad 0 = a + (5 - 1)(-2)$

$\Rightarrow \quad 0 = a - 8$

$\Rightarrow \quad a = 8$

### Short answer type questions (3 marks)

**17.** Find the value of the middlemost term (s) of the AP : –11, – 7, – 3, ...49.

**Sol.** $\because \quad a = -11,\ a_n = 49$ and $d = (-7) - (-11) = 4$

$\therefore \quad a_n = a + (n - 1)d$

$\Rightarrow \quad 49 = -11 + (n - 1) \times 4$

$\Rightarrow \quad n = 16$

Since, n is an even number

$\therefore$ There will be two middle terms, which are

$\frac{16}{2}$th and $\left(\frac{16}{2}+1\right)$th

or $8^{th}$ and $9^{th}$

Now, $a_8 = a + (8 - 1)d$

$= -11 + 7 \times 4 = 17$

$a_9 = a + (9 - 1)d$

$= -11 + 8 \times 4 = 21$

Thus, the values of the two middlemost terms are 17 and 21.

**18.** The $17^{th}$ term of an A.P. exceeds its $10^{th}$ term by 7. Find the common difference.

**Sol.** According to question, $a_{17} - a_{10} = 7$

i.e., $a + 16d - (a + 9d) = 7$

where a = first term, d = common difference

$\Rightarrow$ $7d = 7$

$\Rightarrow$ $d = 1$

**19.** If $n^{th}$ term of an A.P. is $(2n + 1)$, then find the sum of its first three terms.

**Sol.** Let a and d be the first term and the common difference respectively.

$a_n = 2n + 1$ (Given)

$\Rightarrow$ $a_1 = 2 + 1 = 3$, $a_2 = 4 + 1 = 5$ and

$a_3 = 2(3) + 1 = 7$

$\therefore$ Sum of three terms $= a_1 + a_2 + a_3 = 3 + 5 + 7 = 15$

**20.** Find how many integers between 200 and 500 are divisible by 8.

**Sol.** Numbers divisible by 8 between 200 and 500 are 208,216,224 .....496 which forms an A.P.

$\therefore$ First term (a) = 208, common difference (d) = 8

$n^{th}$ term of an A.P. is $a_n = a + (n - 1)d$

$\Rightarrow$ $496 = 208 + (n - 1)8$

$\Rightarrow$ $288 = (n - 1)\,8$

$\Rightarrow$ $n - 1 = 36$

$\Rightarrow$ $n = 37$

**21.** Which term of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots$ is the first negative term?

**Sol.** Given sequence is an A.P. in which a = 20

and $d = 19\frac{1}{4} - 20 = \frac{77}{4} - 20 = \frac{-3}{4}$.

Let $n^{th}$ term of the given A.P. be the first negative term.

i.e., $a_n < 0$

$\Rightarrow$ $a + (n - 1)\,d < 0$

$\Rightarrow$ $20 + (n-1)\left(\frac{-3}{4}\right) < 0$

$\Rightarrow$ $80 - 3n + 3 < 0$

$\Rightarrow$ $83 - 3n < 0$

$\Rightarrow$ $3n > 83$

$\Rightarrow$ $n > 27\frac{2}{3}$

$\Rightarrow$ $n = 28$

$28^{th}$ term be the first negative term.

**22.** Find the middle term (s) of the A.P. 6,13,20.... 216.

**Sol.** We have, first term, a = 6,

common difference

$d = 13 - 6 = 7$

Last term, $216 = \ell = a_n = a + (n-1)d$

$\Rightarrow$ $216 = 6 + (n-1)7$

$\Rightarrow$ $216 - 6 = 7n - 7 \Rightarrow 210 + 7 = 7n$

$\Rightarrow$ $n = \frac{217}{7} = 31$, which is odd.

$\therefore$ Middle term = $\left(\frac{n+1}{2}\right)^{th}$ term $= \left(\frac{31+1}{2}\right)^{th}$ term

$= 16^{th}$ term

$a_{16} = a + (16-1)d = 6 + 15 \times 7 = 111$

**23.** Find whether −150 is a term of the A.P. 17, 12, 7, 2 ..... ?

**Sol.** Given A.P. is 17,12,7,2, ....

$\therefore$ First term, a = 17,

common difference, $d = 12 - 17 = -5$

Let if possible, for a natural number n, $a_n = -150$

$\Rightarrow$ $a + (n-1)d = -150$

$\Rightarrow$ $17 + (n-1)(-5) = -150$

$\Rightarrow$ $(-5)(n-1) = -150 - 17 = -167$

$\Rightarrow$ $n - 1 = \frac{167}{5} \Rightarrow n = \frac{167}{5} + 1 = \frac{172}{5} = 34.4$

Now, 34.4 is not a natural number.

Thus, −150 is not a term of the given A.P.

**24.** The ninth term of an A.P. is −32 and the sum of its eleventh and thirteenth term is −94. Find the common difference of the A.P.

**Sol.** Let the first term be a and d be common difference of the A.P.

Given, $a_9 = -32 \Rightarrow a + 8d = -32$ ... (i)

Also, $a_{11} + a_{13} = -94$

$\Rightarrow$ $a + 10d + a + 12d = -94$

$\Rightarrow$ $2a + 22d = -94$

$\Rightarrow$ $a + 11d = -47$ ... (ii)

Subtracting (ii) from (i), we have

$-3d = 15$

$\Rightarrow$ $d = -5$

**25.** How many three-digit natural numbers are divisible by 7 ?

**Sol.** The three-digit natural numbers which are divisible by 7 are 105, 112, 119, ....994 which forms an A.P. with a = 105, Common difference, d = 7 and $a_n$ = 994

So, 994 = a + (n – 1)d

$\Rightarrow$ 994 = 105 + (n – 1)7

$\Rightarrow$ 889 = (n – 1)7

$\Rightarrow$ $(n-1)=\frac{889}{7}=127$

$\Rightarrow$ n = 128

**26.** In an A.P., the first term is 12 and the common difference is 6. If the last term of the A.P. is 252, find its middle term

**Sol.** Given, first term, a = 12

common difference d = 6,

last term $\ell$ = $a_n$ = 252

$\therefore$ 252 = a + (n – 1)d

$\Rightarrow$ 252 = 12 + (n – 1)6

$\Rightarrow$ 240 = (n – 1)6

$\Rightarrow$ n – 1 = 40

$\Rightarrow$ n = 41

$\therefore$ $\text{Middle term} = \left(\frac{41+1}{2}\right)^{th}$ term = $21^{st}$ term

Now, $a_{21}$ = a + 20d = 12 + 20(6) = 132

**27.** Which term of the A.P. 3, 15, 27, 39.... will be 240 more than its $11^{th}$ term?

**Sol.** We have, first term, a = 3,

common difference d = 15 – 3 = 12

$n^{th}$ term of an A.P. is given by $a_n$ = a + (n – 1)d

$\therefore$ $a_{11}$ = 3 + (10) × 12 = 3 + 120 = 123

Let the $r^{th}$ term of the A.P. be 240 more than the $11^{th}$ term.

$a_r = a_{11} + 240$

$\Rightarrow$ a + (r – 1)d = 123 + 240

$\Rightarrow$ 3 + (r – 1) 12 = 363

$\Rightarrow$ (r – 1) 12 = 360

$\Rightarrow$ r – 1 = 30

$\Rightarrow$ r = 31

**28.** If 9th term of an A.P. is zero, prove that its 29th term is double of its 19th term.

**Sol.** Given 9th term of an A.P. is 0.

i.e., $a_9 = a + 8d = 0$

$\Rightarrow$ $a = -8d$ ... (i)

Now, $a_{29} = a + 28d = -8d + 28d = 20d$ [using (i)]

$a_{19} = a + 18d = -8d + 18d = 10d$ [using (i)]

$\therefore$ $a_{29} = 20d = 2 \times 10d = 2 \times a_{19}$ Hence proved.

**29.** For what value of n are the nth terms of two A.P.'s 63, 65, 67 ... and 3, 10, 17 ... equal ?

**Sol.** For the A.P., 63,65,67...., we have

First term, a = 63, common difference,

$d = 65 - 63 = 2$ and nth term of an A.P. is given by $a_n = a + (n - 1)\, d$

$\therefore$ $a_n = 63 + (n - 1)2 = 2n + 61$

For the A.P., 3, 10, 17 ......, we have

First term a = 3, common difference, $d = 10 - 3 = 7$

$\therefore$ $a_n = 3 + (n - 1)7 = 7n - 4$

Now, it is given that the nth terms of the two A.P.s are equal.

$\therefore$ $2n + 61 = 7n - 4$

$\Rightarrow$ $5n = 65$

$\Rightarrow$ $n = 13$

## Long answer type questions (5 marks)

**30.** If m times the mth term of an A.P. is equal to n times its nth term, then find the (m + n)th term of the A.P.

**Sol.** Let the first term and the common difference of the A.P. are a and d respectively.

We know that the nth term of an A.P. is given by

$a_n = a + (n - 1)d$

Now, it is given that m times the mth term of the A.P. is equal to n times its nth term.

i.e., $m.a_m = n.a_n$

$\Rightarrow$ $m.[a + (m - 1)d] = n\,[a + (n - 1)d]$

$\Rightarrow$ $ma + (m^2 - m)d = na + (n^2 - n)d$

$\Rightarrow$ $(m - n)a + (m^2 - m - n^2 + n)d = 0$

$\Rightarrow$ $(m - n)a + [(m^2 - n^2) - (m - n)]\, d = 0$

$\Rightarrow$ $(m - n)\, a + (m - n)(m + n - 1)\, d = 0$

$\Rightarrow$ $(m - n)\, [a + (m + n - 1)d] = 0$

$\Rightarrow$ $a + (m + n - 1)d = 0$ $(\because\ m \neq n)$

$\Rightarrow$ $a_{m+n} = 0$

Thus, the (m + n)th term of the A.P. is 0.

**31.** Find the sum of n terms of the series $\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+\ldots.$

**Sol.** We have, $\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+\ldots.$

which forms an A.P where first term $(a)=\left(4-\frac{1}{n}\right)$

Common difference (d) $=\left(4-\frac{2}{n}\right)-\left(4-\frac{1}{n}\right)=-\frac{1}{n}$

and last term $(\ell)=\left(4-\frac{n}{n}\right)=(4-1)=3$ $\quad$ ($\because$ Series has n terms)

$\therefore$ Sum of n terms $(S_n)=\frac{n}{2}(a+\ell)$

$$=\frac{n}{2}\left(4-\frac{1}{n}+3\right)=\frac{n}{2}\left(7-\frac{1}{n}\right)=\frac{7n}{2}-\frac{1}{2}\Rightarrow\left(\frac{7n-1}{2}\right)$$

**32.** If the sum of first 7 terms of an A.P. is 49 and that of its first 17 terms is 289, find the sum of first n terms of the A.P.

**Sol.** Let a be the first term and d be the common difference of the A.P.

Sum of n terms, $S_n=\frac{n}{2}[2a+(n-1)d]$

We have, $S_7 = 49$

$\Rightarrow \frac{7}{2}[2a+6d]=49$

$\Rightarrow$ 14a + 42d = 98

$\Rightarrow$ a + 3d = 7 $\quad$ ... (i)

and $S_{17} = 289$

$\Rightarrow \frac{17}{2}[2a+16d]=289$

$\Rightarrow$ 34a + 272d = 578

$\Rightarrow$ a + 8d = 17 $\quad$ ... (ii)

On solving (i) and (ii), we get a = 1, d = 2

$\therefore \quad S_n=\frac{n}{2}[2+(n-1)2]=n^2$

**33.** If $S_n$ denotes the sum of first n terms of an A.P., then prove that $S_{30} = 3\,[S_{20} - S_{10}]$

**Sol.** Let a be the first term and d be the common difference of the A.P.

Now, sum of n terms, $S_n = \frac{n}{2}\,[2a + (n - 1)d]$

$\therefore\quad S_{30} = \frac{30}{2}\,[2a + (30 - 1)d] = 15\,[2a + 29d] = 30a + 435d$

$\therefore\quad S_{20} = \frac{20}{2}\,[2a + (20 - 1)d] = 10\,[2a + 19d] = 20a + 190d$

$\therefore\quad S_{10} = \frac{10}{2}\,[2a + (10 - 1)d] = 5[2a + 9d] = 10a + 45d$

$\therefore\quad 3[S_{20} - S_{10}] = 3[20a + 190d - 10a - 45d] = 30a + 435d = S_{30}$

**34.** The ratio of the sums of first m and first n terms of an A.P. is $m^2 : n^2$. Show that the ratio of its $m^{th}$ and $n^{th}$ terms is $(2m - 1) : (2n - 1)$.

**Sol.** Lat a be the first term and d be the common difference of the given A.P. Then, the sums of m and n terms are given by

$S_m = \frac{m}{2}[2a+(m-1)d] \text{ and } S_n = \frac{n}{2}[2a+(n-1)d]$

Also, $\frac{S_m}{S_n} = \frac{m^2}{n^2}$ (Given)

$$\Rightarrow \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$$

$$\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$$

$\Rightarrow\ \{2a + (m - 1)d\}\, n = \{2a + (n - 1)d\}m$

$\Rightarrow\ 2a(n - m) = \{d[(n - 1)m - (m - 1)n]\}$

$\Rightarrow\ 2a(n - m) = d(n - m) \Rightarrow d = 2a$

$$\therefore\ \frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{a+(m-1)2a}{a+(n-1)2a} = \frac{2m-1}{2n-1}$$

**35.** The house in row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses proceeding the house numbered X is equal to sum of the numbers of houses following X.

**Sol.** $\underbrace{1,2,3,....,X-1}_{S}, X, \underbrace{X+1,.....,49}_{S'}$

$S = 1 + 2 + 3 + .... + (X - 1)$

$=\left(\frac{X-1}{2}\right)[1+X-1]=\left(\frac{X-1}{2}\right)(X)$

$S' = (X + 1) + (X + 2) +...+ 49$

$=\left(\frac{49-X}{2}\right)(X+1+49)=\left(\frac{49-X}{2}\right)(X+50)$

For S = S', we have

$X^2 - X = 49X + 49 \times 50 - X^2 - 50X$

$\Rightarrow$ $2X^2 = 49 \times 50$

$\Rightarrow$ $X^2 = 49 \times 25$

$\therefore$ $X = 35$

**36.** If the $p^{th}$ term of an A.P. is q and $q^{th}$ term is p, prove that its $n^{th}$ term is $(p + q - n)$.

**Sol.** Let a be the first term and d be the common difference of the given A.P.

Then,

$p^{th}$ term $= q \Rightarrow a + (p - 1) d = q$ ... (i)

$q^{th}$ term $= p \Rightarrow a + (q - 1) d = p$ ... (ii)

Subtract equation (ii) from equation (i), we get

$(p - q) d = (q - p) \Rightarrow d = -1$

Putting $d = -1$ in equation (i), we get

$a + (p - 1) \times (-1) = q \Rightarrow a = (p + q - 1)$

$n^{th}$ term $= a + (n - 1) d$

$= (p + q - 1) + (n - 1) \times (-1)$

$= (p + q - n)$

**37.** In an A.P., the sum of first n terms is $\frac{3n^2}{2}+\frac{5n}{2}$. Find its 25th term.

**Sol.** Let $S_n$ denote the sum of n terms of an A.P. whose nth term is $a_n$.

We have,

$$S_n = \frac{3n^2}{2}+\frac{5n}{2}$$

$$\Rightarrow S_{n-1} = \frac{3}{2}(n-1)^2+\frac{5}{2}(n-1) \qquad \text{[Replacing n by (n – 1)]}$$

$$\therefore S_n - S_{n-1} = \left\{\frac{3n^2}{2}+\frac{5n}{2}\right\}-\left\{\frac{3}{2}(n-1)^2+\frac{5}{2}(n-1)\right\}$$

$$\Rightarrow a_n = \frac{3}{2}\{n^2-(n-1)^2\}+\frac{5}{2}\{n-(n-1)\} \qquad [\because a_n = S_n - S_{n-1}]$$

$$\Rightarrow a_n = \frac{3}{2}(2n-1)+\frac{5}{2}$$

$$\Rightarrow a_{25} = \frac{3}{2}(2\times 25-1)+\frac{5}{2} = \frac{3}{2}\times 49+\frac{5}{2} = 76$$

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**38.** Each year, a tree grows 5 cm less than it did the preceding year. If it grew by 1 m in the first year, then in how many years will it have ceased growing?

**Sol.** Given, that tree grow 5 cm or 0.05 m less than preceding year.

$\therefore$ The following sequence can be formed.

$1, (1 - 0.05), (1 - 2 \times 0.05), \ldots\ldots 0$

i.e. $1, 0.95, 0.90, \ldots, 0$ which is an A.P.

Here, $a = 1$, $d = 0.95 - 1 = -0.05$ and $\ell = 0$

Let $\ell = a_n = a + (n - 1)d$

Then, $0 = 1 + (n - 1)(-0.05)$

$\Rightarrow \quad n - 1 = \dfrac{1}{0.05}$

$\Rightarrow \quad n = \dfrac{100}{5} + 1 = 20 + 1 = 21$

Hence, in 21 yr, tree will have ceased growing.

**39.** The sum of first n terms of three APs are $S_1$, $S_2$ and $S_3$. The first term of each AP is unity and their common difference are 1, 2 and 3, respectively.

Prove that $S_1 + S_3 = 2S_2$.

**Sol.** $S_1$ = Sum of first n terms of an AP whose first term is 1 and common difference is 1.

$\therefore \quad S_1 = \dfrac{n}{2}[2 \times 1 + (n - 1) \times 1]$

$\dfrac{n}{2}(n + 1) \quad \left[\because S_n = \dfrac{n}{2}\{2a + (n-1)d\}\right]$

$S_2$ = Sum of first n terms of an AP whose first term is 1 and common difference is 2

$\therefore \quad S_2 = \dfrac{n}{2}[2 \times 1 + (n-1) \times 2] = n^2$

and $S_3$ = Sum of first n terms of an AP whose first term is 1 and common difference is 3.

$S_3 = \dfrac{n}{2}[2 \times 1 + (n-1)3]$

$= \dfrac{n}{2}(3n - 1)$

Now, $S_1 + S_3 = \dfrac{n}{2}(n+1) + \dfrac{n}{2}(3n - 1)$

$= \dfrac{n}{2}(4n) = 2n^2$

$\Rightarrow \quad S_1 + S_3 = 2S_2 \qquad [\because S_2 = n^2]$

Hence proved.

**40.** How many terms of the AP 20, $19\frac{1}{3}$, $18\frac{2}{3}$, ... must be taken, so that their sum is 300 ?

**Sol.** Given AP is $20, 19\frac{1}{3}, 18\frac{2}{3}, ....$

Here, a = 20 and d = $19\frac{1}{3} - 20$

$= \frac{58}{3} - 20 = \frac{58-60}{3} = \frac{-2}{3}$

Let n terms of given AP be required to get sum 300.

We know that, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 300 = \frac{n}{2}\left[2(20) + (n-1)\left(\frac{-2}{3}\right)\right]$ [$\because$ a = 20 and d = −2/3]

$\Rightarrow 600 = n\left[40 - \frac{2}{3}n + \frac{2}{3}\right]$

$\Rightarrow 600 = \frac{1}{3}[120n - 2n^2 + 2n]$

$\Rightarrow 600 \times 3 = 122n - 2n^2$

$\Rightarrow 1800 + 2n^2 - 122n = 0$

$\Rightarrow 2[n^2 - 61n + 900] = 0$

$\Rightarrow n^2 - 61n + 900 = 0$

$\Rightarrow n^2 - 36n - 25n + 900 = 0$

$\Rightarrow n(n - 36) - 25(n - 36) = 0$

$\Rightarrow (n - 36)(n - 25) = 0$

$\Rightarrow n = 25, 36$

Since, a is positive and d is negative, so both values of n are possible.
Hence, sum of 25 terms of given AP
= Sum of 36 terms of given AP = 300.

**Case Study type questions** **(5 marks)**

**1.** Meena's mother start a new shoe shop. To display the shoes, she put 3 pairs of shoes in 1st row, 5 pairs in 2$^{nd}$ row, 7 pairs in 3$^{rd}$ row and so on.

On the bases of above information, answer the following questions

(i) Find the pairs of shoes in 30$^{th}$ row.

(ii) Find the total number of pairs of shoes in 5th and 8th row.

(iii) Find the number of rows required if she puts a total of 120 pairs of shoes.

**[OR]**

(iii) On next day, she arranges x pairs of shoes in 15 rows. Find the value of x.

**Sol.** (i) Number of pair of shoes in the row forms an AP as follows 3, 5, 7, 9……

Number of pairs in 30th row = 30th term of AP

$A_{30}$ = a + 29d = 3 + 29(2) = 3 + 58 = 61

(ii) Number of pair in 5th row = $A_5$ = a + 4d = 3 + 4(2) = 11

Number of pair in 8th row = $A_8$ = a + 7d = 3 + 7(2) = 17

Total pair in 5th and 8th row = 11 + 17 = 28

(iii) Total pair of shoes = 120

$\Rightarrow$ Let n rows are required

3 + 5 + 7 + …… + n terms = 120

$$\frac{n}{2}\{2(3)+(n-1)(2)\} = 120$$

$$\frac{n}{2}\times2\times3+\frac{n}{2}\times(n-1)\times2 = 120$$

3n + n(n – 1) = 120

$3n + n^2 - n = 120$

$n^2 + 2n - 120 = 0$

$n^2 + 12n - 10n - 120 = 0$

n(n +12) – 10(n + 12) = 0

x = –12, 10

Number of rows = 10

**[OR]**

(iii) number of rows = 15

Total pairs = x

3 + 5 + 7 + …. 15 terms = x

$$\frac{15}{2}\{2(3)+(15-1)(2)\} = x$$

$$\frac{15}{2}\{6+14\times2\} = x$$

$$\frac{15}{2}\{6+28\} = x$$

$$\frac{15}{2}\times34 = x$$

15 × 17 = x

x = 255

**2.** Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of Rs. 1,18,000 by paying every month starting with the first instalment of Rs.1000. If he increases the instalment by 100 every month, answer the following.


(i) Find the amount paid by him in 30th instalment.

(ii) Find the total amount paid by him upto 30 instalments.

(iii) If total instalments are 40, then, find the amount paid in the last instalment.

**[OR]**

(iii) Find the ratio of the 1st instalment to the last instalment, if total instalments are 40.

**Sol.** (i) a = 1000, d = 100, 30th instalment

= 30th term of this AP.

= a + 29d

= 1000 + 29(100)

= 1000 + 2900

= Rs. 3900

(ii) Total amount bill of 30 instalments :

= Sum of first 30 terms

$S_{30} = \frac{30}{2}\{2(1000) + (30-1)(100)\}$

$S_{30} = 15\{2000 + 29 \times 100\}$

= 15{2000 + 2900}

= 15{4900}

= Rs. 73500

(iii) Amount paid in last instalment = 40th term

$A_{40} = a + 39d$

= 1000 + 39(100)

= 1000 + 3900

= Rs. 4900

**[OR]**

(iii) First instalment = a = Rs. 1000

Last instalment = $A_{40}$ = a + 39d = 1000 + 39(100)

1000 + 3900 = 4900

$\frac{a}{A_{40}} = \frac{1000}{4900} \Rightarrow \frac{10}{49} \Rightarrow 10 : 49$

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**3.** Aditya is celebrating his birthday. He invited his friends. He bought a packet of toffees/candies which contains 120 candies. He arranges the candies such that in the first row there are 3 candies, in second there are 5 candies, in third there are 7 candies and so on.

(i) Find the total number of rows of candies.

(ii) Find the difference in number of candies placed in 7th and 3rd row.

(iii) Find number of toffees in last 3rd row.

**[OR]**

(iii) Check if 16 is a term of this AP or not.

**Sol.** (i) There is an AP: 3, 5, 7, ...

$a = 3, d = 2$. Let there be n rows

$$120 = \frac{n}{2}\{2\times3+(n-1)\times2\}$$

$n^2 + 2n - 120 = 0$

solving we get $n = 10$ & $-12$

So, there are 10 rows.

(ii) Candies in 3rd row $= 3 + (2) \times 2 = 7$

Candies in 7th row $= 3 + (6) \times 2 = 15$

Difference $= 15 - 7 = 8$.

(iii) 10th terms of AP $= a + 9d = 3 + 9(2) = 21$

writing AP in reverse 21, 19, 17, ...7, 5, 3

So, for new AP, $a = 21, d = -2$

So, no. of toffees in last 3rd row = 3rd term of new AP

$= a + 2d = 21 + 2(-2) = 21 - 4 = 17$

**[OR]**

(iii) Let 16 is nth term of this AP.

$a_n = 16$

$a + (n - 1)d = 16$

$3 + (n - 1)\,2 = 16$

$(n - 1)2 = 13$

$n - 1 = 6.5$

$n = 7.5$

Since n cannot be a decimal so 16 is not a term of this AP.

# 6 Similar Triangles

**Multiple choice questions** **(1 mark)**

**1.** In $\Delta ABC$ and $\Delta PQR$, we have : $\dfrac{AB}{QR}=\dfrac{BC}{PR}=\dfrac{CA}{PQ}$, then

(1) $\Delta PQR \sim \Delta CAB$ (2) $\Delta PQR \sim \Delta ABC$

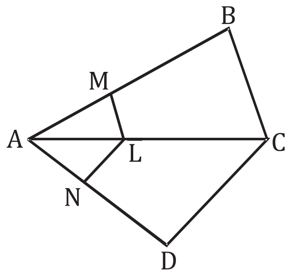
(3) $\Delta CBA \sim \Delta PQR$ (4) $\Delta BCA \sim \Delta PQR$

**Sol. Option (1)**

If $\dfrac{AB}{QR}=\dfrac{BC}{PR}=\dfrac{CA}{PQ}$

then $\Delta PQR \sim \Delta CAB$

**2.** In figure, LM || CB and LN || CD then $\dfrac{AM}{AB}$ equals to



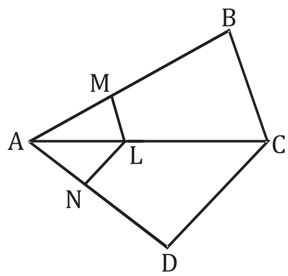

(1) $\dfrac{AL}{LC}$ (2) $\dfrac{AN}{ND}$ (3) $\dfrac{AN}{AD}$ (4) $\dfrac{BC}{AC}$

**Sol. Option (3)**

In $\Delta ABC$,

LM || BC by basic proportionality theorem.

$$\frac{AM}{AB}=\frac{AL}{AC} \quad \text{... (1)}$$




In $\Delta ACD$, LN || CD

by basic proportionality theorem.

$$\frac{AN}{AD}=\frac{AL}{AC} \quad \text{... (2)}$$

From (1) and (2)

$$\frac{AM}{AB}=\frac{AN}{AD}$$

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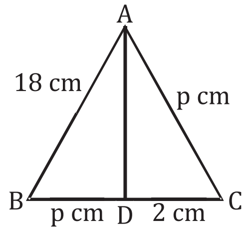
**3.** In the given figure, $\Delta ADB \sim \Delta ADC$ then value of p is –

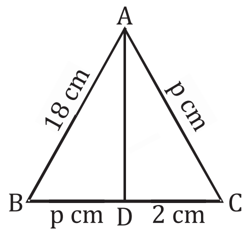



(1) 6 cm (2) 7 cm (3) 8 cm (4) 9 cm

**Sol.** **Option (1)**

$\Delta ADB \sim \Delta ADC$ [Given]

$\frac{AB}{AC} = \frac{BD}{CD}$




$\Rightarrow \frac{18}{p} = \frac{p}{2}$

$\Rightarrow p^2 = 2 \times 18$

$\Rightarrow p^2 = 36$

$\Rightarrow p = 6$ cm

**4.** In $\Delta ABC$, right angled at A, perpendicular is drawn from the point A to BC, meeting BC at D. Then which of the following is true?

(1) $\Delta ADC \sim \Delta ABD$ (2) $\Delta DAC \sim \Delta DBA$

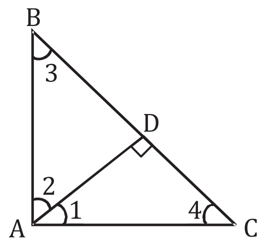
(3) $\Delta DCA \sim \Delta ABD$ (4) $\Delta DAC \sim \Delta ABD$

**Sol.** **Option (2)**

$\angle 1 + \angle 2 = 90°$ ... (1)

$\angle 1 + \angle 4 = 90°$ ... (2)

$\angle 2 + \angle 3 = 90°$ ... (3)




From (1) and (2)

$\angle 2 = \angle 4$

From (1) and (3)

$\angle 1 = \angle 3$

In $\Delta ADC$ and $\Delta BAD$

$\angle 1 = \angle 3$

$\angle 4 = \angle 2$ (Proved above)

$\Delta DAC \sim \Delta DBA$ [by AA Similarity]

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**5.** The areas of two similar triangles are 81 $cm^2$ and 49 $cm^2$ respectively, then the ratio of their corresponding medians is

(1) $\frac{7}{9}$ (2) $\frac{5}{9}$ (3) $\frac{9}{5}$ (4) $\frac{9}{7}$

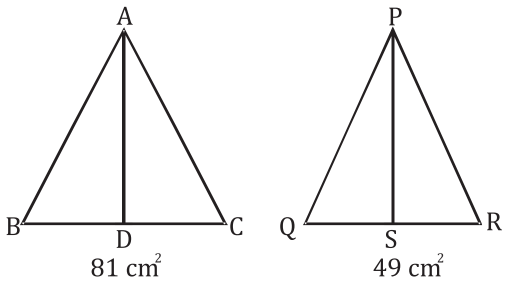
**Sol. Option (4)**

Ratio of area of two similar Δs is equal to ratio of squares of their corresponding medians. Therefore,

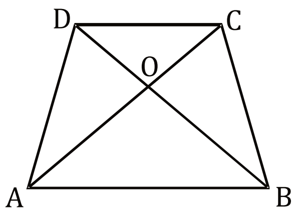
$$\Rightarrow \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{AD^2}{PS^2}$$

$$\Rightarrow \frac{81}{49} = \frac{AD^2}{PS^2}$$

$$\Rightarrow \frac{AD}{PS} = \sqrt{\frac{81}{49}} = \frac{9}{7}$$




**6.** In fig. ABCD is a trapezium, whose diagonals AC and BD intersect at O, such that OA = (3x – 1) cm, OB = (2x + 1) cm, OC = (5x – 3) cm and OD = (6x – 5) cm, then x is –




(1) 2 cm (2) 3 cm (3) 2.5 cm (4) 4 cm

**Sol. Option (1)**

In ΔDOC and ΔBOA

$\angle CDO = \angle ABO$ [Alternate angles]

$\angle DOC = \angle BOA$ [Vertically opposite angles]

$\therefore$ $\Delta DOC \sim \Delta BOA$ [By AA similarity]

$$\Rightarrow \frac{DO}{OB} = \frac{OC}{OA}$$

$$\Rightarrow \frac{OD}{OC} = \frac{OB}{OA}$$

$$\frac{6x-5}{5x-3} = \frac{2x+1}{3x-1}$$

$(6x – 5)(3x –1) = (2x + 1)(5x – 3)$

$18x^2 – 6x – 15x + 5 = 10x^2 – 6x + 5x – 3$

$18x^2 – 10x^2 – 6x – 15x + 5 + 6x – 5x + 3 = 0$

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$8x^2 - 20x + 8 = 0$

$2x^2 - 5x + 2 = 0$

$2x^2 - 4x - x + 2 = 0$

$2x(x - 2) - 1(x - 2) = 0$

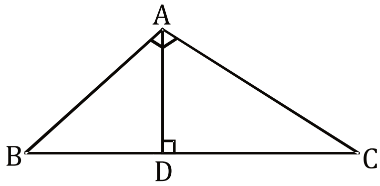
$(2x - 1)(x - 2) = 0$

$x = \frac{1}{2}, 2$

$x = \frac{1}{2}$ is not possible because sides can't be negative.

$\therefore \quad x = 2$

**7.** In the given figure, $\angle BAC = 90°$ and $AD \perp BC$ then, which of the following relation is correct?




(1) $BC \times CD = BC^2$ (2) $AB \times AC = BC^2$

(3) $BD \times CD = AD^2$ (4) $AB \times AC = AD^2$

**Sol. Option (3)**

$\angle C = 90° - \angle DAC$

$\angle C = \angle BAD$ [$\because$ $90° - \angle DAC = \angle BAD$]

In $\Delta ADC$ and $\Delta BAD$

$\angle ADC = \angle ADB$ [Each 90°]

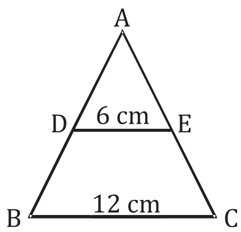
$\angle C = \angle BAD$

$\Delta ACD \sim \Delta BAD$ [by AA Similarity]

$\frac{AD}{BD} = \frac{CD}{AD}$

$\Rightarrow AD^2 = BD \times CD$

**8.** In the given figure, DE = 6 cm, BC = 12 cm, if DE || BC, find the ratio of ar($\Delta ADE$) and ar(DECB).




(1) 1 : 3 (2) 1 : 2 (3) 2 : 5 (4) 3 : 5

**Sol. Option (1)**

Given, DE || BC, DE = 6 cm and BC = 12 cm

In $\Delta ABC$ and $\Delta ADE$,

$\angle ABC = \angle ADE$ [Corresponding angles]

$\angle ACB = \angle AED$ [Corresponding angles]

and $\angle A = \angle A$ [Common angle]

$\therefore$ $\Delta ABC \sim \Delta ADE$ [by AAA similarity criterion]

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

$$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{DE^2}{BC^2} = \frac{6^2}{12^2} = \left(\frac{1}{2}\right)^2$$

$$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Let ar($\Delta ADE$) = k, then ar($\Delta ABC$) = 4k

Now, ar(DECB) = ar($\Delta ABC$) – ar($\Delta ADE$)

= 4k – k = 3k

$\therefore$ Required ratio = ar($\Delta ADE$) : ar(DECB) = k : 3k = 1 : 3

**Assertion reason questions** **(1 marks)**

**9. Assertion (A):** If two sides of a right angled triangle are 7 cm and 8 cm, then its third side will be 9 cm.

**Reason (R):** In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol. Option (4)**

$\because$ $7^2 + 8^2 = 49 + 64 = 113 \neq 9^2$

Assertion (A) is false but Reason (R) is true.

**10.** **Assertion (A):** If $\Delta ABC$ and $\Delta PQR$ are congruent triangles, then they are also similar triangles.

**Reason (R):** All congruent triangles are similar but the similar triangles need not be congruent.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

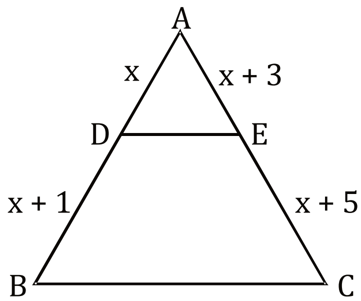
(4) Assertion (A) is false but Reason (R) is true.

**Sol.** **Option (1)**

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

**Very short answer type questions** **(2 mark)**

**11.** In $\Delta ABC$, $DE \parallel BC$, find the value of x.




**Sol.** As $DE \parallel BC$

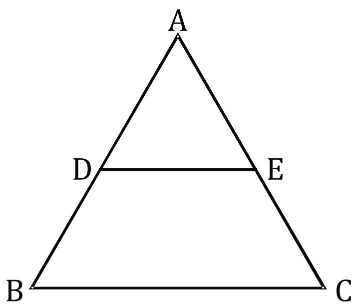
$\therefore \quad \dfrac{AD}{DB} = \dfrac{AE}{EC}$

$\Rightarrow \quad \dfrac{x}{x+1} = \dfrac{x+3}{x+5}$

$\Rightarrow \quad x^2 + 5x = x^2 + 4x + 3$

$\Rightarrow \quad x = 3$

**12.** In given figure, $DE \parallel BC$. If AD = 3 cm, DB = 4 cm and AE = 6 cm, then find EC.

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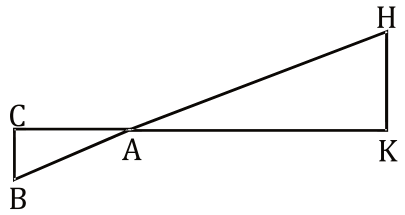
**Sol.** Since DE ∥ BC

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \quad \frac{3}{4} = \frac{6}{EC}$

$\therefore \quad EC = 8$ cm

**13.** In the given figure, ΔAHK is similar to ΔABC, If AK = 10 cm, BC = 3.5 cm and HK = 7 cm, find AC.



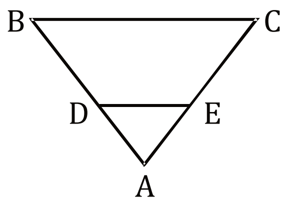

**Sol.** Since, ΔAHK ~ ΔABC.

Corresponding sides of similar triangles are in proportion.

i.e, $\frac{AK}{AC} = \frac{HK}{BC} \Rightarrow \frac{10}{AC} = \frac{7}{3.5}$

$\Rightarrow \quad AC = 5$ cm

**14.** In the given figure DE ∥ BC and in ΔABC such that BC = 8 cm. AB = 6 cm and DA = 1.5 cm. Find DE.




**Sol.** In ΔABC and ΔADE, we have

$\angle ABC = \angle ADE$ {Corresponding angles}

$\angle BAC = \angle DAE$ {Common}

$\therefore \quad \Delta ABC \sim \Delta ADE$ {by AA similarity Criterion}

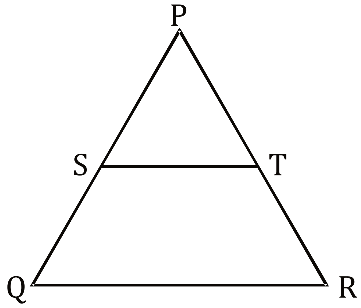
As corresponding sides of similar triangles are proportional.

$\therefore \quad \frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$

Now, $\frac{AB}{AD} = \frac{BC}{DE} \Rightarrow \frac{6}{1.5} = \frac{8}{DE}$

$\Rightarrow \quad DE = \frac{8 \times 1.5}{6} \Rightarrow DE = 2\text{cm}$

**15.** In the given figure, S and T are points on the sides PQ and PR, respectively of ΔPQR, such that PT = 2 cm, TR = 4 cm and ST is parallel to QR. Find the ratio of the areas of ΔPST and ΔPQR.




**Sol.** In ΔPST and ΔPQR

$\angle SPT = \angle QPR$ (Common)

$\angle PST = \angle PQR$ (Corresponding angles)

$\therefore$ $\Delta PST \sim \Delta PQR$ (By AA similarity criterion)

It is known that the ratio of areas of similar triangles is equal to the ratio of the squares of their corresponding sides.

$$\therefore \quad \frac{\text{Area of } \Delta PST}{\text{Area of } \Delta PQR} = \frac{(PT)^2}{(PR)^2} = \frac{(PT)^2}{(PT+TR)^2}$$

$$= \frac{(2)^2}{(2+4)^2} = \frac{4}{36} = \frac{1}{9}$$

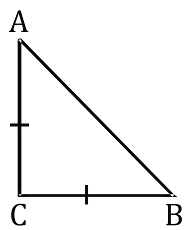
Thus, the ratio of the areas of ΔPST and ΔPQR is 1 : 9.

**Short answer type questions** **(3 marks)**

**16.** ABC is an isosceles triangle right-angled at C. Prove that $AB^2 = 2AC^2$.

**Sol.** Given : ΔABC is right-angled at C.

To prove : $AB^2 = 2AC^2$




Proof : In ΔACB,

$\therefore$ $AB^2 = AC^2 + BC^2$ [By pythagoras theorem]

$\Rightarrow$ $AB^2 = AC^2 + AC^2$ [$\because$ AC = BC]

$\Rightarrow$ $AB^2 = 2AC^2$

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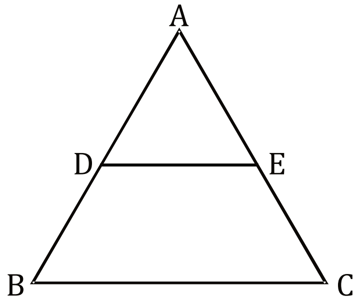
**17.** In figure below, DE || BC if AB = 7.6 cm, AD = 1.9 cm, then find AE : EC.




**Sol.** DB = AB – AD = 7.6 – 1.9

DB = 5.7 cm

As DE || BC

$\therefore$ By B.P.T

$$\frac{AD}{DB} = \frac{AE}{EC}$$

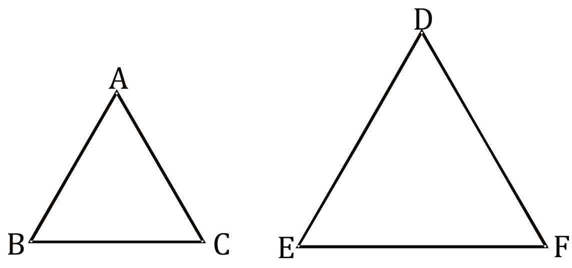
$$\Rightarrow \frac{1.9}{5.7} = \frac{AE}{EC}$$

$$\Rightarrow \frac{AE}{EC} = \frac{1}{3}$$

**18.** Let $\Delta ABC \sim \Delta DEF$ and their areas be respectively 64 cm$^2$ and 121 cm$^2$. If EF = 15.4 cm, find BC.

**Sol.** It is known that the ratio of areas of similar triangles is equal to the ratio of the squares of their corresponding sides.

We have, $\dfrac{\text{area of }\Delta ABC}{\text{area of }\Delta DEF} = \dfrac{BC^2}{EF^2}$




(as $\Delta ABC \sim \Delta DEF$)

$$\Rightarrow \frac{64}{121} = \frac{BC^2}{EF^2} \Rightarrow \frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

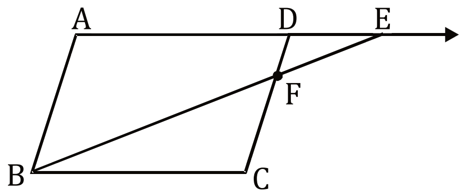
$$\Rightarrow \frac{BC}{15.4} = \frac{8}{11}$$

$$\therefore \quad BC = \frac{8}{11} \times 15.4 = 11.2\text{cm}$$

**19.** E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$.

**Sol.** Given : E is a point on the side AD produced of parallelogram ABCD and BE intersects CD at F.

To prove : $\Delta ABE \sim \Delta CFB$




Proof : In $\Delta ABE$ and $\Delta CFB$,

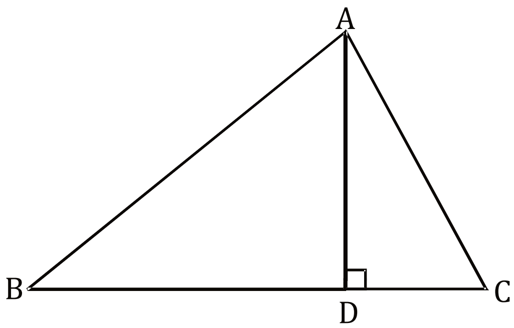
$\angle BAE = \angle FCB$ (Since opposite angles of a parallelogram are equal)

$\angle AEB = \angle CBF$ (Alternate interior angles are equal as AE ∥ BC)

$\therefore$ $\Delta ABE \sim \Delta CFB$ (By AA similarity criterion)

**20.** In $\Delta ABC$, $AD \perp BC$, such that $AD^2 = BD \times CD$.

Prove that $\Delta ABC$ is right angled at A.




**Sol.** Given : A $\Delta ABC$ with $AD \perp BC$ and $AD^2 = BD \times CD$

To prove : $\Delta ABC$ is right angled at A

Proof : $AD^2 = BD \times CD$ (Given)

$\Rightarrow \quad \frac{AD}{CD} = \frac{BD}{AD}$

$\angle ADB = \angle ADC = 90°$

$\therefore \quad \Delta ADC \sim \Delta BDA$ (by SAS)

$\Rightarrow \quad \angle BAD = \angle ACD$ ; $\angle DAC = \angle DBA$ (Corresponding angles of similar triangles)

$\angle BAD + \angle ACD + \angle DAC + \angle DBA = 180°$

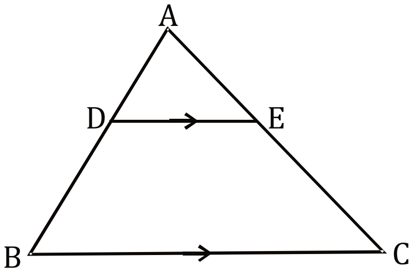
$\Rightarrow \quad 2\angle BAD + 2\angle DAC = 180°$

$\Rightarrow \quad \angle BAD + \angle DAC = 90°$

$\therefore \quad \angle A = 90°$

Thus, $\Delta ABC$ is right angled at A.

**21.** In the given figure, DE || BC. If AD = 1.5 cm, BD = 2AD, then find $\frac{ar(\Delta ADE)}{ar(trapezium\,BCED)}$.




**Sol.** Given : AD = 1.5 cm, $\therefore$ BD = 3 cm and

AB = AD + BD = 1.5 + 3.0 = 4.5 cm.

Given, In triangle ADE and ABC, $\angle A$ is common and DE || BC

$\Rightarrow$ $\angle ADE = \angle ABC$ (Corresponding angles)

$\therefore$ $\Delta ADE \sim \Delta ABC$, (AA similarity)

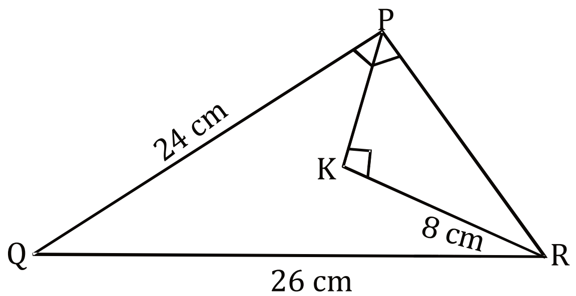
It is known that the ratio of areas of similar triangles is equal to the ratio of the squares of their corresponding sides.

$$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{AD^2}{AB^2} = \frac{(1.5)^2}{(4.5)^2} = \frac{1}{9}$$

$$\Rightarrow \frac{ar(\Delta ADE)}{ar(\Delta ABC) - ar(\Delta ADE)} = \frac{1}{9-1}$$

$$\Rightarrow \frac{ar(\Delta ADE)}{ar(trapezium\,BCED)} = \frac{1}{8}$$

**22.** In the given triangle PQR, $\angle QPR = 90°$, PQ = 24 cm and QR = 26 cm and in $\Delta PKR$, $\angle PKR = 90°$ and KR = 8 cm, find PK.




**Sol.** According to the question, (Given)

$\therefore$ $\angle QPR = 90°$

$QR^2 = QP^2 + PR^2$ [By Pythagoras theorem]

$\therefore$ PR = $\sqrt{26^2 - 24^2} = \sqrt{100} = 10\text{cm}$ [By pythagoras theorem]

$\angle PKR = 90°$ (Given)

$\therefore$ $PK = \sqrt{10^2 - 8^2} = \sqrt{100 - 64}$ [By pythagoras theroem]

$= \sqrt{36} = 6\text{ cm}$

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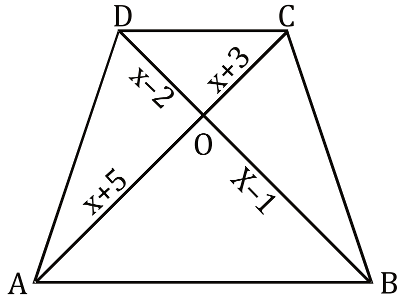
**23.** In the given figure, if AB || DC, find the value of x.




**Sol.** In $\Delta AOB$ & $\Delta DOC$

$\angle ODC = \angle OBA$ [$\because$ AB || DC, alternate interior angles]

$\angle AOB = \angle DOC$ [Vertically opposite angles]

$\Delta AOB \sim \Delta COD$ [By AA similarity]

$\frac{OA}{OC} = \frac{BO}{OD}$

$\Rightarrow \frac{x+5}{x+3} = \frac{x-1}{x-2}$

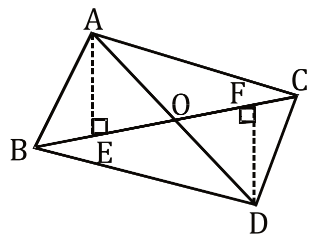
$\Rightarrow (x + 5)(x - 2) = (x - 1)(x + 3)$

$\Rightarrow x^2 - 2x + 5x - 10 = x^2 + 3x - x - 3$

$\Rightarrow 3x - 2x = 10 - 3$

$\therefore x = 7$

**24.** In the figure given below, ABC and DBC are two triangles on the same base BC. If AD intersect BC at O then show that : $\frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$




**Sol.** Given : ABC and DBC are two triangles on same base BC.

To prove : $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Construction : Draw AE and DF perpendicular to BC

Proof : In $\Delta AEO$ and DFO

$\angle AEO = \angle DFO$ (Each 90°)

$\angle AOE = \angle FOD$ (Vertically opposite $\angle$s)

$\therefore \Delta AEO \sim \Delta DFO$ (AA similarity)

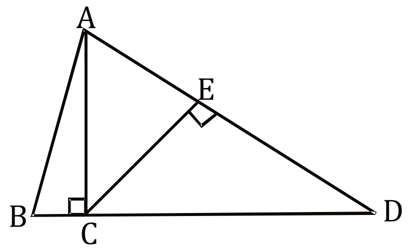
$\therefore \quad \frac{AE}{DF} = \frac{AO}{DO}$ ... (1)

$$\frac{Ar(\Delta ABC)}{Ar(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AE}{\frac{1}{2} \times BC \times DF} = \frac{AE}{DF}$$

$\frac{Ar(\Delta ABC)}{Ar(\Delta DBC)} = \frac{AO}{DO}$ [From (1)]

Hence proved.

**25.** In the given figure, $AC \perp BD$ and $CE \perp AD$. Prove that $AC^2 = DA.AE$




**Sol.** Given : In $\Delta ABC$

$AC \perp BD$ and $CE \perp AD$

To prove : $AC^2 = DA.AE$

Proof : In $\Delta AEC$ & $\Delta ACD$

$\angle AEC = \angle ACD$ [Each 90°]

$\angle EAC = \angle DAC$ [Common]

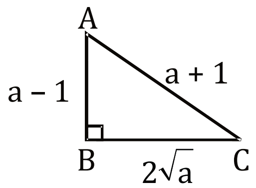
$\Delta AEC \sim \Delta ACD$ (By AA similarity)

$\therefore \quad \frac{AC}{AD} = \frac{AE}{AC}$

$\Rightarrow \quad AC^2 = AD.AE$ Hence proved

**26.** Determine whether the triangle having sides (a – 1) cm, $2\sqrt{a}$ cm and (a + 1) cm is a right angled triangle.

**Sol.**




$= AB^2 + BC^2$

$= (a-1)^2 + (2\sqrt{a})^2$

$= a^2 - 2a + 1 + 4a$

$= (a^2 + 2a + 1)$

$= (a+1)^2 = AC^2$

By converse of Pythogoras theorem, $\Delta ABC$ is a right angle triangle.

Hence (a – 1) cm, $2\sqrt{a}$ cm and (a + 1) cm are sides of right angled triangle.

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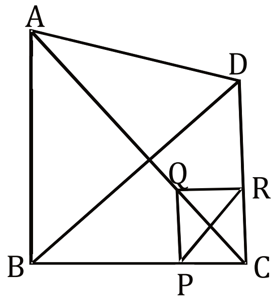
**27.** In the given figure, two triangles ABC and DBC lie on the same side of base BC. P is a point on BC such that PQ || BA and PR || BD. Prove that QR || AD.




**Sol.** Given : Two triangle ABC and DBC lie on the same side of base BC. P is a point on BC such that PQ || BA and PR || BD

To prove : QR || AD

Proof : In ΔABC, PQ || BA

∴ By basic proportionality theorem, we get

$$\frac{CP}{PB}=\frac{CQ}{QA} \quad \text{... (i)}$$

In ΔBDC, RP || DB

∴ By basic proportionality theorem, we get

$$\frac{CP}{PB}=\frac{CR}{RD} \quad \text{... (ii)}$$

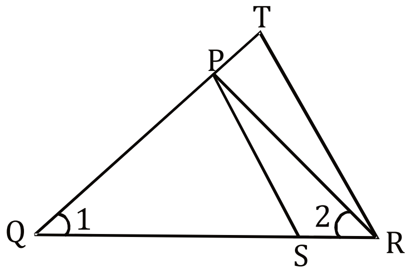
On comparing (i) and (ii), we get

$$\frac{CQ}{QA}=\frac{CR}{RD}$$

In ΔACD, $\frac{CQ}{QA}=\frac{CR}{RD}$

Therefore, by converse of basic proportionality theorem, QR || AD. Hence proved.

**28.** In figure if $\frac{QT}{PR}=\frac{QR}{QS}$ and ∠1 = ∠2. Prove that ΔPQS ~ ΔTQR.

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**Sol.** Given : $\frac{QT}{PR}=\frac{QR}{QS}$ and $\angle 1 = \angle 2$

To prove : $\Delta PQS \sim \Delta TQR$

Proof : $\frac{QT}{PR}=\frac{QR}{QS}$ [Given]

$\Rightarrow \frac{QT}{QR}=\frac{PR}{QS}$ ... (i)

We also have,

$\angle 1 = \angle 2$

$\Rightarrow$ PR = PQ ... (ii)

[Sides opposite to equal angles are equal]

From (i) and (ii), we get

$\frac{QT}{QR}=\frac{PQ}{QS} \Rightarrow \frac{PQ}{QT}=\frac{QS}{QR}$

Thus, in triangle PQS and TQR, we have

$\frac{PQ}{QT}=\frac{QS}{QR}$ and $\angle PQS = \angle TQR = \angle Q$

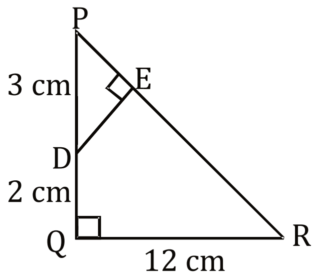
So, by SAS-criterion of similarity, we have

$\Delta PQS \sim \Delta TQR$ Hence proved

**Long answer type questions** **(5 marks)**

**29.** In the given figure, $\Delta PQR$ is right angled triangle right angled at Q. DE $\perp$ PR.

Prove $\Delta PQR \sim \Delta PED$ and find the lengths of PE and DE if PD = 3 cm, QD = 2 cm and QR = 12 cm.




**Sol.** In $\Delta PQR$, by pythagoras theorem

$PR^2 = PQ^2 + QR^2 = 5^2 + 12^2$

PR = 13 cm

In $\Delta PQR$ & $\Delta PED$

$\angle QPR = \angle DPE$ [Common]

$\angle PQR = \angle PED$ [Each 90°]

$\therefore$ $\Delta PQR \sim \Delta PED$ [By AA similarity]

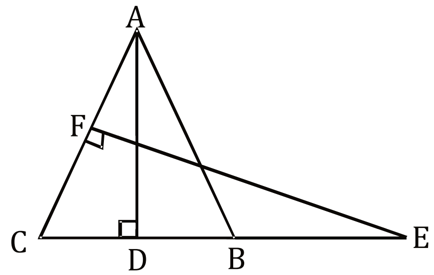
$\therefore \quad \frac{PQ}{PE}=\frac{QR}{ED}=\frac{PR}{PD}$

$\frac{5}{PE}=\frac{12}{ED}=\frac{13}{3}$

$\therefore \quad PE=\frac{15}{13}=1.15\text{ cm}\ \&\ ED=\frac{36}{13}=2.76\text{ cm}$

**30.** In figure, ABC is an isosceles triangle in which AB = AC. E is a point on the side CB produced, such that FE $\perp$ AC. If AD $\perp$ CB

Prove that : AB × EF = AD × EC.




**Sol.** Given : ABC is isosceles $\Delta$ with AB = AC

To prove : AB × EF = AD × EC

Proof : Given that AB = AC

$\therefore \quad \angle ACB = \angle ABC$ .... (i)

[$\because$ angles opposite to equal sides of a triangle are equal]

In $\Delta$ADB and $\Delta$EFC

$\angle ADB = \angle EFC$ [Each 90°]

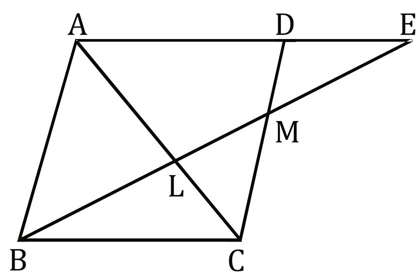
$\angle ABD = \angle ECF$ [using (i)]

$\Delta ADB \sim \Delta EFC$ [By AA similarity criterion]

$\therefore \quad \frac{AB}{EC}=\frac{AD}{EF}$

$\Rightarrow$ AB × EF = AD × EC Hence proved

**31.** In the given figure, M is mid-point of side CD of a parallelogram ABCD. The line BM is drawn intersecting AC at L and AD produced at E. Prove that EL = 2BL.



**Sol.** Given : M is mid point of CD of parallelogram ABCD. Line BM is produced intersecting AC at L and AD produced at E.

To prove : EL = 2BL

In $\Delta$BCM and $\Delta$EDM, we have

CM = DM [$\therefore$ M is the mid-point of CD]

$\angle$BMC = $\angle$EMD [Vertically opposite angles]

$\angle$BCM = $\angle$EDM [Alternate interior angles as AE $\parallel$ BC]

$\therefore$ $\Delta BCM \cong \Delta EDM$ [By ASA congruence criterion]

$\Rightarrow$ BC = ED [By CPCT]

Now, BC = AD = ED ..... (i) [Opposite sides of parallelogram]

In $\Delta$BLC and $\Delta$ELA

$\angle$LBC = $\angle$LEA [Alternate interior angles as AE || BC]

$\angle$BLC = $\angle$ELA [Vertically opposite angles]

$\therefore$ $\Delta BLC \sim \Delta ELA$ [By AA similarity criterion]

Since, sides of similar triangle are proportional.

$\therefore \frac{BL}{EL} = \frac{BC}{AE}$

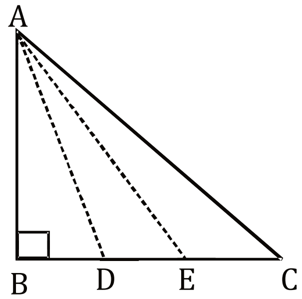
$\Rightarrow \frac{BL}{EL} = \frac{BC}{AD + DE}$ [$\because$ AE = AD + DE]

$\Rightarrow \frac{BL}{EL} = \frac{DE}{DE + DE}$ [By (i)]

$\Rightarrow \frac{BL}{EL} = \frac{DE}{2DE} \Rightarrow \frac{BL}{EL} = \frac{1}{2}$

$\Rightarrow$ EL = 2BL Hence proved.

**32.** In the given figure, a triangle ABC is right angled at B. Side BC is trisected at points D and E. Prove that 8 $AE^2 = 3AC^2 + 5AD^2$



**Sol.** Given : ABC is right angled at B and BD = DE = EC

To prove : $8AE^2 = 3AC^2 + 5\ AD^2$

Proof : Since D and E are the points of trisection of BC.

$\therefore$ BD = DE = CE

Let BD = DE = CE = x

$\therefore$ BE = 2x and BC = 3x.

Now, In $\Delta$ABD, $\Delta$ABE and $\Delta$ABC

$AD^2 = AB^2 + BD^2 = AB^2 + x^2$ .... (i)

$AE^2 = AB^2 + BE^2 = AB^2 + 4x^2$ ... (ii)

$AC^2 = AB^2 + BC^2 = AB^2 + 9x^2$ .... (iii)

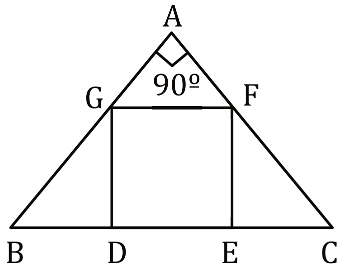
Now, $8AE^2 - 3AC^2 - 5AD^2$

$= 8\ (AB^2 + 4x^2) - 3(AB^2 + 9x^2) - 5\ (AB^2 + x^2) = 0$

Hence, $8AE^2 = 3AC^2 + 5AD^2$

Hence proved.

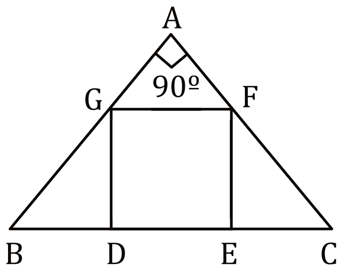
**33.** In figure, DEFG is a square and $\angle BAC = 90°$. Prove that




(i) $\Delta AGF \sim \Delta DBG$ (ii) $\Delta AGF \sim \Delta EFC$

(iii) $\Delta DBG \sim \Delta EFC$ (iv) $DE^2 = BD \times EC$

**Sol.**




Given : DEFG is a square & $\angle A = 90°$

To prove :

(i) $\Delta AGF \sim \Delta DBG$ (ii) $\Delta AGF \sim \Delta EFC$

(iii) $\Delta DBG \sim \Delta EFC$ (iv) $DE^2 = BD \times EC$



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Proof :

(i) In $\Delta AGF$ & $\Delta DBG$

$\angle GAF = \Delta GDB$ [Each 90°]

$\angle AGF = \angle GBD$ [Corresponding Angles]

$\therefore$ $\Delta AGF \sim \Delta DBG$ (By AA similarity)

(ii) In $\Delta AGF$ & $\Delta EFC$

$\angle GAF = \angle FEC$ [Each 90°]

$\angle AFG = \angle FCE$ [Corresponding Angles]

$\therefore$ $\Delta AGF \sim \Delta EFC$ (By AA similarity)

(iii) From (i) & (ii)

As $\Delta AGF \sim \Delta DBG$

and $\Delta AGF \sim \Delta EFC$

$\therefore$ $\Delta DBG \sim \Delta EFC$

(iv) From (iii)

Corresponding side of similar triangles are proportional.

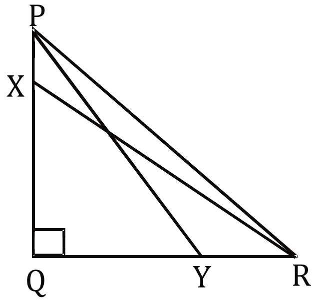
$$\frac{DB}{EF} = \frac{DG}{EC}$$

$BD \times EC = DG \times EF$

$\therefore$ $DE^2 = BD \times EC$ [$\therefore$ DEFG is a square]

Hence proved.

**34.** In given figure, PQR is a right triangle right angled at Q. X and Y are the points on PQ and QR such that PX : XQ = 1 : 2 and QY : YR = 2 : 1. Prove that $9(PY^2 + XR^2) = 13\ PR^2$.




**Sol.** Given : In $\Delta PQR$, $\angle Q = 90°$

PX : XQ = 1 : 2

QY : YR = 2 : 1

To prove : $9(PY^2 + XR^2) = 13PR^2$

Proof : LHS = $9(PY^2 + XR^2)$

$= 9[PQ^2 + QY^2 + XQ^2 + QR^2]$

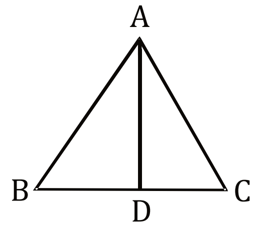
$$= 9\left[PR^2 + \left(\frac{2}{3}QR\right)^2 + \left(\frac{2}{3}PQ\right)^2\right]$$

$$= 9\left[\frac{9PR^2 + 4PR^2}{9}\right]$$

$= 13\ PR^2$

= RHS Hence proved

**35.** In $\Delta ABC$, $AD \perp BC$ and point D lies on BC such that 2 DB = 3 CD. Prove that $5AB^2 = 5AC^2 + BC^2$.




**Sol.** Given : $AD \perp BC$

$2DB = 3CD$

To prove : $5AB^2 = 5AC^2 + BC^2$

Proof : $2DB = 3CD$

$\Rightarrow \dfrac{DB}{CD} = \dfrac{3}{2}$

$\Rightarrow DB = 3x, CD = 2x$ and $BC = 5x$

In $\Delta ADB$, $\angle D = 90°$

$AB^2 = AD^2 + DB^2$

$\Rightarrow AB^2 = AD^2 + (3x)^2$

$\Rightarrow AB^2 = AD^2 + 9x^2$

$\Rightarrow 5AB^2 = 5AD^2 + 45x^2$

$\Rightarrow 5AD^2 = 5AB^2 - 45x^2$ ... (i)

and $AC^2 = AD^2 + CD^2$

$\Rightarrow AC^2 = AD^2 + (2x)^2$

$\Rightarrow AC^2 = AD^2 + 4x^2$

$\Rightarrow 5AC^2 = 5AD^2 + 20x^2$

$\Rightarrow 5AD^2 = 5AC^2 - 20x^2$ ... (ii)

On comparing eq.(i) and eq.(ii),

$5AB^2 - 45x^2 = 5AC^2 - 20x^2$

$\Rightarrow 5AB^2 = 5AC^2 - 20x^2 + 45x^2$

$\Rightarrow 5AB^2 = 5AC^2 + 25x^2$

$\Rightarrow 5AB^2 = 5AC^2 + (5x)^2$

$\therefore 5AB^2 = 5AC^2 + BC^2 [BC = 5x]$ Hence proved

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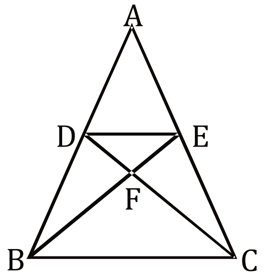
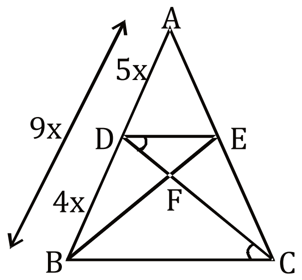
**36.** In the given figure, if DE || BC and AD : DB = 5 : 4, then find $\frac{ar(\Delta DFE)}{ar(\Delta CFB)}$




**Sol.**




In $\Delta ABC$,

DE || BC

So, $\angle ADE = \angle ABC$ [Corresponding angles]

and $\angle DAE = \angle BAC$ [Common]

$\Rightarrow \Delta ADE \sim \Delta ABC$ [By AA similarity]

$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$ [CPCT]

$\Rightarrow \frac{5x}{9x} = \frac{DE}{BC}$

$\Rightarrow \frac{DE}{BC} = \frac{5}{9}$ ... (i)

Now, in $\Delta DFE$ and $\Delta CFB$

$\angle EDF = \angle BCF$ [Alternate angles]

$\angle DFE = \angle CFB$ [Vertically opposite angles]

$\Rightarrow \Delta DFE \sim \Delta CFB$ [By AA similarity]

$\Rightarrow \frac{ar(DFE)}{ar(CFB)} = \left(\frac{DE}{BC}\right)^2$

$= \left(\frac{5}{9}\right)^2 = \frac{25}{81}$

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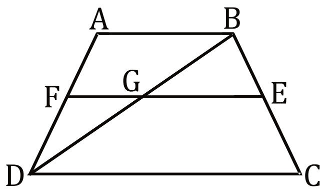
**37.** In trapezium ABCD, AB || DC and DC = 2AB. EF || AB, where E and F lie on BC and AD respectively, such that $\frac{BE}{EC} = \frac{3}{4}$. Diagonal DB intersects EF at G. Prove that 7EF = 10AB.

**Sol.**




Given : ABCD is a trapezium with AB || CD and CD = 2 AB and EF || AB ||CD with $\frac{BE}{EC} = \frac{3}{4}$ and EF intersect BD at G.

To prove : 7EF = 10 AB

Proof : In $\Delta$BCD, GE || CD

$\angle BGE = \angle BDC$ (corresponding angles)

$\angle GBE = \angle DBC$ (common)

$\therefore \quad \Delta BGE \sim \Delta BDC$ (By AA)

$$\therefore \quad \frac{BE}{BC} = \frac{BG}{BD} = \frac{GE}{CD} = \frac{3}{7}$$

$$\Rightarrow \quad \frac{GE}{2AB} = \frac{3}{7}$$

$$\Rightarrow \quad GE = \frac{6}{7}AB$$

Similarly $\Delta DFG \sim \Delta DAB$

$$\therefore \quad \frac{DG}{DB} = \frac{FG}{AB} = \frac{4}{7}$$

$$\Rightarrow \quad FG = \frac{4}{7}AB$$

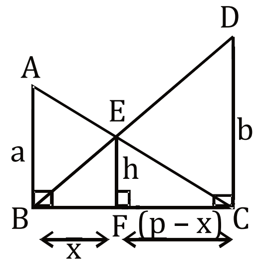
So, EF = EG + FG

$$= \frac{6}{7}AB + \frac{4}{7}AB = \frac{10}{7}AB$$

Hence 7EF = 10AB

**38.** Two poles of heights a m and b m are standing vertically on a level ground, p metre apart. Prove that the height of the point of intersection of the line segments joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ m

**Sol.** Let the height of the two poles be 'a' and 'b' meters respectively and let they be 'p' meters apart.




Height of the pole AB = a meter

Height of the pole CD = b meter

Distance between the poles = p meters

Let the point of intersection of lines joining the top of the poles be 'E' and its height be 'h' meters. Suppose BF = x meter.

Draw EF $\perp$ BC

In $\Delta ABC$ and $\Delta EFC$,

$\angle ACB = \angle ECF$ (Common)

$\angle ABC = \angle EFC$ (90°)

$\therefore$ $\Delta ABC \sim \Delta EFC$ (By AA similarity)

$\Rightarrow \dfrac{AB}{EF} = \dfrac{AC}{EC} = \dfrac{BC}{FC}$ (Corresponding sides are proportional)

$\Rightarrow \dfrac{a}{h} = \dfrac{p}{(p-x)}$

$\Rightarrow$ ap – ax = ph ... (1)

Similarly, $\Delta DCB \sim \Delta EFB$

$\Rightarrow \dfrac{b}{h} = \dfrac{p}{x}$

$\Rightarrow x = \dfrac{ph}{b}$ ... (2)

From (1) and (2), we get

$$ap - \frac{aph}{b} = ph$$

$$a - \frac{ah}{b} = h$$

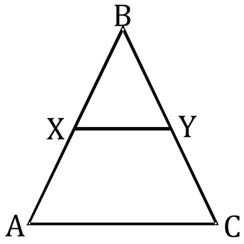
$$\Rightarrow a = h\left(1 + \frac{a}{b}\right)$$

$$\Rightarrow h\left(\frac{a+b}{b}\right) = a$$

$$\Rightarrow h = \frac{ab}{a+b}$$

Hence proved.

**39.** In the given figure, in $\Delta ABC$, $XY \parallel AC$ and XY divides the $\Delta ABC$ into two regions such that ar $(\Delta BXY) = 2$ ar $(ACYX)$. Determine $\frac{AX}{AB}$.




**Sol.** In $\Delta BXY$ and $\Delta ABC$

$\angle BXY = \angle BAC$ (Corresponding angles)

$\angle XBY = \angle ABC$

$\Delta BXY \sim \Delta BAC$ (By AA similarity)

$ar(BXY) = 2ar(ACXY)$ (Given)

Also, $ar(BXY) + ar(ACXY) = ar(ABC)$

$$\frac{3}{2}\, ar(BXY) = ar(ABC)$$

$$\frac{ar(ABC)}{ar(BXY)} = \frac{3}{2} = \left(\frac{AB}{BX}\right)^2$$

$$\frac{BX}{AB} = \sqrt{\frac{2}{3}}$$

Multiply both side by $-1$

$$-\frac{BX}{AB} = -\sqrt{\frac{2}{3}}$$

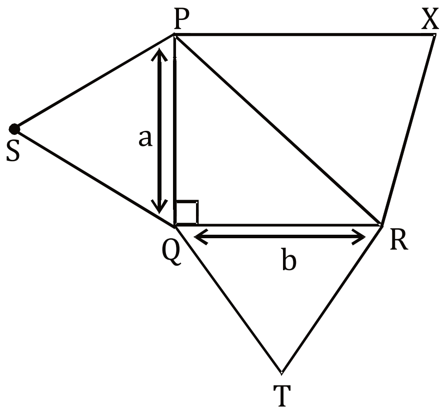
Add 1 both side

$$\frac{AB - BX}{AB} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}$$

$$\frac{AX}{AB} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}$$

**40.** Prove that the equilateral triangles described on the two sides of a right angled triangle are together equal to the equilateral triangle described on the hypotenuse in terms of their areas.

**Sol.**



Given : In $\Delta PQR$, $\angle Q = 90°$

Three equilateral triangles PQS, QRT, PRX are described on three sides of $\Delta PQR$.

To prove : ar ($\Delta PQS$) + ar ($\Delta QRT$) = ar ($\Delta PRX$)

Proof : Let PQ = a units

QR = b units

Then $PR^2 = a^2 + b^2$

ar ($\Delta PQS$) + ar ($\Delta QRT$)

$$= \frac{\sqrt{3}}{4}a^2 + \frac{\sqrt{3}}{4}b^2$$

$$= \frac{\sqrt{3}}{4}(a^2 + b^2)$$

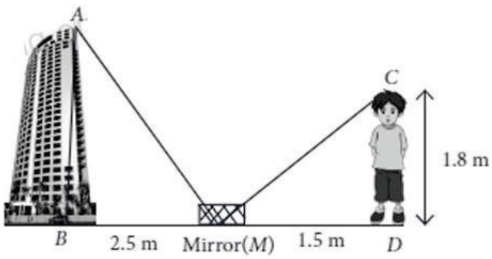
$$\text{ar}(\Delta PRX) = \frac{\sqrt{3}}{4}PR^2$$

$$= \frac{\sqrt{3}}{4}(a^2 + b^2)$$

$\Rightarrow$ ar ($\Delta PQS$) + ar ($\Delta QRT$) = ar ($\Delta PRX$)   Hence proved

**Case Study type questions** **(4 marks)**

**1.** Rohit's father is a mathematician. One day he gave Rohit an activity to measure, the height of building. Rohit accepted the challenge and placed a mirror on ground level to determine the height of building. He is standing at a certain distance so that he can see the top of the building reflection from mirror. Rohit's eye level is at 1.8 m above ground. The distance of Rohit from mirror and that of building from mirror are 1.5 m and 2.5 m respectively.




Based on the above information, answer the following questions.

(i) Determine the two similar $\Delta$s in the above figure and write the similarity criteria which can be applied here.

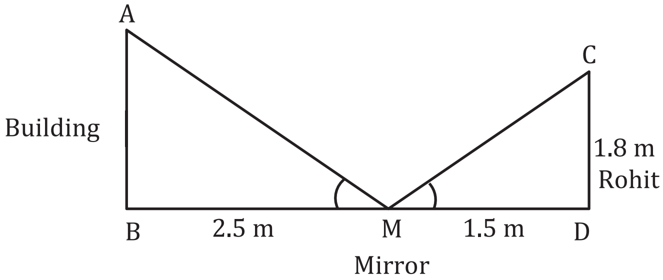
(ii) Find the height of the building.

(iii) In $\Delta ABM$, if $\angle BAM = 30°$, then find the measurement of $\angle MCD$.

**[OR]**

(iii) If $\Delta ABM$ and $\Delta CDM$ are similar where CD = 6 cm, MD = 8 cm and BM = 24 cm, then find AB.

**Sol.**




(i) Since $\angle B = \angle D = 90°, \angle AMB = \angle CMD$ {angle of incident = angle of reflection}

So, by AA similarity criterion, $\Delta ABM \sim \Delta CDM$

(ii) $\Delta ABM \sim \Delta CDM$, by property of similar $\Delta$s,

$$\frac{AB}{CD} = \frac{BM}{DM} \Rightarrow \frac{AB}{1.8} = \frac{2.5}{1.5}$$

$$AB = \frac{2.5 \times 1.8}{1.5} = 3m$$

(iii) $\angle BAM = 30°$

$\Delta ABM \sim \Delta CDM$, by properties of similar $\Delta$s,

$\angle BAM = \angle DCM = 30°$

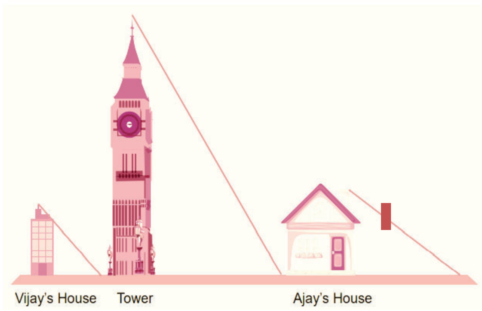
So, $\angle MCD = 30°$

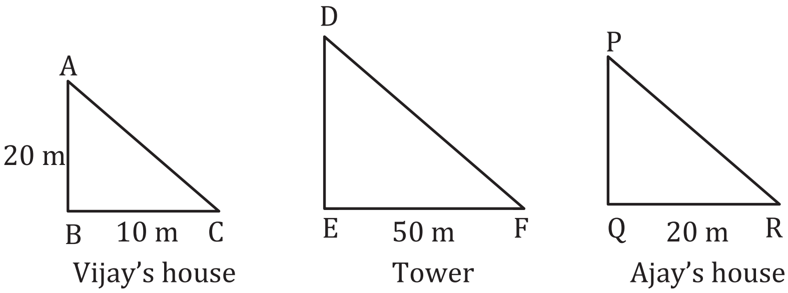
**[OR]**

(iii) $\frac{AB}{CD} = \frac{BM}{DM} \Rightarrow \frac{AB}{6} = \frac{24}{8}$

$$AB = \frac{6 \times 24}{8} = 18 \text{ cm}$$

**2.** Vijay is trying to find the average height of a tower near his house. He is using the properties of similar triangles. The height of Vijay's house is 20 m. When Vijay's house cast a shadow 10 m long on the ground, at the same time, the tower cast a shadow 50 m long on the ground and the house of Ajay casts 20 m shadow on the ground.



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(i) What is the height of the tower?

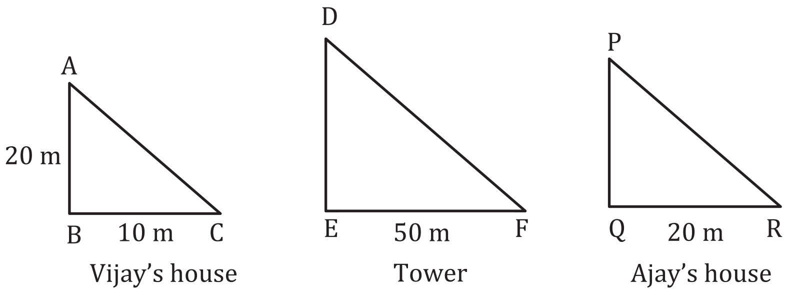
(ii) What will be the length of the shadow of the tower when Vijay's house casts a shadow of 12 m?

(iii) What is the height of Ajay's house?

**[OR]**

(iii) When the tower casts a shadow of 40 m, at the same time what will be the length of the shadow of Ajay's house?

**Sol.** (i)




In $\Delta ABC$ & $\Delta DEF$

$\angle A = \angle D$ (inclination of sun)

$\angle B = \angle E$ $(90°)$

By AA similarity,

$\Delta ABC \sim \Delta DEF$

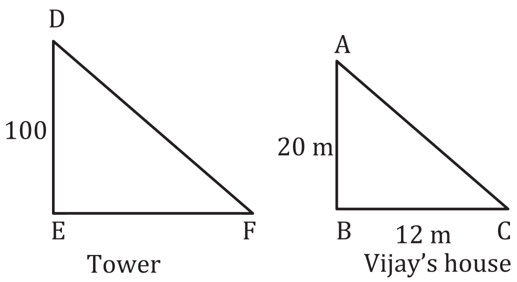
By property of similar $\Delta S$,

$$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{20}{DE} = \frac{10}{50}$$

$\Rightarrow$ $DE = 100$ m

(ii)




As, $\Delta DEF \sim \Delta ABC$

$$\therefore \frac{DE}{AB} = \frac{EF}{BC}$$

$$\frac{100}{20} = \frac{EF}{12}$$

$FE = 60$ m

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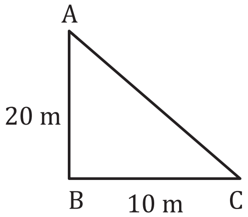
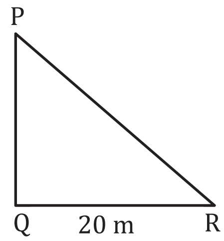
(iii)




Vijay's house




Ajay's house

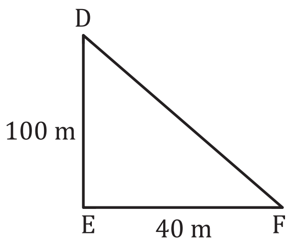
As, $\Delta ABC \sim \Delta PQR$

$\therefore \dfrac{AB}{PQ} = \dfrac{BC}{QR}$; $\dfrac{20}{PQ} = \dfrac{10}{20}$

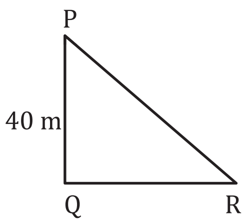
PQ = 40 m

**[OR]**

(iii)




Tower




Ajay's house

$\Delta DEF \sim \Delta PQR$ (AA similarity)

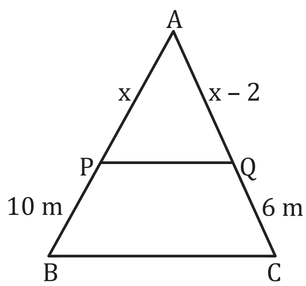
By properties of similar $\Delta^s$

$\dfrac{DE}{PQ} = \dfrac{EF}{QR}$; $\dfrac{100}{40} = \dfrac{40}{QR}$

$100QR = 40 \times 40 \Rightarrow QR = \dfrac{40 \times 40}{100} = 16m$

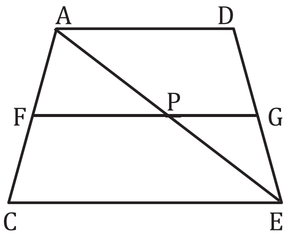
**3.** A frame-house is a house constructed from a wooden skeleton, typically covered with timber board. The concept of similar triangles is used to construct it. Look at the picture below.

(i) The front view of house is shown below in which point P on AB is joined with point Q.

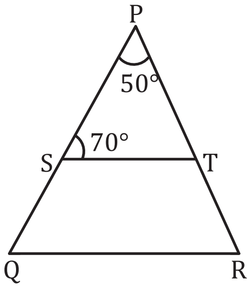



If PQ||BC, AP = x m, PB = 10 m. AQ = (x – 2) m QC = 6 m, then find the value of x.

(ii) The side view of house is shown below in which point F on AC is joined with point G on DE. If ACED is a trapezium with AD||CE, G and F are points on non- parallel side DE and AC respectively such that FG is parallel to AD then $\frac{AF}{FC} = ?$




(iii) The front view of house is shown below in which points S on PQ is joined with point T on PR.




If $\frac{PS}{QS} = \frac{PT}{TR}$ and $\angle PST = 70°$, $\angle QPR = 50°$ then the find the angle $\angle QRP$.

**[OR]**

(iii) Consider the front view of house. If S and T are points on side PQ and PR respectively such that ST||QR and PS : SQ = 3 : 1. Also, TP = 6.6 m, then find the value of PR.

**Sol.** (i) PQ||BC so by basic proportionality theorem

$$\frac{AP}{PB} = \frac{AQ}{QC} \Rightarrow \frac{x}{10} = \frac{x-2}{6}$$

$6x = 10(x - 2)$

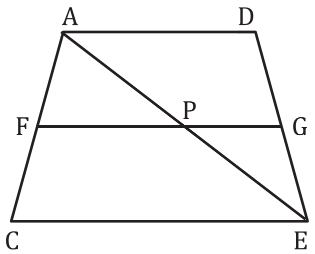
$6x = 10x - 20$

$6x - 10x = -20$

$-4x = -20$

$x = 5$ m

(ii)




Let us join AE and let AE intersects FG at point P.

In $\Delta ACE$, FP||CE, by Basic Proportionality theorem,

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$$\frac{AF}{FC}=\frac{AP}{PE} \qquad \text{... (1)}$$

Now in ΔEAD, GP||DA. So, again by Basic Proportionality Theorem,

$$\frac{EG}{GD}=\frac{EP}{AP}$$

Taking reciprocal

$$\frac{GD}{EG}=\frac{AP}{EP} \qquad \text{... (2)}$$

From (1) & (2)

$$\frac{AF}{FC}=\frac{GD}{EG}$$

(iii) If $\frac{PS}{QS}=\frac{PT}{TR}$ then by converse of basic proportionality theorem, ST||QR.

Now in ΔPST by angle sum property,

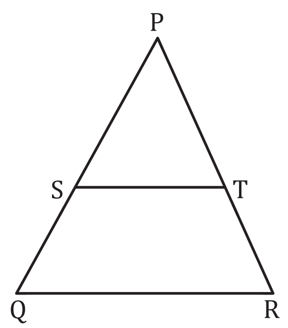
$50° + 70° + \angle PTS = 180°$

$\angle PTS = 60°$

Also, $\angle QRP = \angle PTS = 60°$ (corresponding $\angle$s)

**[OR]**

(iii)




ST||QR, so by basic proportionality theorem

$$\frac{PS}{SQ}=\frac{PT}{TR}$$

$$\frac{3}{1}=\frac{6.6}{TR}$$

TR = 2.2 m

PR = TR + TP

PR = 6.6 + 2.2

PR = 8.8 m

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# 7 Co-ordinate Geometry

**Multiple choice questions** **(1 mark)**

**1.** The point on the x – axis which is equidistant from P (–2 , 9) and Q (2, –5) is

(1) (0, 7) (2) (–7, 0) (3) (7, 0) (4) (7, –7)

**Sol. Option (2)**

Let R(a, 0) be equidistant from P(–2, 9) and Q(2, –5)

Now, PR = QR $\Rightarrow PR^2 = QR^2$

$\Rightarrow (-2-a)^2 + (9-0)^2 = (2-a)^2 + (-5-0)^2$

$\Rightarrow 4 + a^2 + 4a + 81 = 4 + a^2 - 4a + 25$

$\Rightarrow 8a = -56$

$\Rightarrow a = -7$

The required point is (–7, 0).

**2.** If the points A (4, 3) and B (x, 5) are on the circle with centre O (2, 3), find the value of x.

(1)1 (2) 2 (3) 3 (4) 4

**Sol. Option (2)**

Since the point A(4, 3) and B(x, 5) are equidistant from point O(2, 3) as point A & B are on the circle.

$\therefore$ AO = BO

$\Rightarrow AO^2 = BO^2$

$\Rightarrow (4-2)^2 + (3-3)^2 = (x-2)^2 + (5-3)^2$

$\Rightarrow 4 + 0 = x^2 + 4 - 4x + 4$

$\Rightarrow 0 = x^2 - 4x + 4 = (x-2)^2$

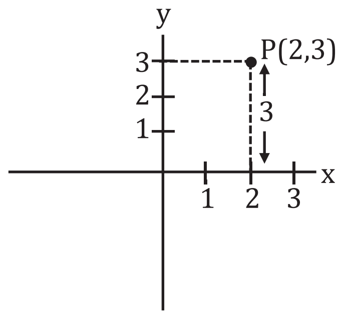
$\Rightarrow x = 2$

**3.** The distance of the point P (2, 3) from the x-axis is:

(1) 2 units (2) 3 units (3) 1 unit (4) 5 units

**Sol. Option (2)**

The distance of the point P(2, 3) from the x – axis is 3.

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**4.** Vertices of a quadrilateral ABCD are A(0, 0), B(4, 5), C(9, 9) and D(5, 4). What is the shape of the quadrilateral?

(1) Square
(2) Rectangle but not a square
(3) Rhombus
(4) Parallelogram but not a rhombus

**Sol.** **Option (3)**

First, check the lengths of sides of quad. ABCD

AB = $\sqrt{(4-0)^2+(5-0)^2} = \sqrt{16+25} = \sqrt{41}$

BC = $\sqrt{(9-4)^2+(9-5)^2} = \sqrt{25+16} = \sqrt{41}$

CD = $\sqrt{(5-9)^2+(4-9)^2} = \sqrt{16+25} = \sqrt{41}$

DA = $\sqrt{(5-0)^2+(4-0)^2} = \sqrt{25+16} = \sqrt{41}$

Since the sides are equal, So it can be either rhombus or square.

Second, check the diagonals of quad. ABCD

AC = $\sqrt{(9-0)^2+(9-0)^2} = 9\sqrt{2}$

BD = $\sqrt{(5-4)^2+(4-5)^2} = \sqrt{2}$

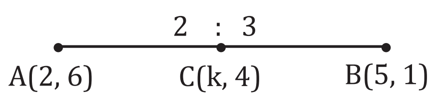
Since, the diagonals are not equal,

So the given quad. ABCD is a rhombus.

**5.** If the point C(k, 4) divides the join of the points A(2, 6) and B(5, 1) in the ratio 2 : 3 then the value of k is

(1) 16
(2) $\frac{28}{5}$
(3) $\frac{16}{5}$
(4) $\frac{8}{5}$

**Sol.** **Option (3)**




$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$k = \frac{2(5)+3(2)}{2+3}$$

$$k = \frac{10+6}{5}$$

$$k = \frac{16}{5}$$

**6.** If the points A (2, 3), B (5, K) and C(6, 7) are collinear, then the value of K is

(1) 1 (2) $\frac{11}{4}$ (3) 6 (4) $-\frac{3}{2}$

**Sol. Option (3)**

A(2, 3), B(5, K), C(6, 7) are collinear

So, ar($\Delta$ABC) = 0

$\frac{1}{2}|2(K-7)+5(7-3)+6(3-K)| = 0$

$|2K-14+5(4)+18-6K| = 0$

$|-4K-14+20+18| = 0$

$|-4K+24| = 0$

$-4K = -24$

K = 6

**7.** Find the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and B(2, –5).

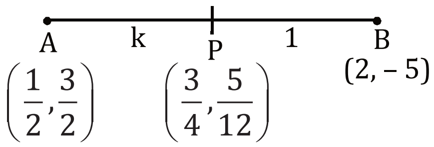
(1) 2 : 3 (2) 1 : 5 (3) 5 : 2 (4) 3 : 5

**Sol. Option (2)**

$A(x_1, y_1) = \frac{1}{2}, \frac{3}{2}$

$B(x_2, y_2) = 2, -5$

$P(x, y) = \left(\frac{3}{4}, \frac{5}{12}\right)$




Let the ratio = k : 1

By section formula x = $\frac{3}{4} = \frac{2k+\frac{1}{2}}{k+1}$

$\frac{3}{4}$ (k + 1) = 2k + $\frac{1}{2}$

$\frac{3}{4}$k – 2k = $\frac{1}{2}$ – $\frac{3}{4}$

$\frac{3k-8k}{4} = \frac{2-3}{4}$

$-5k = -1$

k = $\frac{1}{5}$

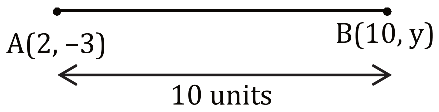
Therefore required ratio = k : 1

= $\frac{1}{5}$ : 1 = 1 : 5

**8.** A line segment is of length 10 units. If the coordinates of its one end are (2, –3) and the abscissa of the other end is 10, then its ordinate is

(1) 9, 6 (2) 3, –9 (3) –3, 9 (4) 9, –6

**Sol. Option (2)**




AB = 10 units $\Rightarrow$ $AB^2 = 100$

$(10 - 2)^2 + (y + 3)^2 = 100$

$(8)^2 + y^2 + 9 + 6y = 100$

$y^2 + 9 + 6y = 100 - 8^2$

$y^2 + 9 + 6y = 100 - 64$

$y^2 + 6y + 9 = 36$

$y^2 + 6y + 9 - 36 = 0$

$y^2 + 6y - 27 = 0$

$y^2 + 9y - 3y - 27 = 0$

$y(y + 9) - 3(y + 9) = 0$

$(y + 9)(y - 3) = 0$

$y = -9, y = 3$

## Assertion Reason questions (1 mark)

**9.** **Assertion (A):** The distance point P(2, 3) from the x-axis is 3.

**Reason (R):** The distance from x-axis is equal to its ordinate.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol. Option (1)**

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

**10.** **Assertion (A):** The point (4, 0) lies on y-axis.

**Reason (R):** The x-coordinate of the point on y-axis is not zero.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

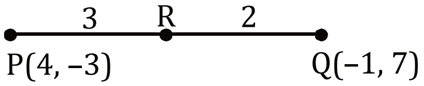
**Sol. Option (4)**

Assertion (A) is false but Reason (R) is true.

**Very short answer type questions** **(2 mark)**

**11.** The coordinates of the points P and Q are (4, – 3) and (–1, 7) respectively. Find the abscissa of a point R on the line segment PQ such that $\frac{PR}{PQ}=\frac{3}{5}$.

**Sol.**




$$\frac{PQ}{PR}=\frac{5}{3} \Rightarrow \frac{PQ-PR}{PR}=\frac{5-3}{3}$$

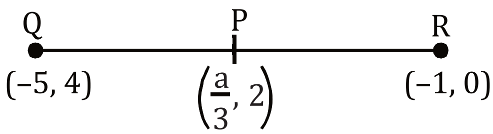
$$\Rightarrow \quad \frac{RQ}{PR}=\frac{2}{3} \text{ or } \frac{PR}{RQ}=\frac{3}{2}$$

i.e, R divides PQ in the ratio 3 : 2

$$\text{Abscissa of } R=\frac{3\times -1+2\times 4}{3+2}=\frac{-3+8}{5}=1$$

**12.** Find the value of a, for which point $P\left(\frac{a}{3}, 2\right)$ is the midpoint of the line segment joining the points Q(–5, 4) and R(–1, 0).

**Sol.**




P is mid-point of QR

$$\Rightarrow \quad \frac{a}{3}=\frac{-5+(-1)}{2}$$

$$\Rightarrow \quad \frac{a}{3}=\frac{-6}{2}$$

$\Rightarrow$ a = –9

**13.** The ordinate of a point A on y-axis is 5 and B has co-ordinates (–3, 1). Find the length of AB.

**Sol.** Here, A $\rightarrow$ (0, 5) and B $\rightarrow$ (–3, 1)

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(-3-0)^2+(1-5)^2}$$

$$= \sqrt{9+16}=\sqrt{25}$$

= 5 units

**14.** Find the perpendicular distance of A(5, 12) from the y-axis.

**Sol.** The point on the y-axis is (0, 12)

$\therefore$ Distance between (5, 12) and (0, 12)

$$d = \sqrt{(0-5)^2+(12-12)^2}$$

$$= \sqrt{25+0}$$

= 5 units

**15.** If the centre and radius of circle is (3, 4) and 7 units respectively, then what is the position of the point A(5, 8) with respect to circle?

**Sol.** Distance of the point,

$$a = \sqrt{(5-3)^2+(8-4)^2}$$

$$= \sqrt{4+16}=\sqrt{20}=2\sqrt{5}$$

$\because$ $2\sqrt{5}$ is less than 7.

$\therefore$ The point lies inside the circle.

**Short answer type questions** **(3 marks)**

**16.** Find the ratio in which the line segment joining the points P (3, –6) and Q(5, 3) is divided by the x-axis.

**Sol.** Let the required ratio be $\lambda : 1$

Then, the point of division is $\left(\dfrac{5\lambda+3}{\lambda+1}, \dfrac{3\lambda-6}{\lambda+1}\right)$

Given that this point lies on the x-axis.

Hence, its ordinate is 0 i.e. (x, 0)

$\therefore$ $\dfrac{3\lambda-6}{\lambda+1}=0$ or $3\lambda = 6$ or $\lambda = 2$

Thus, the required ratio is 2 : 1

**17.** Point P (5, –3) is one of the two points of trisection of the line segment joining the points A(7, –2) and B (1, –5). State true or false and justify your answer.

**Sol.** Points of trisection of line segment AB are given by $\left(\dfrac{2\times1+1\times7}{3}, \dfrac{2\times-5+1\times-2}{3}\right)$


and $\left(\dfrac{1\times1+2\times7}{3}, \dfrac{1\times-5+2\times-2}{3}\right)=\left(\dfrac{9}{3}, \dfrac{-12}{3}\right)$ and $\left(\dfrac{15}{3}, \dfrac{-9}{3}\right)$

or $(3, -4)$ and $(5, -3)$

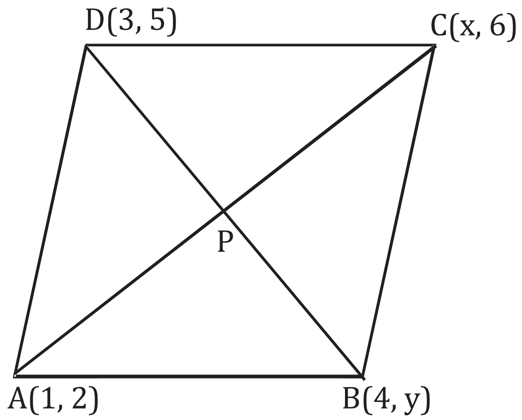
$\therefore$ Given statement is true.

**18.** If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

**Sol.** Let A(1, 2), B(4, y), C(x, 6) and D(3, 5) be the vertices of a parallelogram ABCD.

Since, the diagonals of a parallelogram bisect each other.

So, P is the mid-point of AC and BD




$\therefore \quad \left(\frac{x+1}{2},\frac{6+2}{2}\right)=\left(\frac{3+4}{2},\frac{5+y}{2}\right)$

$\Rightarrow \quad \frac{x+1}{2}=\frac{7}{2}$

$\Rightarrow \quad x + 1 = 7$ or $x = 6$

$\Rightarrow \quad 4=\frac{5+y}{2}\Rightarrow 5+y=8 \quad \text{or } y=8-5=3$

Hence, x = 6 and y = 3.

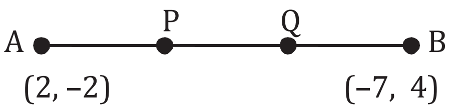
**19.** Let P and Q be the points of trisection of the line segment joining the points A(2, –2) and B(–7, 4) such that P is nearer to A. Find the coordinates of P and Q.

**Sol.** $\because$ P divides AB in the ratio 1 : 2.

$\therefore \quad$ Coordinates of P $=\left(\frac{1\times(-7)+2\times 2}{1+2}\right),\left(\frac{1\times 4+2\times -(2)}{1+2}\right)$

$=\left(\frac{-7+4}{3},\frac{4-4}{3}\right)=(-1,0)$

$\because$ Q is the mid point of PB




$\therefore \quad$ Coordinates of Q $=\frac{-1+(-7)}{2},\frac{0+4}{2}=\left(\frac{-8}{2},2\right)=(-4,2)$

**20.** The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from Q(2, –5) and R(–3, 6), find the coordinates of P.

**Sol.** Let the point be P (2y, y)

PQ = PR

$\Rightarrow \sqrt{(2y-2)^2+(y+5)^2} = \sqrt{(2y+3)^2+(y-6)^2}$

Squaring both sides

$\Rightarrow 4y^2 + 4 - 8y + y^2 + 25 + 10y$

$= 4y^2 + 9 + 12y + y^2 + 36 - 12y$

$\Rightarrow 2y + 29 = 45$

$\Rightarrow y = 8$

Hence, coordinates of point P are (16, 8).

**21.** Prove that the area of a triangle with vertices (t, t – 2), (t + 2, t + 2) and (t + 3, t) is independent of t.

**Sol.** Area of a triangle

$= \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

Area of the triangle $= \frac{1}{2}|t(t+2-t)+(t+2)(t-t+2)+(t+3)(t-2-t-2)|$

$= \frac{1}{2}|2t+2t+4-4t-12|$

= 4 sq. units

**22.** Prove that the points (7, 10), (–2, 5) and (3, –4) are the vertices of an isosceles right triangle.

**Sol.** Let the given points be A(7, 10), B(–2, 5) and C(3, –4)

Using distance formula, we have

$AB = \sqrt{(7+2)^2+(10-5)^2} = \sqrt{81+25} = \sqrt{106}$

$BC = \sqrt{(-2-3)^2+(5+4)^2} = \sqrt{25+81} = \sqrt{106}$

$CA = \sqrt{(7-3)^2+(10+4)^2} = \sqrt{16+196} = \sqrt{212}$

Since, AB = BC, $\therefore$ ABC is an isosceles triangle.

Also, $AB^2 + BC^2 = 106 + 106 = 212 = AC^2$

So, ABC is an isosceles right angled triangle at B.

**23.** Find the point on y-axis which is equidistant from the point (5, –2) and (–3, 2).

**Sol.** Let P(0, y) be the point on the y-axis which is equidistant from A(5, –2) and B(–3, 2).

$\Rightarrow$ AP = BP

$\Rightarrow$ $(AP)^2 = (BP)^2$

$\Rightarrow$ $(5-0)^2 + (-2-y)^2 = (-3-0)^2 + (2-y)^2$

$\Rightarrow$ $25 + 4 + y^2 + 4y = 9 + 4 + y^2 - 4y$

$\Rightarrow$ $8y = 9 - 25$

$\Rightarrow$ $y = -\frac{16}{8} = -2$

$\therefore$ Point P (0, – 2) is equidistant from the point (5, –2) and (–3, 2).

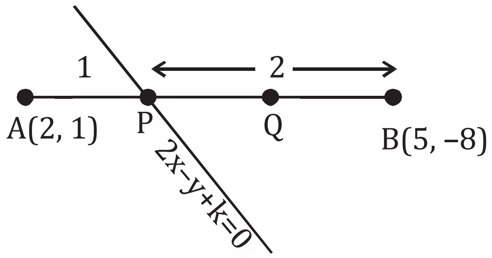
**24.** The line joining the points A (2, 1) and B (5, –8) is trisected by the points P and Q. If the point P lies on the line 2x – y + k = 0, find the value of k.

**Sol.** As line segment AB is trisected by the points P and Q. Therefore,

**Case I :** when AP : PB = 1 : 2.

Then, coordinates of P are $\left\{\frac{1\times5+2\times2}{1+2}, \frac{1\times-8+1\times2}{1+2}\right\}$

$\Rightarrow$ P(3 – 2)




Since the point P(3, –2) lies on the line

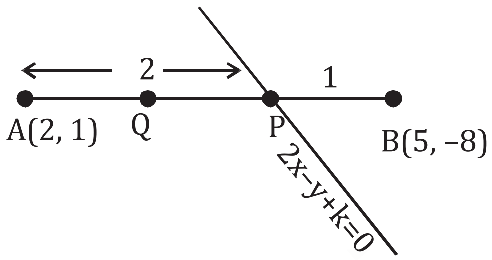
$2x - y + k = 0$

$\Rightarrow$ $2 \times 3 - (-2) + k = 0$

$\Rightarrow$ $k = -8$

**Case II:** When AP : PB = 2 : 1

Coordinates of point P are




$$\left\{\frac{2\times5+1\times2}{2+1}, \frac{2\times-8+1\times1}{2+1}\right\} = \{4, -5\}$$

Since the point P(4, –5) lies on the line

$2x - y + k = 0$

$\therefore$ $2 \times 4 - (-5) + k = 0$

$\Rightarrow$ $k = -13.$

**25.** Prove that the points A(0, –1), B(–2, 3), C(6, 7) and D(8, 3) are the vertices of a rectangle ABCD.

**Sol.** Given: A(0, –1), B(–2, 3), C(6, 7), D(8, 3)

To prove: ABCD is a rectangle.

Proof : Now, $AB=\sqrt{(0+2)^2+(-1-3)^2}=\sqrt{4+16}=\sqrt{20}=2\sqrt{5}\text{units}$

$BC=\sqrt{(-2-6)^2+(3-7)^2}=\sqrt{64+16}=\sqrt{80}=4\sqrt{5}\text{units}$

$CD=\sqrt{(6-8)^2+(7-3)^2}=\sqrt{4+16}=\sqrt{20}=2\sqrt{5}\text{units}$

$AD=\sqrt{(0-8)^2+(-1-3)^2}=\sqrt{64+16}=\sqrt{80}=4\sqrt{5}\text{units}$

Diagonal $AC=\sqrt{(0-6)^2+(-1-7)^2}=\sqrt{36+64}=\sqrt{100}=10\text{units}$

$BD=\sqrt{(-2-8)^2+(3-3)^2}=\sqrt{(-10)^2+0^2}=\sqrt{100}=10\text{units}$

Since, AB = CD and BC = AD i.e., opposite sides of given quadrilateral are of the same length and also AC = BD i.e., diagonals are of the same length. Therefore, the given points are the vertices of a rectangle.

**26.** The base BC of an equilateral triangle ABC lies on y-axis. The coordinates of point C are (0, 3). The origin is the mid-point of the base. Find the coordinates of the points A and B. Also find the coordinates of another point D such that BACD is a rhombus.

**Sol.** $\because$ O is the mid-point of the base BC,

$\therefore$ coordinates of point C are (0, 3)

So, BC = 6 units

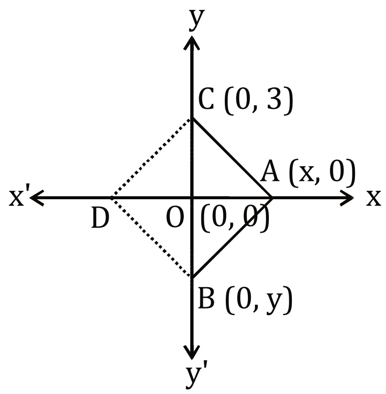
O is midpoint of BC

$\frac{3+y}{2}=0$

$\Rightarrow$ y = – 3

$\therefore$ Coordinates of B (0, – 3)

Let the coordinates of point A be (x, 0)




$\therefore\ AC=\sqrt{(0-x)^2+(3-0)^2}=\sqrt{x^2+9}$

Also, AB = BC ($\because$ ΔABC is an equilateral triangle)

$\Rightarrow\ \sqrt{x^2+9}=6\Rightarrow x^2+9=36$

$\Rightarrow\ x^2=27\Rightarrow x=\pm3\sqrt{3}$

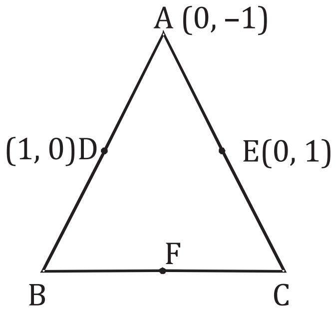
$\therefore$ coordinates of point A = (x, 0) = $(3\sqrt{3},0)$

Since BACD is a rhombus.

$\therefore$ AB = AC = CD = DB

$\therefore$ Coordinates of point D = $\left(-3\sqrt{3},0\right)$

**27.** In the given figure, ABC is a triangle, coordinates of whose vertex A is (0, –1). D and E are the mid-points of the sides AB and AC respectively and their coordinates are (1, 0) and (0, 1) respectively. If F is the mid-point of BC, find the areas of ΔABC and ΔDEF.




**Sol.** ABC is a triangle and D, E, F are mid points of ΔABC.

∴ By using mid point formula,

Co-ordinates of B are given by $\frac{0+x_1}{2}=1$

⇒ $x_1 = 2$; $\frac{-1+y_1}{2}=0$

⇒ $x_1 = 2$; $y_1 = 1$

∴ Coordinates of B are (2, 1)

Similarly, coordinates of C are

$0=\frac{0+x_2}{2} \Rightarrow x_2=0$ and $1 = \frac{-1+y_2}{2} \Rightarrow y_2 = 3$

∴ coordinates of C are (0, 3) and coordinates of F are

$x=\frac{2+0}{2}=1$ and $y=\frac{1+3}{2}=2$

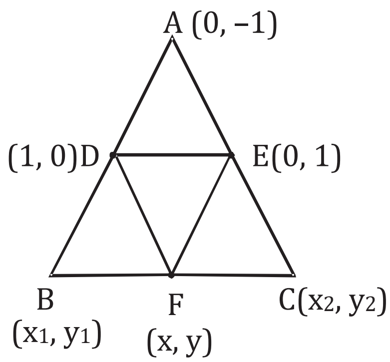
∴ coordinates of F are (1, 2)

Area of ΔABC with A(0, –1), B(2, 1) and C(0, 3)

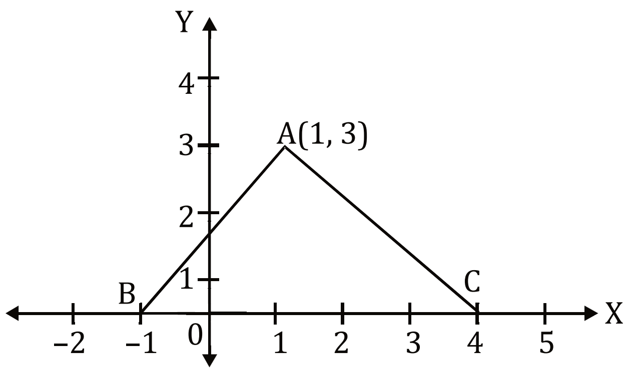
$=\frac{1}{2}|0+2(3+1)+0|=4$ sq.units

Area of ΔDEF with D(1, 0), E(0, 1) and F(1,2)

$=\frac{1}{2}|1(1-2)+0+1(0-1)|=1$ sq.unit




**28.** In the given figure, the area of triangle ABC (in sq. units) is

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**Sol.** Area of $\Delta ABC$ where A(1, 3), B(–1, 0) & C(4, 0) is

$$\Delta = \frac{1}{2}|1(0-0)-1(0-3)+4(3-0)|$$

$$\frac{1}{2}|0+3+12| = \frac{1}{2}\times 15 = 7.5 \text{sq.units}$$

**29.** Find the relation between x and y if the points A(x, y), B(–5, 7) and C(–4, 5) are collinear.

**Sol.** Since, A(x, y), B(–5, 7) and C(–4, 5) are collinear

$\therefore$ Area of $\Delta ABC = 0$

$\Rightarrow \frac{1}{2}[|x(7-5)+(-5)(5-y)+(-4)(y-7)|]=0$

$\Rightarrow$ 2x – 25 + 5y – 4y + 28 = 0

$\Rightarrow$ 2x + y = – 3

**30.** If the points A(x, y), B(3, 6) and C(–3, 4) are collinear, show that x – 3y + 15 = 0.

**Sol.** $\because$ A(x, y), B(3, 6) and C(–3, 4) are collinear, then

$\therefore$ Area of the $\Delta ABC = 0$

$\Rightarrow \frac{1}{2}[x(6-4)+3(4-y)+(-3)(y-6)]=0$

$\Rightarrow$ 2x + 12 – 3y – 3y + 18 = 0

$\Rightarrow$ 2x – 6y + 30 = 0 $\Rightarrow$ 2(x – 3y + 15) = 0

$\Rightarrow$ x – 3y + 15 = 0

**31.** A line intersects y-axis and x-axis at the points P and Q respectively. If (2, –5) is the mid-point of PQ, then find the coordinates of P and Q.

**Sol.** Let coordinates of P and Q be (0, y) and (x, 0) respectively.

Let M(2, – 5) be the mid-point of PQ.

$\therefore$ By mid point formula

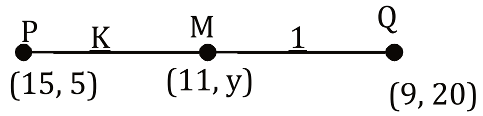
$$2=\frac{x+0}{2}, \qquad -5=\frac{0+y}{2}$$

$\Rightarrow$ x = 4, y = – 10

$\therefore$ Points are P(0, –10) and Q(4, 0).

**32.** Point M(11, y) lies on the line segment joining the point P(15, 5) and Q(9, 20). Find the ratio in which point M divides the line segment PQ. Also find the value of y.

**Sol.** Let M(11, y) divides the line segment joining P(15, 5) and Q(9, 20) in K : 1.




$\therefore$ Coordinates of $M=\left(\dfrac{9\times K+15\times 1}{K+1},\dfrac{20\times K+1\times 5}{K+1}\right)$ [By using section formula]

$\Rightarrow \quad (11,y)=\left(\dfrac{9K+15}{K+1},\dfrac{20K+5}{K+1}\right)$

$\therefore \quad 11=\dfrac{9K+15}{K+1}$ ... (i)

and $\quad y=\dfrac{20K+5}{K+1}$ ... (ii)

Using (i) we get,

$9K + 15 = 11K + 11$

$-2K = -4 \Rightarrow K = 2$

using (i), we get

$\therefore \quad y=\dfrac{20\times 2+5}{2+1}=\dfrac{40+5}{3}=\dfrac{45}{3}=15$

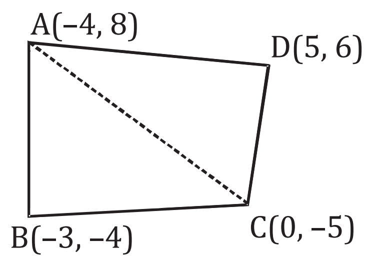
Thus, the value of y is 15 and M divides the line segment in 2 : 1.

## Long answer type questions (5 marks)

**33.** If A(–4, 8), B(–3, –4), C(0, –5) and D(5, 6) are the vertices of a quadrilateral ABCD, find its area.

**Sol.** Join AC.

Here, we get two triangles ABC and ADC.




Area of $\Delta ABC=\dfrac{1}{2}|-4(-4+5)+(-3)(-5-8)+0\times(8+4)|$

$=\dfrac{1}{2}|(-4\times 1)+39+0|=\dfrac{35}{2}\text{sq.units}$

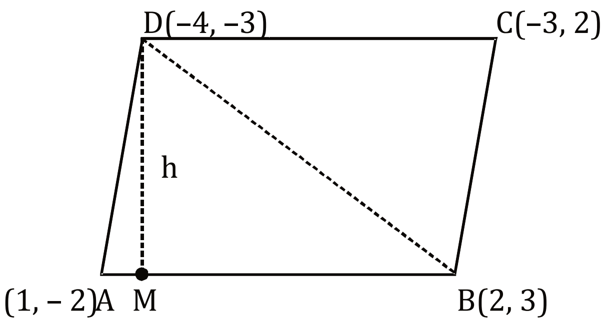
Area of $\Delta ADC=\dfrac{1}{2}|-4(6+5)+5(-5-8)+0(8-6)|$

$=\dfrac{1}{2}|-44-65+0|=\dfrac{1}{2}|-109|=\dfrac{109}{2}\text{sq.units}$

Now, area of quadrilateral ABCD = Area of $\Delta ABC$ + Area of $\Delta ADC$

$=\dfrac{35}{2}+\dfrac{109}{2}=\dfrac{144}{2}=72\text{ sq.units}$

**34.** If the points A(1, –2), B(2, 3), C(–3, 2) and D(–4, –3) are the vertices of parallelogram ABCD, then taking AB as the base, find the height of the parallelogram.




**Sol.** Let DM = h be the height of parallelogram ABCD when AB is taken as the base.

$$\text{Area of } \Delta ABD = \frac{1}{2}(AB \times DM) \qquad [\because \text{area of } \Delta = \frac{1}{2} \times \text{base} \times \text{height}]$$

$$\Rightarrow \quad \text{Area of } \Delta ABD = \frac{1}{2} \times AB \times h$$

$$\Rightarrow \quad h = \frac{2(\text{area of } \Delta ABD)}{AB} \qquad \text{... (i)}$$

Now, first find the length of AB by using distance formula,

$$AB = \sqrt{(2-1)^2 + (3+2)^2} = \sqrt{26} \qquad [\because d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)}]$$

Now, the coordinates of vertices of ΔABD are A(1, –2), B(2, 3) and D(–4, –3).

$$\therefore \quad \text{Area of } \Delta ABD = \frac{1}{2}|[1(3+3) + 2(-3+2) + (-4)(-2-3)]|$$

$$\left[\text{Area of triangle} = \frac{1}{2}|[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3 + (y_1 - y_2)]|\right]$$

$$= \frac{1}{2}|[6 - 2 + 20]| = \frac{24}{2} = 12 \text{ sq.units}$$

Now, putting the value of AB and area of ΔABD in Eq. (i) we get

$$h = \frac{2 \times 12}{\sqrt{26}} = \frac{24}{\sqrt{26}} \times \frac{\sqrt{26}}{\sqrt{26}}$$

$$h = \frac{24\sqrt{26}}{26} = \frac{12\sqrt{26}}{13} \text{ units}$$

**35.** Two vertices of a triangle are (8, – 6) and (–4, 6). The area of the triangle is 120 sq. units. Find the third vertex if it lies on x – 2y = 6.

**Sol.** Let A(8, –6), B(–4, 6) and C(x, y) be the vertices of $\Delta$ABC.

Area of $\Delta ABC = \frac{1}{2}\,|8(6 - y) - 4(y + 6) + x(-6 - 6)| = 120$

$= \frac{1}{2}\,|48 - 8y - 4y - 24 - 12x| = 120$

$\Rightarrow\quad -12x - 12y + 24 = -240$ ... (i)

or $\quad -12x - 12y + 24 = 240$ ... (ii)

Solving eq.(ii) and x – 2y = 6, we get

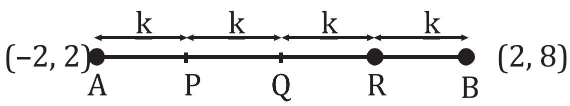
x = –10 and y = –8

Solving eq.(i) and x – 2y = 6, we get

$x = \frac{50}{3}$ and $y = \frac{16}{3}$

**36.** Find the coordinates of the points which divide the line segment joining A(–2, 2) and B(2, 8) into four equal parts.

**Sol.** Let P, Q and R be the points on line segment AB such that AP = PQ = QR = RB




Now, $\frac{AP}{PB} = \frac{k}{3k} = \frac{1}{3}$

Therefore, P divides AB internally in the ratio 1 : 3.

$\therefore\quad$ Coordinates of $P = \frac{1\times 2 + 3(-2)}{1+3}, \frac{1\times 8 + 3\times 2}{1+3}$

using section formula $\left[\text{i.e.}\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)\right]$

$= \left(\frac{2-6}{4}, \frac{8+6}{4}\right) = \left(\frac{-4}{4}, \frac{14}{4}\right) = \left(-1, \frac{7}{2}\right)$

Again, $\frac{AR}{RB} = \frac{3k}{k} = \frac{3}{1}$

Therefore, R divides AB internally in the ratio 3 : 1.

$\therefore$ Coordinates of $R = \left[\frac{3\times2+1\times(-2)}{3+1}, \frac{3\times8+1\times2}{3+1}\right]$

$= \left(\frac{6-2}{4}, \frac{24+2}{4}\right) = \left(\frac{4}{4}, \frac{26}{4}\right) = \left(1, \frac{13}{2}\right)$

Also, $\frac{AQ}{QB} = \frac{2k}{2k} = \frac{1}{1} \Rightarrow Q$ is the mid-point of AB.

$\therefore$ Coordinates of $Q = \left(\frac{-2+2}{2}, \frac{2+8}{2}\right)$

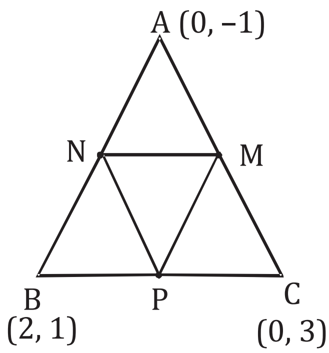
$\left[\therefore \text{coordinates of midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$

$\Rightarrow$ $Q = \left(\frac{0}{2}, \frac{10}{2}\right) = (0,5)$

So, the required points are $\left(-1, \frac{7}{2}\right), \left(1, \frac{13}{2}\right)$ and $(0, 5)$

**37.** Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of this area of this triangle to the area of the given triangle.

**Sol.** Let A(0, –1), B(2, 1) and C(0, 3) be the vertices of a $\Delta ABC$ and M, N and P be the mid-point of AC, AB and BC, respectively.




$\therefore$ Coordinate of mid-point of AC,

$M = \left(\frac{0+0}{2}, \frac{3-1}{2}\right) = (0,1)$

$\left[\therefore \text{coordinates of mid-point} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$

Coordinate of mid-point of AB,

$N = \left(\frac{0+2}{2}, \frac{-1+1}{2}\right) = (1,0)$

and coordinate of mid-point of BC,

$P = \left(\frac{0+2}{2}, \frac{3+1}{2}\right) = (1,2)$

Let $N(x_1, y_1) = N(1, 0)$, $P(x_2, y_2) = P(1, 2)$ and $M(x_3, y_3) = M(0, 1)$

$\therefore$ Area of $\Delta NPM$

$= \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

$= \frac{1}{2}|1(2-1)+1(1-0)+0(0-2)|$

$=\frac{1}{2}|1+1+0|=\frac{2}{2}$

= 1 sq. unit

Again, let $A(x_1, y_1)=A(0, -1)$, $B(x_2, y_2)=B(2, 1)$ and $C(x_3, y_3) = C(0, 3)$

∴ Area of ΔABC

$=\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

$=\frac{1}{2}|0(1-3)+2(3+1)+0(-1-1)|$

$=\frac{1}{2}|0+8+0|=4$ sq units

∴ Required ratio = Area of ΔNPM : Area of ΔABC = 1 : 4

**38.** Find the centre of a circle passing through the points (6, –6), (3, –7) and (3, 3).

**Sol.** Let C (x, y) be the centre of the circle passing through the points P(6, –6), Q(3, –7) and R (3, 3).

Then PC = QC = CR [radii of circle]

Now, PC = QC

$\Rightarrow$ $PC^2 = QC^2$ [on squaring both sides]

$\Rightarrow$ $(x-6)^2 + (y+6)^2 = (x-3)^2 + (y+7)^2$ $[\because \text{distance}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}]$

$\Rightarrow$ $x^2 - 12x + 36 + y^2 + 12y + 36$

$= x^2 - 6x + 9 + y^2 + 14y + 49$

$\Rightarrow$ $-12x + 6x + 12y - 14y + 72 - 58 = 0$

$\Rightarrow$ $-6x - 2y + 14 = 0$

$\Rightarrow$ $3x + y - 7 = 0$ ... (i) [dividing by – 2]

and QC = CR $\Rightarrow$ $QC^2 = CR^2$ [on squaring both sides]

$\Rightarrow$ $(x-3)^2 + (y+7)^2 = (x-3)^2 + (y-3)^2$

$\Rightarrow$ $(y+7)^2 = (y-3)^2$

$\Rightarrow$ $y^2 + 14y + 49 = y^2 - 6y + 9$

$\Rightarrow$ $20y + 40 = 0 \Rightarrow y=-\frac{40}{20}=-2$ ... (ii)

On putting y = – 2 in Eq. (i), we get

$3x - 2 - 7 = 0$

$\Rightarrow$ $3x = 9$

$\Rightarrow$ $x = 3$

Hence, the centre of circle is (3, – 2).

**39.** Find the value of k, for which points

$(k, 2 - 2k)$, $(-k + 1, 2k)$ and $(-4 - k, 6 - 2k)$ are collinear.

**Sol.** Let $A(k, 2 - 2k)$, $B(-k + 1, 2k)$ and $C(-4 - k, 6 - 2k)$.

Here, k is an unknown.

Since, given points are collinear.

So, area of $\Delta ABC = 0$

i.e. $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$\Rightarrow$ $k(2k - 6 + 2k) + (-k + 1)(6 - 2k - 2 + 2k) + (-4 - k)(2 - 2k - 2k) = 0$

$\Rightarrow$ $4k^2 - 6k + (-k + 1)(4) + (-4 - k)(2 - 4k) = 0$

$\Rightarrow$ $4k^2 - 6k - 4k + 4 - 8 + 16k - 2k + 4k^2 = 0$

$\Rightarrow$ $8k^2 + 4k - 4 = 0$

$\Rightarrow$ $2k^2 + k - 1 = 0$ [divide by 4]

$\Rightarrow$ $2k^2 + 2k - k - 1 = 0$ [by factorisation]

$\Rightarrow$ $(2k - 1)(k + 1) = 0$

$\Rightarrow$ $2k - 1 = 0$ or $k + 1 = 0$

$\Rightarrow$ $k = \frac{1}{2}$ or $k = -1$

Hence, for $k = \frac{1}{2}$ and $k = -1$, given points are collinear.

**40.** If the line joining the points $A(a^2,1)$ and $B(b^2,1)$ is divided in the ratio $b : a$ at the point P whose x-coordinate is 7, then find the value of $a^2 - ab + b^2$.

**Sol.** Using the section formula, if a point $(x, y)$ divides the line joining the points $(x_1, y_1)$ and $(x_2, y_2)$ in the ratio $m : n$, then

$$(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Since P divides the line in ratio $b : a$.

Now using section formula, we have

$$7 = \frac{b \times b^2 + a \times a^2}{a+b}$$

$$7 = \frac{b^3 + a^3}{a+b}$$

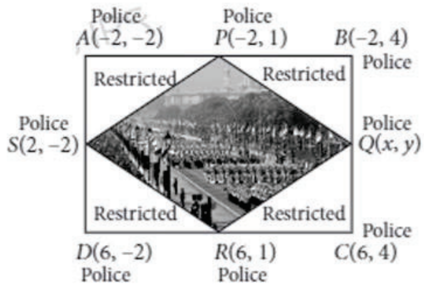
$$7 = \frac{(a+b)(a^2 - ab + b^2)}{a+b}$$

$7 = a^2 - ab + b^2$

**Case Study type questions** **(4 marks)**

**1.** Republic Day programme

In order to facilitate smooth passage of the parade, movement of traffic on certain roads leading to the route of the parade and tableaux is always restricted. To avoid traffic on the road, Delhi police decided to construct a rectangular route plan, as shown in the following figure. Based on above information answer the following questions.




(d) (−1, 1)

(i) If Q is the midpoint of BC, then find the coordinates of Q.

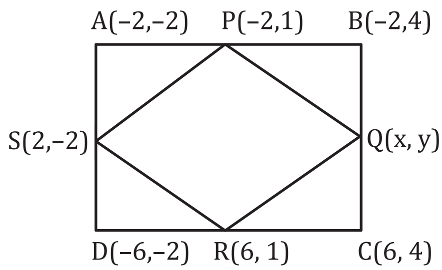
(ii) What is the length of sides of quadrilateral PQRS?

**[OR]**

(ii) What is the length of route PQRS?

(iii) Check whether quadrilateral PQRS is a trapezium, square, rectangle or a rhombus?

**Sol.**




(i) By mid point section formula,

$$x = \frac{-2+6}{2} = \frac{4}{2} = 2$$

$$y = \frac{4+4}{2} = \frac{8}{2} = 4$$

Coordinates of Q(2, 4).

(ii) $PQ = \sqrt{(2+2)^2+(4-1)^2} = \sqrt{(4)^2+(3)^2} = \sqrt{16+9} = \sqrt{25} = 5$ units

$QR = \sqrt{(6-2)^2+(1-4)^2} = \sqrt{(4)^2+(3)^2} = \sqrt{16+9} = \sqrt{25} = 5$ units

$RS = \sqrt{(6-2)^2+(1+2)^2} = \sqrt{(4)^2+(3)^2} = \sqrt{16+9} = \sqrt{25} = 5$ units

$SP = \sqrt{(-2-2)^2+(1+2)^2} = \sqrt{(-4)^2+(3)^2} = \sqrt{16+9} = \sqrt{25} = 5$ units

**[OR]**

(ii) Length of route PQRS = PQ + QR + RS + SP

= 5 + 5 + 5 + 5 = 20 units

(iii) In quadrilateral PQRS,

PQ = QR = RS = SP = 5 units

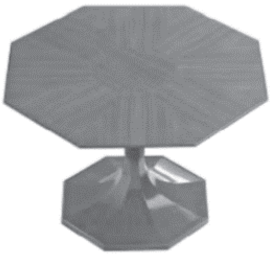
and PR = $\sqrt{(6+2)^2+(0)^2} \Rightarrow \sqrt{8^2}$ = 8 units

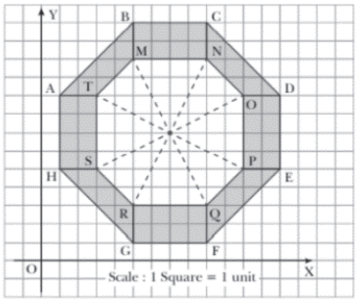
QS = $\sqrt{(2-2)^2+(-2-4)^2}$ = $\sqrt{(-6)^2}$ = 6 units

PR $\neq$ QS {diagonals are not equal}

$\Rightarrow$ PQRS will be a rhombus.

**2.** The top of a table is shown in the figure given below.






(i) Find the coordinates of the points H and G.

(ii) Calculate the distance between the points A and B.

(iii) Find the coordinates of the midpoint of line segment joining points M and Q.

**[OR]**

(iii) If G is taken as origin, and x, y axes put along GF and GB, then find the point denoted by coordinate (4, 2).

**Sol.** (i) Coordinates of H = (1, 5)

coordinates of G = (5, 1)

(ii) Distance between A(1, 9) and B (5, 13) will be

AB = $\sqrt{(5-1)^2+(13-9)^2}$

= $\sqrt{(4)^2+(4)^2}$

= $\sqrt{16+16}$

= $\sqrt{32}$

= $4\sqrt{2}$ units

(iii) Midpoint of line segment joining M (5, 11) and Q(9, 3) will be

$$\left(\frac{5+9}{2}, \frac{3+11}{2}\right) \Rightarrow \left(\frac{14}{2}, \frac{14}{2}\right) \Rightarrow (7, 7)$$

**[OR]**

(iii) G is (0,0) then the point whose coordinate is (4, 2) will be point Q.

# 8 Introduction to Trigonometry

**Multiple choice questions** **(1 mark)**

**1.** In $\Delta ABC$, right-angled at B, AB = 24 cm, BC = 7 cm. The value of tan C is:

(1) $\frac{12}{7}$ (2) $\frac{24}{7}$ (3) $\frac{20}{7}$ (4) $\frac{7}{24}$

**Sol. Option (2)**

AB = 24 cm and BC = 7 cm

$\tan C = \frac{\text{Opposite side}}{\text{Adjacent side}}$

$\tan C = \frac{24}{7}$

**2.** $(\sin 30° + \cos 60°) - (\sin 60° + \cos 30°)$ is equal to:

(1) 0 (2) $1+2\sqrt{3}$ (3) $1-\sqrt{3}$ (4) $1+\sqrt{3}$

**Sol. Option (3)**

$\sin 30° = \frac{1}{2}$, $\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$ and $\cos 60° = \frac{1}{2}$

Putting these values, we get:

$\left(\frac{1}{2}+\frac{1}{2}\right)-\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right)$

$= 1 - \left[\left(\frac{2\sqrt{3}}{2}\right)\right]$

$= 1 - \sqrt{3}$

**3.** The value of $\frac{\tan 60°}{\cot 30°}$ is equal to:

(1) 0 (2) 1 (3) 2 (4) 3

**Sol. Option (2)**

$\tan 60° = \sqrt{3}$ and $\cot 30° = \sqrt{3}$

Hence, $\frac{\tan 60°}{\cot 30°} = \frac{\sqrt{3}}{\sqrt{3}} = 1$

**4.** $1 - \cos^2 A$ is equal to:

(1) $\sin^2 A$ (2) $\tan^2 A$ (3) $1 - \sin^2 A$ (4) $\sec^2 A$

**Sol. Option (1)**

We know, by trigonometry identities,

$\sin^2 A + \cos^2 A = 1$

$1 - \cos^2 A = \sin^2 A$

**5.** sin (90° – A) and cos A are:

(1) Different (2) Same (3) Not related (4) None of the above

**Sol. Option (2)**

By trigonometry identities.

Sin (90°– A) = cos A {since 90° – A comes in the first quadrant of unit circle}

**6.** If $\cos X = \frac{2}{3}$ then tan X is equal to:

(1) $\frac{5}{2}$ (2) $\sqrt{\left(\frac{5}{2}\right)}$ (3) $\frac{\sqrt{5}}{2}$ (4) $\frac{2}{\sqrt{5}}$

**Sol. Option (3)**

By trigonometry identities, we know:

$1 + \tan^2 X = \sec^2 X$

And $\sec X = \frac{1}{\cos X} = \frac{1}{\left(\frac{2}{3}\right)} = \frac{3}{2}$

Hence,

$1 + \tan^2 X = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$\tan^2 X = \left(\frac{9}{4}\right) - 1 = \frac{5}{4}$

$\tan X = \frac{\sqrt{5}}{2}$

**7.** If $\cos X = \frac{a}{b}$, then sin X is equal to:

(1) $\frac{(b^2 - a^2)}{b}$ (2) $\frac{(b - a)}{b}$ (3) $\frac{\sqrt{(b^2 - a^2)}}{b}$ (4) $\frac{\sqrt{(b - a)}}{b}$

**Sol. Option (3)**

$\cos X = \frac{a}{b}$

By trigonometry identities, we know that:

$\sin^2 X + \cos^2 X = 1$

$\sin^2 X = 1 - \cos^2 X = 1 - \left(\frac{a}{b}\right)^2$

$\sin X = \frac{\sqrt{(b^2 - a^2)}}{b}$

**8.** The value of sin 60° cos 30° + sin 30° cos 60° is:

(1) 0 (2) 1 (3) 2 (4) 4

**Sol. Option (2)**

$\sin 60° = \frac{\sqrt{3}}{2}$, $\sin 30° = \frac{1}{2}$, $\cos 60° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$

Therefore,

$$\left(\frac{\sqrt{3}}{2}\right)\times\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)$$

$$= \left(\frac{3}{4}\right)+\left(\frac{1}{4}\right) = \frac{4}{4}$$

= 1

**Assertion Reason questions** **(1 marks)**

**9.** **Assertion (A):** sinA is the product of sin and A.

**Reason (R):** The value of sinθ increases as θ increases.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol. Option (4)**

Assertion (A) is false but Reason (R) is true.

**10.** **Assertion (A):** In a right ΔABC, right angled at B, if tanA = 1, then 2sinA.cosA = 1

**Reason (R):** cosecA is the abbreviation used for cosecant of angle A.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol. Option (2)**

$\tan A = 1 \Rightarrow A = 45°$

$2\sin A\cos A = 2 \times \sin 45° \cos 45°$

$$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{2}{2} = 1$$

Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

**Very short answer type questions** **(2 mark)**

**11.** In a triangle ABC, write $\cos\left(\frac{B+C}{2}\right)$ in terms of angle A.

**Sol.** A + B + C = 180°

$\Rightarrow$ B + C = 180° – A

$$\therefore \quad \cos\left(\frac{B+C}{2}\right)=\cos\left[\frac{180^\circ-A}{2}\right]$$

$$=\cos\left(90^\circ-\frac{A}{2}\right)$$

$$=\sin\frac{A}{2}$$

**12.** If $\sin\alpha=\frac{1}{2}$, then find value of $3\sin\alpha - 4\sin^3\alpha$.

**Sol.** Given, $\sin\alpha=\frac{1}{2}$,

then $3\sin\alpha - 4\sin^3\alpha = 3\times\frac{1}{2}-4\times\left(\frac{1}{2}\right)^3$

$=\frac{3}{2}-\frac{4}{8}=1$

**13.** Find the value of $\frac{\sin 25^\circ}{\cos 65^\circ}+\frac{\tan 23^\circ}{\cot 67^\circ}$

**Sol.** $\frac{\sin 25^\circ}{\cos 65^\circ}+\frac{\tan 23^\circ}{\cot 67^\circ}$

$$=\frac{\sin 25^\circ}{\sin(90^\circ-65^\circ)}+\frac{\tan 23^\circ}{\tan(90^\circ-67^\circ)}$$

$$=\frac{\sin 25^\circ}{\sin 25^\circ}+\frac{\tan 23^\circ}{\tan 23^\circ}$$

= 1 + 1 = 2

**14.** If $\sin(x - 20)^\circ = \cos(3x - 10)^\circ$, then find the value of x.

**Sol.** $\sin(x - 20)^\circ = \cos(3x - 10)^\circ$

$\Rightarrow$ $\cos[90^\circ - (x - 20)^\circ] = \cos(3x - 10)^\circ$

By comparing the coefficient

$90^\circ - x^\circ + 20^\circ = 3x^\circ - 10^\circ$

$\Rightarrow$ $110^\circ + 10^\circ = 3x^\circ + x^\circ$

$\Rightarrow$ $120^\circ = 4x$

$\Rightarrow$ $x=\frac{120^\circ}{4}=30^\circ$

**15.** If $\sec^2\theta(1 + \sin\theta)(1 - \sin\theta) = k$, then find the value of k.

**Sol.** $\sec^2\theta(1 + \sin\theta)(1 - \sin\theta) = \sec^2\theta(1 - \sin^2\theta)$

$[(a + b)(a - b) = a^2 - b^2]$ $\quad [\because \cos^2\theta + \sin^2\theta = 1]$

$= \sec^2\theta . \cos^2\theta = 1$ $\quad \left[\because \sec\theta = \frac{1}{\cos\theta}\right]$

$\therefore \quad k = 1$

**16.** If $\sec A = \frac{15}{7}$ and $A + B = 90°$, find the value of cosec B.

**Sol.** We have, $\sec A = \frac{15}{7}$ and $A + B = 90°$

$\Rightarrow \quad A = 90° - B$

$\therefore \quad \sec A = \sec(90° - B)$

$\Rightarrow \quad \sec A = \operatorname{cosec} B$ $\quad [\because \sec(90° - \theta) = \operatorname{cosec}\theta]$

$\Rightarrow \quad \operatorname{cosec} B = \frac{15}{7}$ $\quad \left[\because \sec A = \frac{15}{7}\right]$

**Short answer type questions** **(3 marks)**

**17.** Evaluate : $\frac{3\tan^2 30° + \tan^2 60° + \operatorname{cosec} 30° - \tan 45°}{\cot^2 45°}$

**Sol.** $\frac{3\tan^2 30° + \tan^2 60° + \operatorname{cosec} 30° - \tan 45°}{\cot^2 45°}$

$= \frac{3 \times \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 + 2 - 1}{(1)^2}$

$= \frac{3 \times \frac{1}{3} + 3 + 2 - 1}{1}$

$= 1 + 3 + 2 - 1 = 5$

**18.** If $\sin(A + B) = 1$ and $\sin(A - B) = \frac{1}{2}$, $0 \le A, B \le 90°$ and $A > B$, then find A and B.

**Sol.** $\sin(A + B) = 1 = \sin 90°$

$\Rightarrow \quad A + B = 90°$ ... (i)

$\sin(A - B) = \frac{1}{2} = \sin 30°$

$\Rightarrow \quad A - B = 30°$ ... (ii)

Adding equation (i) and (ii)

$2A = 120°$

$\Rightarrow \quad A = 60°$

Putting in equation (i)

$60° + B = 90°$

$\Rightarrow \quad B = 30°$

$\therefore \quad A = 60° \ \& \ B = 30°$

**19.** Evaluate : $\dfrac{6\sin 23^\circ + \sec 79^\circ + 3\tan 48^\circ}{\operatorname{cosec} 11^\circ + 3\cot 42^\circ + 6\cos 67^\circ}$

**Sol.** $\dfrac{6\sin 23^\circ + \sec 79^\circ + 3\tan 48^\circ}{\operatorname{cosec} 11^\circ + 3\cot 42^\circ + 6\cos 67^\circ}$

$= \dfrac{6\cos(90^\circ - 23^\circ) + \operatorname{cosec}(90^\circ - 79^\circ) + 3\cot(90^\circ - 48^\circ)}{\operatorname{cosec} 11^\circ + 3\cot 42^\circ + 6\cos 67^\circ}$

$= \dfrac{6\cos 67^\circ + \operatorname{cosec} 11^\circ + 3\cot 42^\circ}{\operatorname{cosec} 11^\circ + 3\cot 42^\circ + 6\cos 67^\circ}$

= 1

**20.** If tan A + cot A = 2, then find the value of $\tan^2 A + \cot^2 A$.

**Sol.** tan A + cot A = 2

On squaring both sides,

$(\tan A + \cot A)^2 = (2)^2$

$\Rightarrow$ $\tan^2 A + \cot^2 A + 2\tan A . \cot A = 4$

$\Rightarrow$ $\tan^2 A + \cot^2 A + 2\tan A \times \dfrac{1}{\tan A} = 4$

$\Rightarrow$ $\tan^2 A + \cot^2 A + 2 = 4$

$\Rightarrow$ $\tan^2 A + \cot^2 A = 4 - 2$

$\therefore$ $\tan^2 A + \cot^2 A = 2$

**21.** Without using tables, evaluate the following :

$3\cos 68^\circ . \operatorname{cosec} 22^\circ - \dfrac{1}{2}\tan 43^\circ . \tan 47^\circ . \tan 12^\circ . \tan 60^\circ . \tan 78^\circ$

**Sol.** We have,

$3\cos 68^\circ . \operatorname{cosec} 22^\circ - \dfrac{1}{2}\tan 43^\circ . \tan 47^\circ . \tan 12^\circ . \tan 60^\circ . \tan 78^\circ$

$= 3\cos(90^\circ - 22^\circ) . \operatorname{cosec} 22^\circ - \dfrac{1}{2}\{\tan 43^\circ . \tan(90^\circ - 43^\circ)\} \tan 12^\circ \sqrt{3}\tan(90^\circ - 12^\circ)$

$= 3\sin 22^\circ \operatorname{cosec} 22^\circ - \dfrac{1}{2}\{\tan 43^\circ \cot 43^\circ \tan 12^\circ \sqrt{3} \cot 12^\circ\}$

$= 3 \times 1 - \dfrac{1}{2} \times 1 \times 1 \times \sqrt{3} = 3 - \dfrac{\sqrt{3}}{2} = \dfrac{6 - \sqrt{3}}{2}$

**22.** Without using trigonometric tables, evaluate

$$\frac{7\cos 70^\circ}{2\sin 20^\circ}+\frac{3}{2}\frac{\cos 55^\circ \operatorname{cosec} 35^\circ}{\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 85^\circ \tan 65^\circ}$$

**Sol.** $$\frac{7\cos(90^\circ-20^\circ)}{2\sin 20^\circ}+\frac{3}{2}\frac{\cos(90^\circ-35^\circ)\operatorname{cosec} 35^\circ}{\tan(90^\circ-85^\circ).\tan(90^\circ-65^\circ).1.\tan 85^\circ.\tan 65^\circ}$$

$$\frac{7\sin 20^\circ}{2\sin 20^\circ}+\frac{3}{2}\frac{\sin 35^\circ.\operatorname{cosec} 35^\circ}{\cot 85^\circ.\cot 65^\circ.1.\tan 85^\circ.\tan 65^\circ}$$

$$\Rightarrow \quad \frac{7}{2}+\frac{3}{2}=\frac{10}{2}=5$$

**23.** Simplify : $\dfrac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}+\sin\theta\cos\theta$

**Sol.** We have $\dfrac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}+\sin\theta\cos\theta$

$$=\frac{(\sin\theta+\cos\theta)(\sin^2\theta-\sin\theta\cos\theta+\cos^2\theta)}{\sin\theta+\cos\theta}+\sin\theta\cos\theta$$

$= 1 - \sin\theta\cos\theta + \sin\theta\cos\theta = 1$ $\quad [\because \sin^2\theta + \cos^2\theta = 1]$

**24.** Prove that: $\tan 1^\circ \tan 2^\circ \tan 3^\circ ........ \tan 89^\circ = 1$

**Sol.** We have,

LHS = $\tan 1^\circ \tan 2^\circ \tan 3^\circ ...... \tan 89^\circ$

$= \tan(90^\circ - 89^\circ)\tan(90^\circ - 88^\circ)\tan(90^\circ - 87^\circ) ...... \tan(90^\circ - 44^\circ)\tan 45^\circ ..... \tan 87^\circ \tan 88^\circ \tan 89^\circ$

$= \cot 89^\circ \cot 88^\circ \cot 87^\circ .... \tan 45^\circ .... \tan 87^\circ \tan 88^\circ \tan 89^\circ$

$= (\cot 89^\circ \tan 89^\circ)(\cot 88^\circ \tan 88^\circ)(\cot 87^\circ \tan 87^\circ) ... (\cot 44^\circ \tan 44^\circ).\tan 45^\circ$

$[\because \cot\theta\tan\theta = 1 \text{ and } \tan 45^\circ = 1]$

$= 1 \times 1 \times 1 ...... \times 1 = 1$ = RHS

**25.** Prove that : $\dfrac{\cos A}{1-\tan A}+\dfrac{\sin A}{1-\cot A}=\sin A+\cos A$

**Sol.** L.H.S = $\dfrac{\cos A}{1-\tan A}+\dfrac{\sin A}{1-\cot A}$

$$=\frac{\cos A}{1-\left(\frac{\sin A}{\cos A}\right)}+\frac{\sin A}{1-\left(\frac{\cos A}{\sin A}\right)}$$

$$=\frac{\cos^2 A}{\cos A-\sin A}+\frac{\sin^2 A}{\sin A-\cos A}$$

$$=\frac{\cos^2 A}{\cos A-\sin A}-\frac{\sin^2 A}{\cos A-\sin A}$$

$$=\frac{\cos^2 A-\sin^2 A}{\cos A-\sin A}$$

$$=\frac{(\cos A-\sin A)(\cos A+\sin A)}{(\cos A-\sin A)}$$

$= \cos A + \sin A$ = RHS $\quad$ Hence proved

**26.** Prove that : $(\tan\theta + \sec\theta - 1)(\tan\theta + 1 + \sec\theta) = \dfrac{2\sin\theta}{1-\sin\theta}$

**Sol.** LHS = $(\sec\theta + \tan\theta - 1)(\sec\theta + \tan\theta + 1)$

$= (\sec\theta + \tan\theta)^2 - 1$

$= \sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta - 1$

$= 1 + \tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta - 1$

$= 2\tan^2\theta + 2\sec\theta\tan\theta = 2\tan\theta(\tan\theta + \sec\theta)$

$$=\frac{2\sin\theta}{\cos\theta}\left(\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)=\frac{2\sin\theta(\sin\theta+1)}{\cos^2\theta}$$

$$=\frac{2\sin\theta(\sin\theta+1)}{1-\sin^2\theta}=\frac{2\sin\theta(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}=\frac{2\sin\theta}{1-\sin\theta}$$

= RHS Hence proved

**27.** If $\sec\theta = x + \dfrac{1}{4x}$, prove that $\sec\theta + \tan\theta = 2x$ or $\dfrac{1}{2x}$

**Sol.** $\sec\theta = x + \dfrac{1}{4x}$

$\tan^2\theta = \sec^2\theta - 1$

$$=\left(x+\frac{1}{4x}\right)^2-1$$

$$=x^2+\frac{1}{16x^2}+\frac{1}{2}-1$$

$$=x^2+\frac{1}{16x^2}-\frac{1}{2}$$

$$=\left(x-\frac{1}{4x}\right)^2$$

$$\tan\theta=\left(x-\frac{1}{4x}\right)\text{or}-\left(x-\frac{1}{4x}\right)$$

$$\sec\theta + \tan\theta = x+\frac{1}{4x}+x-\frac{1}{4}x = 2x$$

**[OR]**

$$=x+\frac{1}{4x}-x+\frac{1}{4x}=\frac{1}{2x}$$

$\therefore$ $\sec\theta + \tan\theta = 2x$ or $\dfrac{1}{2x}$ Hence proved

**28.** If $x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$ and $x \sin\theta = y \cos\theta$, prove that $x^2 + y^2 = 1$

**Sol.** Given : $x \sin\theta = y \cos\theta$

$\Rightarrow \quad x = \dfrac{y\cos\theta}{\sin\theta}$ ... (i)

and $\quad x\sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$ ... (ii)

Eliminating x from equation (i) and equation (ii),

$\dfrac{y\cos\theta}{\sin\theta}\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow \quad y \cos\theta \sin^2\theta + y \cos^3\theta = \sin\theta \cos\theta$

$\Rightarrow \quad y \cos\theta [\sin^2\theta + \cos^2\theta] = \sin\theta \cos\theta$

$\Rightarrow \quad y \cos\theta = \sin\theta \cos\theta$

$\Rightarrow \quad y = \sin\theta$ ... (iii)

Substituting this value of y in equation (i),

$x = \cos\theta$ ... (iv)

$\therefore$ Squaring and adding equation(iii) and equation(iv), we get

$x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$ Hence proved

**29.** Prove the following : $\dfrac{\tan A}{1-\cot A}+\dfrac{\cot A}{1-\tan A} = 1 + \tan A + \cot A$

**Sol.** L.H.S. = $\dfrac{\tan A}{1-\cot A}+\dfrac{\cot A}{1-\tan A}$

$$=\frac{\tan A}{\left(1-\dfrac{1}{\tan A}\right)}+\frac{\dfrac{1}{\tan A}}{1-\tan A}$$

$$=\frac{\tan A}{\left(\dfrac{\tan A-1}{\tan A}\right)}+\frac{1}{\tan A(1-\tan A)}$$

$$=\frac{\tan^2 A}{\tan A-1}-\frac{1}{\tan A(\tan A-1)}=\frac{\tan^3 A-1}{\tan A(\tan A-1)}$$

$$=\frac{(\tan A-1)(\tan^2 A+\tan A+1)}{\tan A(\tan A-1)}$$

$$=\frac{\tan^2 A+\tan A+1}{\tan A}=\tan A+1+\frac{1}{\tan A}$$

$= 1 + \tan A + \cot A =$ R.H.S. Hence proved.

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**30.** Prove that : $\sec^2\theta - \frac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta - \cos^2\theta} = 1$

**Sol.** L.H.S. = $\sec^2\theta - \frac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta - \cos^2\theta}$

$= \sec^2\theta - \frac{\sin^2\theta(1-2\sin^2\theta)}{\cos^2\theta(2\cos^2\theta - 1)}$ $\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$= \sec^2\theta - \frac{\tan^2\theta(1-2\sin^2\theta)}{2(1-\sin^2\theta)-1}$

$= \sec^2\theta - \frac{\tan^2\theta(1-2\sin^2\theta)}{1-2\sin^2\theta}$

$= \sec^2\theta - \tan^2\theta = 1 =$ R.H.S. Hence proved

**31.** Prove that : $(1 - \sin\theta + \cos\theta)^2 = 2(1 + \cos\theta)(1 - \sin\theta)$

**Sol.** We have,

LHS = $(1 - \sin\theta + \cos\theta)^2$

$\Rightarrow$ $1 + \sin^2\theta + \cos^2\theta - 2\sin\theta + 2\cos\theta - 2\sin\theta\cos\theta$ $\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$\Rightarrow$ $2 - 2\sin\theta + 2\cos\theta - 2\sin\theta\cos\theta$

$\Rightarrow$ $2(1 - \sin\theta) + 2\cos\theta(1 - \sin\theta)$

$\Rightarrow$ $2(1 - \sin\theta)(1 + \cos\theta) =$ RHS Hence proved

**32.** If $\sin\theta + \sin^2\theta = 1$, prove that $\cos^2\theta + \cos^4\theta = 1$

**Sol.** We have,

$\sin\theta + \sin^2\theta = 1$

$\Rightarrow$ $\sin\theta = 1 - \sin^2\theta$ $\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$\Rightarrow$ $\sin\theta = \cos^2\theta$

Now, $\cos^2\theta + \cos^4\theta = \cos^2\theta + (\cos^2\theta)^2$

$\Rightarrow$ $\cos^2\theta + \cos^4\theta = \cos^2\theta + \sin^2\theta$ $[\because \cos^2\theta = \sin\theta]$

$\Rightarrow$ $\cos^2\theta + \cos^4\theta = 1$ Hence proved

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## Long answer type questions (5 marks)

**33.** Prove that : $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}=2\text{cosec}\theta$

**Sol.** $\text{LHS}=\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}$

$=\frac{(\sec\theta-1)+(\sec\theta+1)}{\sqrt{(\sec\theta+1)(\sec\theta-1)}}$

$=\frac{2\sec\theta}{\sqrt{\sec^2\theta-1}}=\frac{2\sec\theta}{\tan\theta}$ $(\because \tan^2\theta = \sec^2\theta - 1)$

$=2\times\frac{1}{\cos\theta}\times\frac{\cos\theta}{\sin\theta}=2\times\frac{1}{\sin\theta}=2\text{cosec}\theta$

= RHS Hence proved

**34.** If $\text{cosec}\theta + \cot\theta = p$, then prove that $\cos\theta=\frac{p^2-1}{p^2+1}$

**Sol.** $\text{RHS} = \frac{p^2-1}{p^2+1}$

$=\frac{(\text{cosec}\theta+\cot\theta)^2-1}{(\text{cosec}\theta+\cot\theta)^2+1}$

$=\frac{\text{cosec}^2\theta+\cot^2\theta+2\text{cosec}\theta\cot\theta-1}{\text{cosec}^2\theta+\cot^2\theta+2\text{cosec}\theta\cot\theta+1}$ $\left[\because 1+\cot^2\theta=\text{cosec}^2\theta\right]$

$=\frac{1+\cot^2\theta+\cot^2\theta+2\text{cosec}\theta\cot\theta-1}{\text{cosec}^2\theta+\text{cosec}^2\theta-1+2\text{cosec}\theta\cot\theta+1}$

$=\frac{2\cot\theta(\cot\theta+\text{cosec}\theta)}{2\text{cosec}\theta(\text{cosec}\theta+\cot\theta)}$

$=\frac{\cos\theta}{\sin\theta}\times\sin\theta=\cos\theta=\text{LHS}$

Hence proved

**35.** If $x = r\sin A\cos C$, $y = r\sin A\sin C$ and $z = r\cos A$, then prove that $x^2 + y^2 + z^2 = r^2$

**Sol.** Since, $x^2 = r^2\sin^2A\cos^2C$

$y^2 = r^2\sin^2A\sin^2C$

and $z^2 = r^2\cos^2A$

L.H.S. $= x^2 + y^2 + z^2$

$= r^2\sin^2A\cos^2C + r^2\sin^2A\sin^2C + r^2\cos^2A$

$= r^2\sin^2A(\cos^2C + \sin^2C) + r^2\cos^2A$ $\left[\because \sin^2\theta+\cos^2\theta=1\right]$

$= r^2\sin^2A + r^2\cos^2A$

$= r^2(\sin^2A + \cos^2A)$

$= r^2$

= R.H.S Hence proved

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**36.** Prove the following identities :

$$\tan^2A - \tan^2B = \frac{\cos^2 B - \cos^2 A}{\cos^2 B\cos^2 A} = \frac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B}$$

**Sol.** LHS = $\tan^2A - \tan^2B$

$$= \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}$$

$$= \frac{\sin^2 A\cos^2 B - \cos^2 A\sin^2 B}{\cos^2 A\cos^2 B} \qquad \left[\because \sin^2\theta + \cos^2\theta = 1\right]$$

$$= \frac{(1-\cos^2 A)\cos^2 B - \cos^2 A(1-\cos^2 B)}{\cos^2 A\cos^2 B}$$

$$= \frac{\cos^2 B - \cos^2 A}{\cos^2 A\cos^2 B} \qquad \text{... (i)}$$

$$= \frac{(1-\sin^2 B)-(1-\sin^2 A)}{\cos^2 A\cos^2 B}$$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B} \qquad \text{... (ii)}$$

From (i) and (ii)

$$\tan^2A - \tan^2B = \frac{\cos^2 B - \cos^2 A}{\cos^2 B\cos^2 A} = \frac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B}$$

Hence proved

**37.** Prove that : $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

**Sol.** L.H.S = $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$$ [Dividing the numerator and denominator by sin A]

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{\{(\cot A + \operatorname{cosec} A) - 1\}(\cot A + \operatorname{cosec} A)}{\{(\cot A - \operatorname{cosec} A) + 1\}(\cot A + \operatorname{cosec} A)}$$

$$= \frac{\{(\cot A + \operatorname{cosec} A) - 1\}(\cot A + \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A + (\cot A + \operatorname{cosec} A)} \qquad \left[\because 1 + \cot^2 A = \operatorname{cosec}^2 A\right]$$

$$= \frac{\{(\cot A + \operatorname{cosec} A) - 1\}(\cot A + \operatorname{cosec} A)}{-1 + (\cot A + \operatorname{cosec} A)} = \cot A + \operatorname{cosec} A = \text{R.H.S.}$$

Hence proved

**38.** If $a\cos\theta - b\sin\theta = c$, prove that $a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$

**Sol.** We have,

$(a\cos\theta - b\sin\theta)^2 + (a\sin\theta + b\cos\theta)^2$

$= (a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta) + (a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta)$

$= a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$ $\quad \left[\because \sin^2\theta+\cos^2\theta=1\right]$

$= a^2 + b^2$

$\Rightarrow c^2 + (a\sin\theta + b\cos\theta)^2 = a^2 + b^2$ $\quad [\because a\cos\theta - b\sin\theta = c]$

$\Rightarrow (a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - c^2$

$\Rightarrow a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$ Hence proved

**OR**

$a\cos\theta - b\sin\theta = c$

$\Rightarrow (a\cos\theta - b\sin\theta)^2 = c^2$ [Squaring both sides]

$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta - 2ab\cos\theta\sin\theta = c^2$

$\Rightarrow a^2(1-\sin^2\theta) + b^2(1-\cos^2\theta) - 2ab\sin\theta\cos\theta = c^2.$

$\Rightarrow a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta = a^2 + b^2 - c^2.$

$\Rightarrow (a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - c^2$

$\Rightarrow a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$ Hence proved

**39.** $\dfrac{1}{\sec A-\tan A}-\dfrac{1}{\cos A}=\dfrac{1}{\cos A}-\dfrac{1}{\sec A+\tan A}$

**Sol.** LHS $= \dfrac{1}{\sec A-\tan A}-\dfrac{1}{\cos A}$

$=\dfrac{1}{\sec A-\tan A}\times\dfrac{\sec A+\tan A}{\sec A+\tan A}-\dfrac{1}{\cos A}$

$=\dfrac{\sec A+\tan A}{\sec^2 A-\tan^2 A}-\dfrac{1}{\cos A}$ $\quad \left[\because 1+\tan^2 A=\sec^2 A\right]$

$= \sec A + \tan A - \sec A = \tan A$

RHS $= \dfrac{1}{\cos A}-\dfrac{1}{\sec A+\tan A}$

$=\sec A-\left(\dfrac{\sec A-\tan A}{\sec^2 A-\tan^2 A}\right)$

$= \sec A - \sec A + \tan A$

$= \tan A$

Since, LHS = RHS Hence proved

**40.** If $\dfrac{\cos\alpha}{\cos\beta} = m$ and $\dfrac{\cos\alpha}{\sin\beta} = n$ show that $(m^2 + n^2)\cos^2\beta = n^2$

**Sol.** We have,

LHS = $(m^2 + n^2)\cos^2\beta$

$\Rightarrow$ LHS = $\left(\dfrac{\cos^2\alpha}{\cos^2\beta}+\dfrac{\cos^2\alpha}{\sin^2\beta}\right)\cos^2\beta$ $\left[\therefore m=\dfrac{\cos\alpha}{\cos\beta} \text{ and } n=\dfrac{\cos\alpha}{\sin\beta}\right]$

$\Rightarrow \left(\dfrac{\cos^2\alpha\sin^2\beta+\cos^2\alpha\cos^2\beta}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$

$\Rightarrow \cos^2\alpha\left(\dfrac{\sin^2\beta+\cos^2\beta}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$ $\left[\because \sin^2\theta+\cos^2\theta=1\right]$

$\Rightarrow \cos^2\alpha\left(\dfrac{1}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$

$\Rightarrow \dfrac{\cos^2\alpha}{\sin^2\beta}=\left(\dfrac{\cos\alpha}{\sin\beta}\right)^2=n^2=\text{RHS}$ Hence proved

**41.** If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$

**Sol.** We have,

LHS = $m^2 - n^2$

$\Rightarrow$ LHS = $(\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$

$= 4\tan\theta\sin\theta$ $[\because (a + b)^2 - (a - b)^2 = 4ab]$

And,

RHS = $4\sqrt{mn}$

$= 4\sqrt{(\tan\theta+\sin\theta)(\tan\theta-\sin\theta)}$

$=4\sqrt{\tan^2\theta-\sin^2\theta}$

$=4\sqrt{\dfrac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta}$

$=4\sqrt{\dfrac{\sin^2\theta-\sin^2\theta\cos^2\theta}{\cos^2\theta}}$

$=4\sqrt{\dfrac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}}$ $\left[\because \sin^2\theta+\cos^2\theta=1\right]$

$=4\sqrt{\dfrac{\sin^4\theta}{\cos^2\theta}}=4\dfrac{\sin^2\theta}{\cos\theta}=4\sin\theta\dfrac{\sin\theta}{\cos\theta}$

$= 4\sin\theta\tan\theta$

Thus, we have

LHS = RHS, i.e., $m^2 - n^2 = 4\sqrt{mn}$ Hence proved

**42.** If cos θ + sin θ = $\sqrt{2}\cos\theta$ then prove that : cosθ – sinθ = $\sqrt{2}\sin\theta$

**Sol.** We have,

$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\Rightarrow (\cos\theta + \sin\theta)^2 = 2\cos^2\theta$

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta = 2\cos^2\theta$

$\Rightarrow \cos^2\theta - 2\cos\theta\sin\theta = \sin^2\theta$

$\Rightarrow \cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta = 2\sin^2\theta$

(Adding $\sin^2\theta$ both side)

$\Rightarrow (\cos\theta - \sin\theta)^2 = 2\sin^2\theta$

$\Rightarrow \cos\theta - \sin\theta = \sqrt{2}\sin\theta$ Hence proved

**43.** $\dfrac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta} = 2 + \dfrac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$

**Sol.** We have to prove that

$$\frac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta} = 2 + \frac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$$

or, $\dfrac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta} - \dfrac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta} = 2$

Now,

$$\Rightarrow \text{LHS} = \frac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta} - \frac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$$

$$\Rightarrow \frac{\sin\theta}{\operatorname{cosec}\theta+\cot\theta} + \frac{\sin\theta}{\operatorname{cosec}\theta-\cot\theta}$$

$$\Rightarrow \sin\theta\left\{\frac{1}{\operatorname{cosec}\theta+\cot\theta} + \frac{1}{\operatorname{cosec}\theta-\cot\theta}\right\}$$

$$\Rightarrow \sin\theta\left\{\frac{\operatorname{cosec}\theta-\cot\theta+\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}^2\theta-\cot^2\theta}\right\}$$

$\Rightarrow$ LHS = $\sin\theta(2\operatorname{cosec}\theta)$ [$\because \operatorname{cosec}^2\theta - \cot^2\theta = 1$]

$\Rightarrow$ LHS = 2 = RHS Hence proved

**44.** If sinθ + cosθ = $\sqrt{2}\sin(90°-\theta)$, determine cot θ.

**Sol.** We have,

$\sin\theta + \cos\theta = \sqrt{2}\sin(90° - \theta)$

$\Rightarrow \sin\theta + \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sin\theta = \sqrt{2}\cos\theta - \cos\theta$

$\Rightarrow \sin\theta = (\sqrt{2}-1)\cos\theta$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = (\sqrt{2}-1) \Rightarrow \tan\theta = (\sqrt{2}-1)$$

$$\Rightarrow \cot\theta = \frac{1}{\sqrt{2}-1} \qquad \left[\because \cot\theta = \frac{1}{\tan\theta}\right]$$

$$\Rightarrow \cot\theta = \frac{(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{\sqrt{2}+1}{2-1} = \sqrt{2}+1$$

**45.** Prove that following identities :

$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

**Sol.** We have,

LHS = $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$

$\Rightarrow 2[(\sin^2\theta)^3 + (\cos^2\theta)^3] - 3(\sin^4\theta + \cos^4\theta) + 1$

$\Rightarrow 2[(\sin^2\theta + \cos^2\theta)\{(\sin^2\theta)^2 + (\cos^2\theta)^2 - \sin^2\theta\cos^2\theta\}] - 3(\sin^4\theta + \cos^4\theta) + 1$

$\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$\Rightarrow 2\{(\sin^2\theta)^2 + (\cos^2\theta)^2 - \sin^2\theta\cos^2\theta\} - 3(\sin^4\theta + \cos^4\theta) + 1$

$\Rightarrow 2\sin^4\theta + 2\cos^4\theta - 2\sin^2\theta\cos^2\theta - 3\sin^4\theta - 3\cos^4\theta + 1$

$\Rightarrow -\sin^4\theta - \cos^4\theta - 2\sin^2\theta\cos^2\theta + 1$

$\Rightarrow -(\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta) + 1$

$\Rightarrow -(\sin^2\theta + \cos^2\theta)^2 + 1 = -1 + 1 = 0 = \text{RHS}$ Hence proved

## Case Study type questions (4 marks)

**1.** A group of students of class X visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All-India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Rajpath (formerly called the Kingsway), is about 138 feet (42 metres) in height.

(i) What is the angle of elevation if they are standing at a distance of 42 m away from the monument?

(ii) They want to see the monument at an angle of $60^\circ$. So, they want to know the distance where they should stand and hence find the distance. (use $\sqrt{3} = 1.732$)

(iii) If the altitude of the Sun is at $60^\circ$, then find the height of the vertical monument that will cast a shadow of length 20 m.

(iv) The ratio of the length of the monument and its shadow is 1 : 1. Find the angle of elevation of the Sun.

**[OR]**

(iv) What is the angle of elevation if they are standing at a distance of $138\sqrt{3}$ feet away from the monument?

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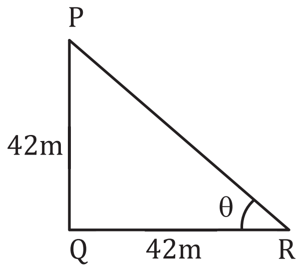
**Sol.** (i)



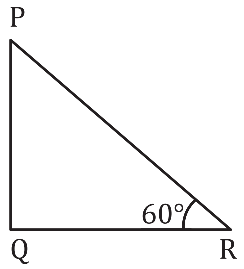

Let PQ = Height of India Gate monument

QR = Distance from the monument

$\Rightarrow$ $\tan\theta = \frac{45}{45} = 1$

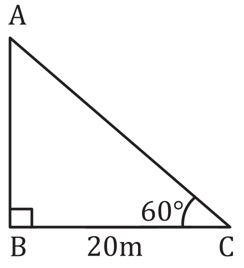
$\Rightarrow$ $\theta = 45°$

(ii)




Required distance, $QR = \frac{PQ}{\tan 60°} = \frac{42}{\sqrt{3}} = 24.24$ m

(iii)



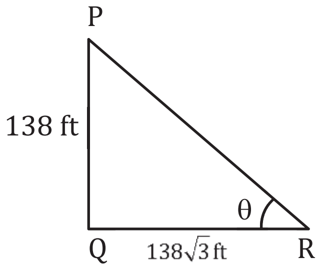

Height of monument, AB = BC × tan60°

$= 20\sqrt{3}\,\text{m}$

(iv) Angle of elevation of the Sun will be 45°.

**[OR]**

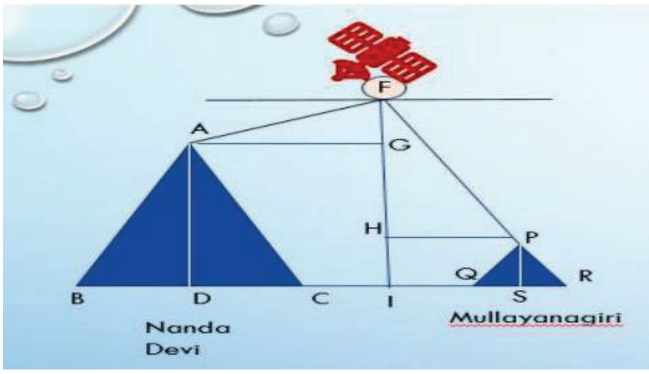
(iv)




Angle of elevation, $\tan\theta = \frac{138}{138\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\theta = 30°$

**2.** A satellite flying at height 'h' is watching the top of the two tallest mountains in Uttarakhand and Karnataka, them being Nanda Devi (height 7,816m) and Mullayanagiri (height 1,930 m). The angles of depression from the satellite to the top of Nanda Devi and Mullayanagiri are 30° and 60° respectively. If the horizontal distance between the peaks of the two mountains is 1937 km, and the satellite is vertically above the midpoint of the distance between the two mountains.




(i) Find the distance of the satellite from the top of Nanda Devi.
(ii) Find the distance of the satellite from the top of Mullayanagiri.
(iii) Find the distance of the satellite from the ground.
(iv) What is the angle of elevation if a man is standing at a distance of 7816m from Nanda Devi?

**[OR]**

(iv) If a mile stone very far away makes $45^o$ to the top of Mullanyangiri mountain, then find the distance of this mile stone from the mountain.

**Sol.** (i) $AG = DI = \frac{1937}{2}m = PH$

In $\Delta FGA$, $\cos 30° = \frac{AG}{AF}$

$\Rightarrow AF = \frac{1937}{2} \times \frac{2}{\sqrt{3}} = 1118.32$ km

(ii) In $\Delta FHP$,

$\cos 60° = \frac{PH}{FP} \Rightarrow FP = \frac{1937}{2} \times \frac{2}{1} = 1937$ km

(iii) In $\Delta AFG$,

$\tan 30° = \frac{FG}{AG} \Rightarrow FG = \frac{1}{\sqrt{3}} \times \frac{1932}{2} = 559.16$ km

Distance of satellite form ground = 559.16 + GI
= 559.16 + 7.816
= 566.776 km

(iv) Angle of elevation, $\tan\theta = \frac{7816}{7816} = 1$

$\Rightarrow \theta = 45°$

**[OR]**

(iv) Distance of mile stone from the mountain = $\frac{PS}{\tan 45°} = \frac{1930}{1} = 1930$ m

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# 9 Heights and Distances

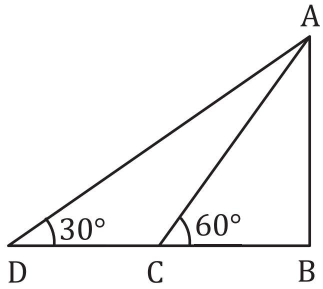
**Multiple choice questions** **(1 mark)**

**1.** If the length of the shadow of a tree is decreasing then the angle of elevation is:

(1) increasing (2) decreasing

(3) remains the same (4) none of the above

**Sol.** **Option (1)**

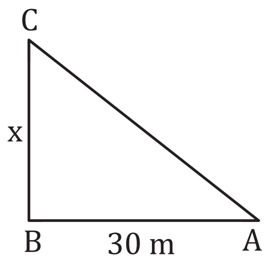



as the shadow reaches from point D to C towards the direction of the tree, the angle of elevation increases from 30° to 60°.

**2.** The angle of elevation of the top of a building from a point on the ground, which is 30 m away from the foot of the building, is 30°. The height of the building is:

(1) 10 m (2) $\frac{30}{\sqrt{3}}$ m (3) $\frac{\sqrt{3}}{10}$ m (d) 30 m

**Sol.** **Option (2)**




Say x is the height of the building.

a is a point 30 m away from the foot of the building.

here, height is the perpendicular distance between point C and foot of building. The angle of elevation formed is 30°.

hence, $\tan 30° = \frac{\text{Perpendicular}}{\text{base}} = \frac{x}{30}$

$$\frac{1}{\sqrt{3}} = \frac{x}{30}$$

$$x = \frac{30}{\sqrt{3}} \text{ m}$$

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3. If the height of the building and distance from the foot of building to a point is increased by 20%, then the angle of elevation on the top of the building:

(1) increases (2) decreases (3) do not change (4) None of the above

**Sol. Option (3)**

We know, for an angle of elevation θ, $\tan\theta = \dfrac{\text{Height of building}}{\text{Distance from the point}}$

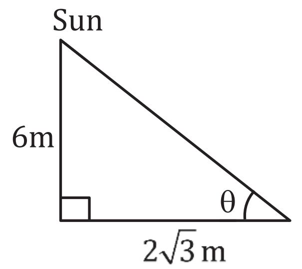
If we increase both the value, the angle of elevation remains unchanged.

4. If a tower 6m high casts a shadow of $2\sqrt{3}$ m long on the ground, then the sun's elevation is:

(1) 60° (2) 45° (3) 30° (4) 90°

**Sol. Option (1)**

As per the given question:




hence, $\tan\theta = \dfrac{6}{2\sqrt{3}}$

$\tan\theta = \sqrt{3}$

$\tan\theta = \tan 60°$

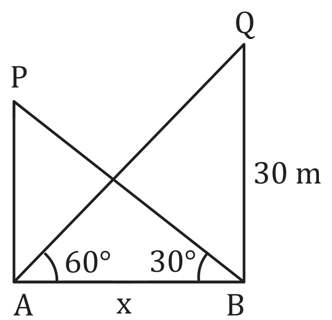
$\Rightarrow \theta = 60°$

5. The angle of elevation of the top of a building 30 m high from the foot of another building in the same plane is 60°, and also the angle of elevation of the top of the second building from the foot of the first building is 30°, then the height of another building is

(1) 10 m (2) $15\sqrt{3}$ m (3) $12\sqrt{3}$ m (4) 36 m

**Sol. Option (1)**

As per the given question:

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Hence,

$\tan 60° = \frac{30}{x}$

$\sqrt{3} = \frac{30}{x}$

$x = \frac{30}{\sqrt{3}}$

$x = \frac{30}{\sqrt{3}}$ m

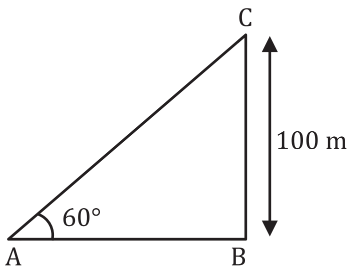
$\tan30° = \frac{PA}{AB}$

$PA = AB \times \frac{1}{\sqrt{3}} = \frac{30}{\sqrt{3}} \times \frac{1}{\sqrt{5}} = \frac{30}{3} = 10$ m

**6.** The length of the string of a kite flying at 100 m above the ground with the elevation of 60° is

(1) 100 m (2) $100\sqrt{2}$ m (3) $\frac{200}{\sqrt{3}}$ m (4) 200 m

**Sol. Option (3)**




AC = length of string of a kite

In right $\Delta ABC$

$\sin60° = \frac{BC}{AC}$

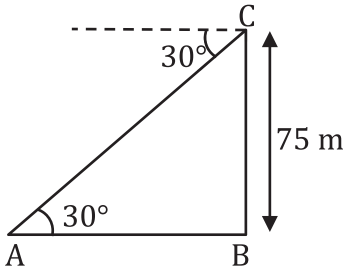
$\frac{\sqrt{3}}{2} = \frac{100}{AC}$

$AC = \frac{200}{\sqrt{3}}$ m

**7.** If the angle of depression of an object from a 75 m high tower is 30°, then the distance of the object from the base of the tower is

(1) $\sqrt{3}$ m (2) $50\sqrt{3}$ m (3) $75\sqrt{3}$ m (4) $150\sqrt{3}$ m

**Sol. Option (3)**




height of tower (BC) = 75 m

Distance between object and base of tower = AB

In right $\Delta ABC$

$\tan 30° = \frac{BC}{AB}$; $\frac{1}{\sqrt{3}} = \frac{75}{AB}$

$AB = 75\sqrt{3}\,m$

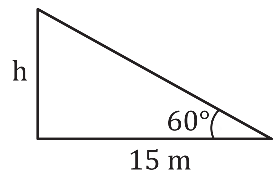
**8.** From a point on the ground, which is 15 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°. The height of the tower (in m) standing straight is:

(1) $15\sqrt{3}\,m$ (2) $10\sqrt{3}\ m$ (3) $12\sqrt{3}\ m$ (4) $20\sqrt{3}\ m$

**Sol. Option (1)**

We know: tan (angle of elevation) = $\frac{\text{height of tower}}{\text{Its distance from the point}}$

$\tan 60° = \frac{h}{15}$




$\sqrt{3} = \frac{h}{15}$

$h = 15\sqrt{3}\,m$

**Assertion reason questions** **(1 marks)**

**9.** **Assertion (A) :** The angle of elevation of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level.

**Reason (R) :** The angle of depression. of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol. Option (2)**

Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

**10.** **Assertion (A) :** The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.

**Reason (R) :** Trigonometric ratios are used to find height or length of an object or distance between two distant objects.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

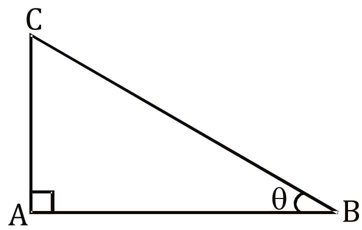
**Sol.** **Option (2)**

Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

## Very short answer type questions (2 mark)

**11.** The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3}:1$. What is the angle of elevation of the sun ?

**Sol.** Let AC be the height of tower, AB be the length of its shadow and $\theta$ be the angle of elevation of the sun.




In right $\Delta CAB$,

$$\tan\theta = \frac{CA}{AB}$$

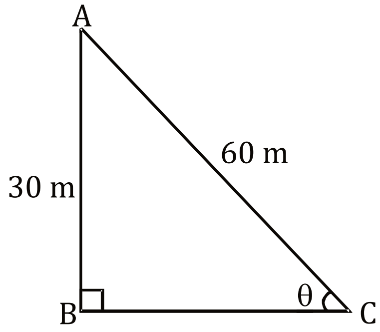
i.e., $\tan\theta = \dfrac{\sqrt{3}}{1}$ (Given)

$\Rightarrow$ $\tan\theta = \tan 60°$

$\Rightarrow$ $\theta = 60°$

**12.** A kite is flying at a height of 30 m from the ground. The length of string from the kite to the ground is 60 m. Assuming that there is no slack in the string. Find the angle of elevation of the kite at the ground.

**Sol.** Let A be the position of the kite. AC be the length of the string of the kite and θ be the angle of elevation of the kite at the ground.




$\therefore$ In right ΔABC, $\sin\theta = \dfrac{AB}{AC} = \dfrac{30}{60} = \dfrac{1}{2}$

$\Rightarrow \sin\theta = \dfrac{1}{2}$

$\Rightarrow \sin\theta = \sin 30°$

$\Rightarrow \theta = 30°$

**Short answer type questions** **(3 marks)**

**13.** From the top of a cliff 20 m high, the angle of elevation of the top of a tower is found to be equal to the angle of depression of the foot of the tower. Find the height of the tower.

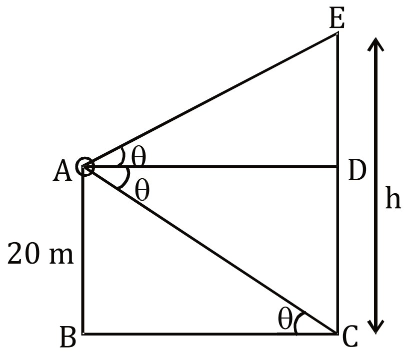
**Sol.** Let AB be the cliff and EC = h m be the height of tower.

$\therefore$ ED = EC – DC = EC – AB = (h – 20) m

In ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{20}{BC}$$

$\Rightarrow BC = \dfrac{20}{\tan\theta}$ ... (i)




In ΔEAD,

$$\tan\theta = \frac{ED}{AD} = \frac{ED}{BC} = \frac{(h-20)\tan\theta}{20} \quad \text{from eq.(i)}$$

$\Rightarrow h - 20 = 20 \Rightarrow h = 40$

$\therefore$ Height of tower is 40 m.

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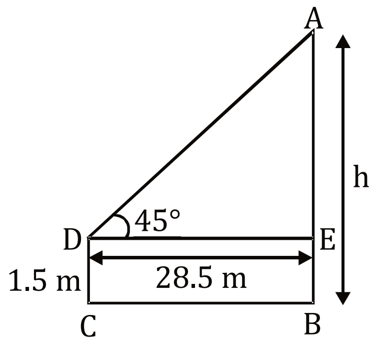
**14.** An observer 1.5 m tall is 28.5 m away from a tower. The angle of elevation of the top of the tower from his eyes is 45°. What is the height of the tower?

**Sol.** Let AB be the tower of height h and CD be the observer of height 1.5 m at a distance of 28.5 m from the tower AB.




In $\Delta AED$, we have

$$\Rightarrow \tan 45° = \frac{AE}{DE}$$

$$\Rightarrow 1 = \frac{AE}{28.5}$$

$$\Rightarrow AE = 28.5 \text{ m}$$

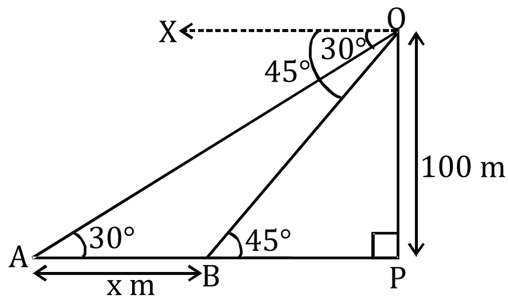
$$\therefore h = AE + BE = (28.5 + 1.5) \text{ m} = 30 \text{ m}$$

Hence, the height of the tower is 30 m.

**15.** As observed from the top of a 100 m high lighthouse above sea level, the angle of depression of ship, sailing directly towards it, changes from 30° to 45°. Determine the distance travelled by the ship during the period of observation.

**Sol.** Let A and B be the two position of the ship. Let x m be the distance travelled by the ship during the period of observation i.e. AB = x metres.

Let the observer be at O, the top of the light house PO.




In $\Delta OPB$, we have

$$\tan 45° = \frac{OP}{BP}$$

$$\Rightarrow 1 = \frac{100}{BP} \Rightarrow BP = 100 \text{ m}$$

In $\Delta OPA$, we have

$$\Rightarrow \tan 30° = \frac{OP}{AP}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{x + BP}$$

$$\Rightarrow x + BP = 100\sqrt{3}$$

$$\Rightarrow x + 100 = 100\sqrt{3}$$

$$\Rightarrow x = 100\sqrt{3} - 100$$

$$\Rightarrow x = 100(\sqrt{3} - 1)$$

$$= 100(1.732 - 1) = 73.2 \text{ m}$$

Hence, the distance travelled by the ship from A to B is 73.2 m.

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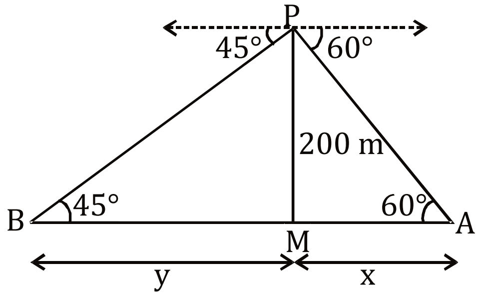
**16.** An aeroplane at an altitude of 200 m observes the angles of depression of two opposite points on two banks of the river to be 45° and 60°. Find, (in metres), the width of the river. [Use $\sqrt{3}$ = 1.732]

**Sol.** Let P be the position of the aeroplane and let A and B be two points on the two banks of a river such that the angles of depression at A and B are 60° and 45° respectively. Let AM = x m and BM = y m. We have to find AB.




In ΔAMP, we have

$$\tan 60° = \frac{PM}{AM} \Rightarrow \sqrt{3} = \frac{200}{x}$$

$$\Rightarrow \quad 200 = \sqrt{3}x \Rightarrow x = \frac{200}{\sqrt{3}} \qquad \text{... (i)}$$

In ΔBMP, we have

$$\tan 45° = \frac{PM}{BM}$$

$$\Rightarrow \quad 1 = \frac{200}{y} \Rightarrow y = 200 \qquad \text{... (ii)}$$

From equation (i) and (ii), we get

$$AB = x + y = \frac{200}{\sqrt{3}} + 200$$

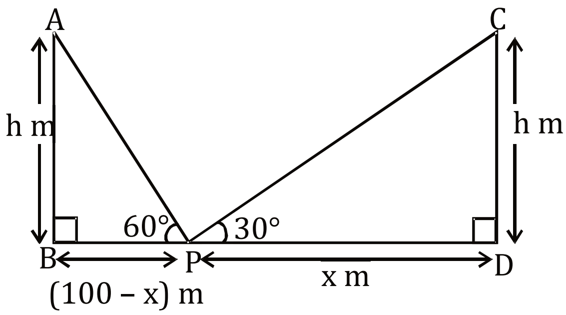
$$= 200\left(\frac{1}{\sqrt{3}} + 1\right) = 315.4\,\text{m}$$

Hence, the width of the river is 315.4 metres.

**17.** Two poles of equal heights are standing opposite to each other on either side of a road, which is 100 metres wide. From a point between them on the road, the angles of elevation of their tops are 30° and 60°. Find the position of the point and the heights of the poles.

**Sol.** Let AB and CD be two poles of equal height. Let P be the given point between the poles.

In ΔCDP, we have



tan 30° = $\frac{h}{x}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow x = h\sqrt{3}$ ... (i)

In ΔABP, we have

tan 60° = $\frac{h}{100-x}$

$\Rightarrow \quad \sqrt{3} = \frac{h}{100-x}$

$\Rightarrow \quad h = \sqrt{3}(100-x)$ ... (ii)

Substituting the value of h from (ii) in (i), we have

$x = \sqrt{3}\,(100-x)\sqrt{3}$

$\Rightarrow \quad x = 300 - 3x$

$\Rightarrow \quad x = \frac{300}{4} = 75$

$\therefore \quad 75 = h\sqrt{3}$ [From (i)]

$\Rightarrow \quad h = \frac{75}{\sqrt{3}} = \frac{75 \times \sqrt{3}}{3} = 25\sqrt{3}$

$= 25 \times 1.73 = 43.25$ m

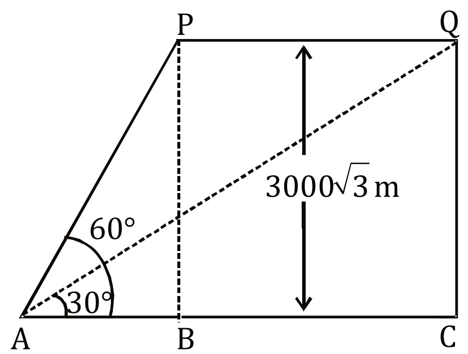
Hence, the point is 75 m away from pole CD and height of each pole is 43.25 m.

**18.** The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 30 seconds the angle of elevation becomes 30°. If the aeroplane is flying at a constant height of $3000\sqrt{3}$ m, find the speed of the aeroplane.

**Sol.** Let A be point of observation and P and Q be positions of the plane. Let ABC be the line through A and it is given that angles of elevation from point A to two position P and Q are 60° and 30°.

$\therefore \quad \angle PAB = 60°, \angle QAB = 30°$

Height = $3000\sqrt{3}$ m




So, in ΔABP, we have

tan 60° = $\frac{PB}{AB}$

$\sqrt{3} = \frac{3000\sqrt{3}}{AB} \Rightarrow AB = 3000$ m

In $\Delta ACQ$,

$$\tan 30° = \frac{QC}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{3000\sqrt{3}}{AC}$$

$\Rightarrow$ AC = 9000 m

$\therefore$ Distance = BC = AC – AB = 9000 m – 3000 m = 6000 m

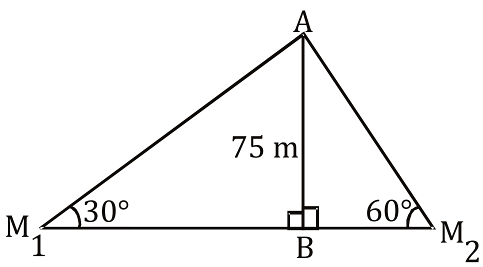
Also, plane travels for 30 seconds.

Hence, speed of plane = $\frac{6000}{30}$ = 200 m/s

= 720 km/h.

**19.** Two men on either side of a 75 m high building and in line with base of building observe the angle of elevation of the top of the building as 30° and 60°. Find the distance between the two men. (Use $\sqrt{3} = 1.73$)

**Sol.**




In $\Delta ABM_1$

$$\frac{AB}{BM_1} = \tan 30° \Rightarrow \frac{75}{BM_1} = \frac{1}{\sqrt{3}} \Rightarrow BM_1 = 75\sqrt{3}\,\text{m} \qquad \text{... (i)}$$

In $\Delta ABM_2$

$$\frac{AB}{BM_2} = \tan 60° = \sqrt{3} \Rightarrow \frac{75}{BM_2} = \sqrt{3}$$

$$\Rightarrow \quad BM_2 = \frac{75}{\sqrt{3}} = 25\sqrt{3}\,\text{m} \qquad \text{... (ii)}$$

$\therefore$ $M_1M_2 = M_1B + BM_2$ (from (i) and (ii))

$= 75\sqrt{3} + 25\sqrt{3} = 100\sqrt{3}\,\text{m} = 173\text{ m}$

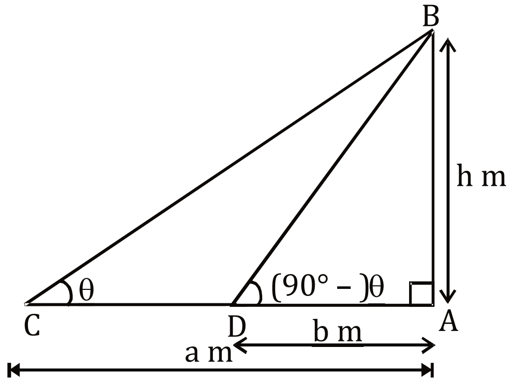
$\therefore$ Distance between two men = 173 m.

**20.** The angles of elevation of the top of a tower from two points on the ground at distance 'a' metres and 'b' metres from the base of the tower and in the same straight line are complementary. Prove that the height of the tower is $\sqrt{ab}$ metres.

**Sol.** Let AB be the tower and let C and D be the two positions of the observer. Then AC = a metres and AD = b metres.

Let $\angle ACB = \theta$. Then, $\angle ADB = (90° - \theta)$.

Let AB = h metres.




From right triangle CAB, we have

$$\tan\theta = \frac{AB}{AC}$$

$$\Rightarrow \tan\theta = \frac{h}{a} \qquad \text{... (i)}$$

From right triangle DAB, we have

$$\frac{AB}{AD} = \tan(90° - \theta) \Rightarrow \frac{h}{b} = \cot\theta$$

$$\Rightarrow \quad h = b\cot\theta$$

$$\Rightarrow \tan\theta = \frac{b}{h} \qquad \text{... (ii)}$$

From (i) and (ii), we have $\frac{h}{a} = \frac{b}{h}$

$$h^2 = ab$$

$$\therefore \quad h = \sqrt{ab}$$

### Long answer type questions (5 marks)

**21.** Two ships are there in the sea on either side of a lighthouse in such a way that the ships and the lighthouse are in the same straight line. The angles of depression of two ships as observed from the top of the lighthouse are 60° and 45°. If the height of the lighthouse is 200 m, find the distance between the two ships. [Use $\sqrt{3} = 1.73$]

**Sol.** Let the distance between the two ships be d metres.

A and B be the position of two ships. Let the distance of ship at A from the lighthouse be x metres. Then, the distance of ship at B from the lighthouse will be (d – x) metres.

In $\Delta ACO$,

In $\Delta OCA$,

$$\tan 45° = \frac{OC}{CA} = \frac{200}{x}$$

$$1 = \frac{200}{x} \Rightarrow x = 200 \text{ m} \qquad \text{... (i)}$$

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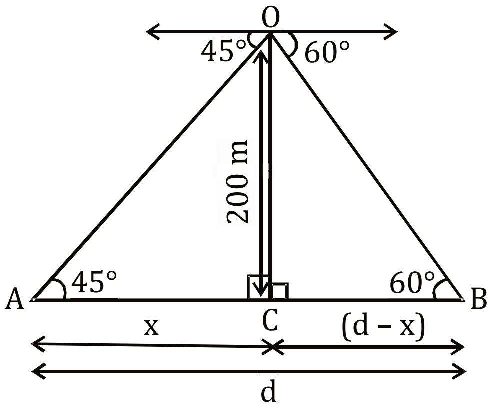
In $\Delta BCO$,

$\tan 60° = \dfrac{OC}{CB}$

$\sqrt{3} = \dfrac{200}{d-x} \Rightarrow d - x = \dfrac{200}{\sqrt{3}}$ ... (ii)




Substituting the value of x from (i) in (ii)

$d - 200 = \dfrac{200}{\sqrt{3}}$

$d = \dfrac{200}{\sqrt{3}} + 200 = 200\left(\dfrac{1}{\sqrt{3}} + 1\right)$

$= 200\left(\dfrac{1+\sqrt{3}}{\sqrt{3}}\right) = 200 \times \dfrac{(1+\sqrt{3})}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

$= 200\dfrac{(\sqrt{3}+3)}{3} = 200 \times \dfrac{(1.73+3)}{3}$

$= 200 \times \dfrac{4.73}{3} = \dfrac{946}{3} = 315.33$ m

$\therefore$ The distance between the two ships = 315.33 m

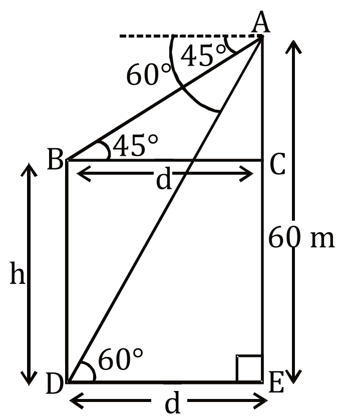
**22.** From the top of a 60 m high building, the angles of depression of the top and the bottom of a tower are 45° and 60° respectively. Find the height of the tower. (Take $\sqrt{3} = 1.73$)

**Sol.** Let the height of the building be AE = 60 m,

the height of the tower is 'h' m. The distance between the base of the building and the tower be 'd' m.

In $\Delta ADE$,

$\tan 60° = \dfrac{AE}{DE} \Rightarrow \sqrt{3} = \dfrac{60}{d}$




$d = \dfrac{60}{\sqrt{3}} = \dfrac{60\sqrt{3}}{3} = 20\sqrt{3}$

$BC = 20\sqrt{3} = 20 \times 1.73 = 34.60$ m

In $\Delta ABC$,

$\tan 45° = \dfrac{AC}{BC}$

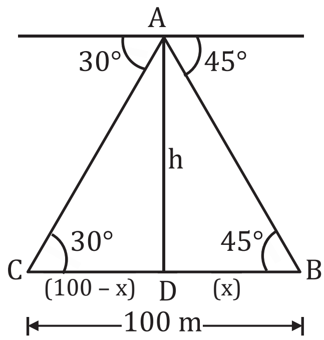
$\Rightarrow \quad 1 = \dfrac{AC}{34.60}$

$\Rightarrow \quad AC = 34.60$ m

Now, height of the tower = BD = AE – AC = 60 – 34.60 = 25.4 m

**23.** Two ships are approaching a light-house from opposite directions. The angles of depressions of the two ships from the top of the light-house are 30° and 45°. If the distance between the two ships is 100 m, find the height of the light-house. [Use $\sqrt{3}=1.732$]

**Sol.** Let AD be the light-house and its height be h metres. The distance of one ship at position B from the light house is x metres and that of other ship at position C is (100 – x) metres.




CD = (100 – x)m
BD = x m

In ΔADB,

$$\tan 45° = \frac{AD}{BD}$$

$$1 = \frac{h}{x} \Rightarrow x = h \quad \text{... (i)}$$

In ΔADC,

$$\tan 30° = \frac{AD}{CD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{100-x}$$

$$100-x=h\sqrt{3}$$

$$100 - h = \sqrt{3}h \quad \text{[From (i)]}$$

$$\sqrt{3}\,h+h=100$$

$$h(\sqrt{3}+1)=100$$

$$h=\frac{100}{\sqrt{3}+1}=\frac{100}{(\sqrt{3}+1)}\times\frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}=\frac{100(\sqrt{3}-1)}{2} = 50 \times (1.732 - 1)$$

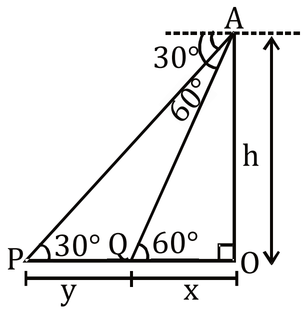
h = 36.6 m

∴ The height of the light house is 36.6 m.

**24.** A man on a cliff observes a boat at an angle of depression of 30° which is approaching the shore to the point immediately beneath the observer with a uniform speed. Six minutes later, the angle of depression of the boat is found to be 60°. Find the time taken by the boat to reach the shore.

**Sol.** Let OA be the cliff and P be the initial position of the boat when the angle of depression is 30°. After 6 minutes, the boat reaches to Q such that the angle of depression at Q is 60°.

In Δ's POA and QOA, we have




$$\tan 30° = \frac{OA}{OP} \text{ and } \tan 60° = \frac{OA}{OQ}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x+y} \text{ and } \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad x+y=\sqrt{3}h \text{ and } h=\sqrt{3}x$$

From these equations

$x + y = \sqrt{3}\,(x\sqrt{3})$

$x + y = 3x$

$y = 2x$

$x = \frac{y}{2}$

We know that, distance $\alpha$ time [speed is constant]

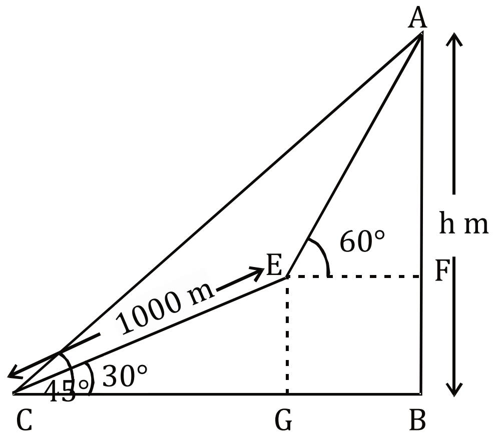
$\Rightarrow \frac{d_1}{d_2} = \frac{t_1}{t_2} \Rightarrow \frac{y}{x} = \frac{6}{t_2} \Rightarrow \frac{y}{\frac{y}{2}} = \frac{6}{t_2}$

$\therefore\ t_2 = 3m$

In moving distance y he takes 6 minutes; In moving distance $\frac{y}{2}$ he takes 3 minutes

Total time = 6 + 3 = 9 minutes

**25.** At the foot of a mountain, the elevation of its summit is 45°. After ascending 1000 m towards the mountain up a slope of 30° inclination, the elevation is found to be 60°. Find the height of the mountain.




**Sol.** Let AB be the mountain of height h metres and C be its foot.

In $\Delta$ACB, we have

$\tan 45° = \frac{h}{CB}$

$\Rightarrow\ 1 = \frac{h}{CB}$

$\Rightarrow\ h = CB$ ... (i)

In $\Delta$CGE, we have

$\sin 30° = \frac{EG}{1000}$

$\Rightarrow\ \frac{1}{2} = \frac{EG}{1000} \Rightarrow EG = 500$ ... (ii)

In ΔCGE, we have

$$\cos 30° = \frac{CG}{1000}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{CG}{1000}$$

$$\Rightarrow \quad CG = 500\sqrt{3} \qquad \text{... (iii)}$$

Now, BG = BC – CG

$$\Rightarrow \quad BG = h - 500\sqrt{3} \qquad \text{[From (i) and (iii)]}$$

In ΔAEF, we have

$$\tan 60° = \frac{AF}{EF}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h - BF}{BG} \qquad [\because BG = EF \text{ and } BF = EG = 500]$$

$$\Rightarrow \quad \sqrt{3} = \frac{h - 500}{h - 500\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}h - (500 \times 3) = h - 500$$

$$\Rightarrow \quad h(\sqrt{3} - 1) = 1500 - 500$$

$$\Rightarrow \quad h \times 0.73 = 1000$$

$$\Rightarrow \quad h = \frac{1000}{0.73} = 1369.86$$

Hence, height of the mountain is 1369.86 m.

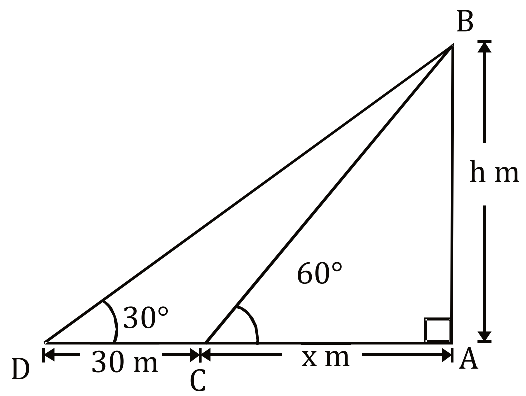
**26.** A person standing on the bank of a river observes that the angle of elevation of the top of the tree standing on the opposite bank is 60°. When he moves 30 m away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the width of the river.

**Sol.** Let AB be the tree and AC be width of the river. Let C and D be the two positions of the person.

Then ∠ACB = 60°, ∠ADB = 30°, ∠DAB = 90° and CD = 30 m.

Let AB = h metres and AC = x metres.

From the right ΔDAB, we have




$$\frac{BA}{AD} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{h}{30 + x} = \frac{1}{\sqrt{3}} \Rightarrow 30 + x = h\sqrt{3}$$

$$\Rightarrow \quad x = \left(h\sqrt{3} - 30\right) \qquad \text{... (i)}$$

From right $\Delta CAB$, we have

$$\frac{AB}{AC} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \frac{h}{x} = \sqrt{3} \Rightarrow x = \frac{h}{\sqrt{3}} \quad \text{... (ii)}$$

From (i) and (ii), we have

$$(h\sqrt{3} - 30) = \frac{h}{\sqrt{3}} \Rightarrow 3h - 30\sqrt{3} = h$$

$$\Rightarrow 2h = 30\sqrt{3} \Rightarrow h = 15\sqrt{3}$$

Putting $h = 15\sqrt{3}$ in (ii), we get $x = \frac{15\sqrt{3}}{\sqrt{3}} = 15$

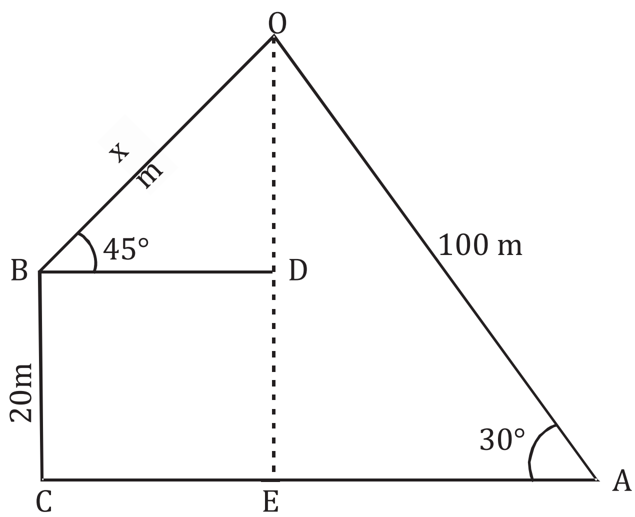
Hence, the height of the tree is $15\sqrt{3}$ m and width of the river is 15 m.

**27.** A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 30°. A girl standing on the roof of 20 m high building, finds the angle of elevation of the same bird to be 45°. Both the boy and the girl are on the opposite sides of the bird. Find the distance of the bird from the girl.

**Sol.** Let the required distance be x m.

Let the position of bird, boy and girl be O,A and B respectively

Then $\Delta BDO$,




$$\sin 45° = \frac{OD}{BO}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{OD}{BO} \Rightarrow OD = \frac{x}{\sqrt{2}} \quad \text{...(i)}$$

In $\Delta OAE$,

$$\sin 30° = \frac{OE}{OA} \Rightarrow \frac{1}{2} = \frac{OE}{100}$$

$\Rightarrow$ OE = 50

$\Rightarrow$ OD = OE – DE

$\Rightarrow$ OD = 50 – DE = (50 – 20) m = 30 m

$\therefore$ From (i), we have x = OD × $\sqrt{2}$

$$= 30 \times \sqrt{2} = 30 \times 1.41 = 42.30$$

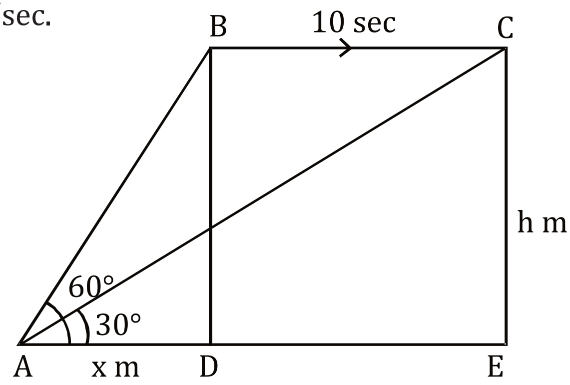
Hence, the required distance = 42.3 m.

**28.** The angle of elevation of a jet fighter from a point A on the ground is 60°. After a flight of 10 seconds, the angle of elevation changes to 30°. If the jet is flying at a speed of 432 km/hr, find the constant height at which the jet is flying. [Use $\sqrt{3}=1.732$]

**Sol.** Let A be the point of observation, B and C be the positions of the jet fighter.

BC = DE = Distance covered by the jet fighter in 10 seconds.

Speed of the jet fighter $=\dfrac{432\times1000}{60\times60}$ m/sec = 120 m/sec.




Distance = Speed × Time

DE = 120 × 10 m = 1200 m

In $\Delta$ACE, $\tan 30^\circ=\dfrac{h}{x+DE}=\dfrac{h}{x+1200}$

$\Rightarrow \dfrac{1}{\sqrt{3}}=\dfrac{h}{x+1200}\Rightarrow x+1200=h\sqrt{3}$

$\Rightarrow x=h\sqrt{3}-1200$ .... (i)

In $\Delta$ADB, $\tan60^\circ=\dfrac{h}{x} \Rightarrow x=\dfrac{h}{\sqrt{3}}$ ... (ii)

Substitute the value of x in (i)

$\Rightarrow \dfrac{h}{\sqrt{3}}+1200=h\sqrt{3}$

$\Rightarrow h+1200\sqrt{3}=3h\Rightarrow 2h=1200\sqrt{3}$

$\Rightarrow h=600\sqrt{3}=600\times1.732=1039.20$

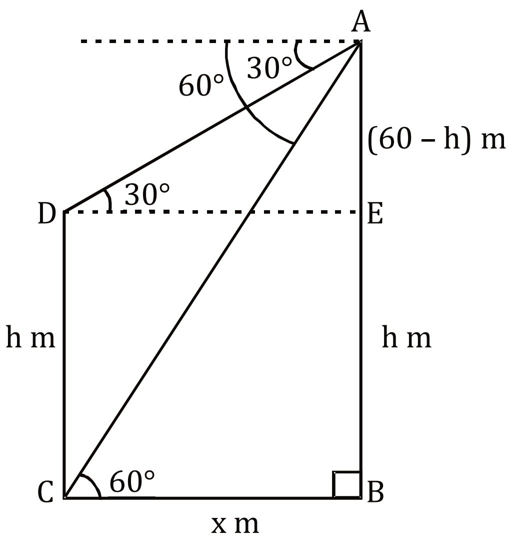
Hence, height of the jet = 1039.20 m

**29.** From the top of a building 60 m high, the angles of depression of the top and bottom of a vertical lamp post are observed to be 30° and 60° respectively. Find

(i) The horizontal distance between the building and the lamp post.

(ii) The height of the lamp post. [Take $\sqrt{3}=1.73$]

**Sol.**



Let AB be the building of height 60 m and CD be the lamp post. Let the height of the lamp post be h m and the distance between the building and the lamp post be x m.

Then, in $\Delta ABC$, we have,

$$\tan 60^\circ = \frac{60}{x}.$$

$$\Rightarrow \quad \sqrt{3} = \frac{60}{x} \Rightarrow x = \frac{60}{\sqrt{3}} \qquad \text{... (i)}$$

In $\Delta ADE$, we have

$$\tan 30^\circ = \frac{60-h}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{60-h}{x}$$

$$x = (60 - h)\sqrt{3} \qquad \text{... (ii)}$$

from (i) & (ii)

$$\frac{60}{\sqrt{3}} = (60-h)\sqrt{3}$$

$$\Rightarrow \quad 60 = (60 - h)3$$

$$\Rightarrow \quad 60 = 180 - 3h$$

$$\Rightarrow \quad h = \frac{120}{3} = 40\,m$$

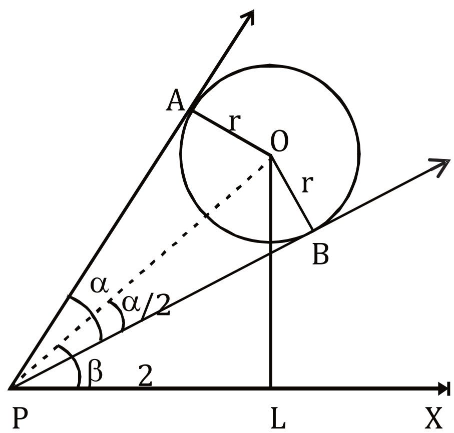
(i) The horizontal distance between the building and the lamp post, x

$= (60-h)\sqrt{3}\,m = 20\sqrt{3}\,m = 34.60$ m

(ii) The height of the lamp post = 40 m.

**30.** A round balloon of radius 'r' subtends an angle '$\alpha$' at the eye of the observer while the angle of elevation of its centre is $\beta$. Prove that the height of the centre of the balloon is r sin $\beta$ cosec $\frac{\alpha}{2}$.

**Sol.** Let O be the centre of the balloon of radius r and P be the eye of the observer. Let PA and PB be tangents to the spherical balloon. Then, $\angle APB = \alpha$.




$$\therefore \angle APO = \angle BPO = \frac{\alpha}{2}$$

Let OL be perpendicular from O on the horizontal PX.

$\therefore$ $\angle OPL = \beta$.

In $\Delta OAP$, we have

$$\sin\frac{\alpha}{2} = \frac{OA}{OP}$$

$$\Rightarrow \quad \sin\frac{\alpha}{2} = \frac{r}{OP}$$

$$\Rightarrow \quad OP = r\,\mathrm{cosec}\frac{\alpha}{2} \quad ....(i)$$

In $\Delta OPL$, we have

$$\sin\beta = \frac{OL}{OP}$$

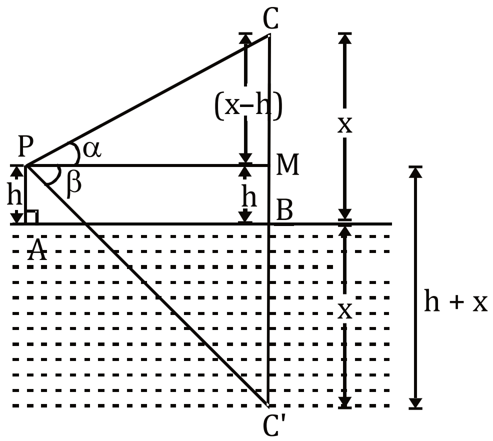
$$\Rightarrow \quad OL = OP\sin\beta = r\,\mathrm{cosec}\frac{\alpha}{2}\sin\beta \qquad [\text{Using (i)}]$$

Hence, the height of the centre of the balloon is $r\sin\beta\,\mathrm{cosec}\frac{\alpha}{2}$.

**31.** If the angle of elevation of a cloud from a point h metres above a lake is $\alpha$ and the angle of depression of its reflection in the lake from the same point is $\beta$, prove that the height of the cloud is $\frac{h(\tan\beta+\tan\alpha)}{\tan\beta-\tan\alpha}$.

**Sol.** Let point P be h metres above the lake and height of the cloud be x metres.

In $\Delta PMC$,




$$\frac{PM}{MC} = \cot\alpha$$

$$\Rightarrow \quad PM = MC\cot\alpha \quad ..... (i)$$

In $\Delta PMC'$,

$$\cot\beta = \frac{PM}{MC'} \Rightarrow PM = MC'\cot\beta \;....(ii)$$

From (i) and (ii), we have

$MC\cot\alpha = MC'\cot\beta$

$\Rightarrow \quad (x - h)\cot\alpha = (x + h)\cot\beta$

$\Rightarrow \quad x\cot\alpha - h\cot\alpha = x\cot\beta + h\cot\beta$

$\Rightarrow \quad x\cot\alpha - x\cot\beta = h\cot\alpha + h\cot\beta$

$\Rightarrow \quad x(\cot\alpha - \cot\beta) = h(\cot\beta + \cot\alpha)$

$$\Rightarrow x = \frac{h(\cot\beta+\cot\alpha)}{\cot\alpha-\cot\beta} = h\frac{\left(\frac{1}{\tan\beta}+\frac{1}{\tan\alpha}\right)}{\left(\frac{1}{\tan\alpha}-\frac{1}{\tan\beta}\right)}$$

$$\Rightarrow x = \frac{h(\tan\alpha+\tan\beta)}{(\tan\beta-\tan\alpha)}$$

Hence, the height of the cloud is $\frac{h(\tan\alpha+\tan\beta)}{(\tan\beta-\tan\alpha)}$.

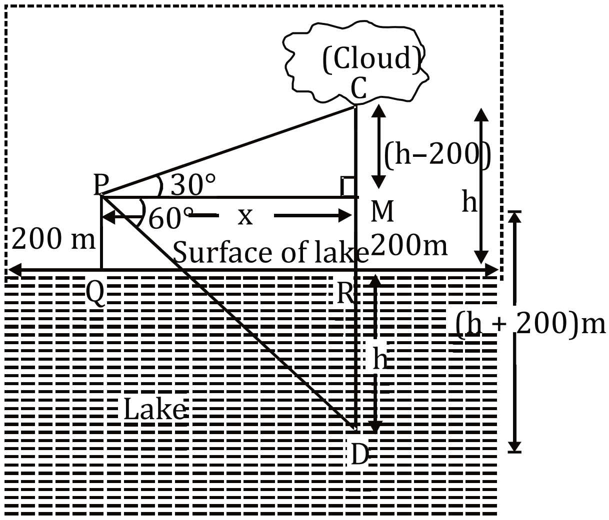
**32.** The angle of elevation of a cloud from a point 200 metres above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60°. Find the height of the cloud.

**Sol.** Let QR be the surface of the lake and P be the point of observation which is 200 m above Q i.e., PQ = 200 m. Let C be the position of the cloud.

Through C draw CR perpendicular to the surface of the lake and D the reflection of the cloud in the lake then CR = RD = h metres. (Assume) (Laws of reflection)

Through P, draw PQ perpendicular on the surface of lake and PM $\perp$ CD.

$\angle CPM = 30°$, $\angle MPD = 60°$




MR = PQ = 200 m, Let PM = x metres

CM = CR – MR = (h – 200 m) ($\because$ CR = h metres)

DM = DR + RM = (h + 200 m) ($\because$ DR = h metres)

| In right $\Delta CMP$, | From right $\Delta PMD$, |
|---|---|
| $\frac{CM}{PM} = \tan 30°$ | $\frac{MD}{PM} = \tan 60°$ |
| $\frac{h-200}{x} = \frac{1}{\sqrt{3}}$ | $\frac{h+200}{x} = \sqrt{3}$ |
| $x = \sqrt{3}(h-200)$ ...(1) | $x = \frac{h+200}{\sqrt{3}}$ ....(2) |

From (1) and (2), we get :

$$\sqrt{3}\ (h - 200) = \frac{h+200}{\sqrt{3}}$$

$\Rightarrow$ 3 (h – 200) = h + 200

$\Rightarrow$ 3h – h = 600 + 200 = 800

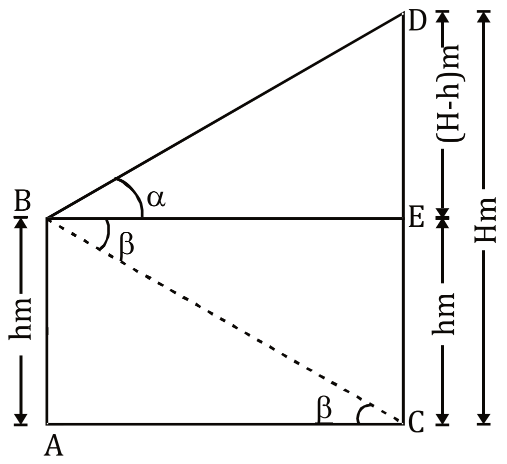
$\Rightarrow$ 2h = 800

$\Rightarrow$ $h = \frac{800}{2} = 400$

Required height of the cloud = h = CR = 400 m.

**33.** From a window (h metres high above the ground) of a house in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are $\alpha$ and $\beta$ respectively. Show that the height of the opposite house is : $h(1 + \tan \alpha \cot \beta)$ metres.

**Sol.**




Let B be the window of a house AB and let CD be the other house of height H m. Then, AB = h metres.

Draw BE ∥ AC, meeting CD at E. Then, $\angle EBD = \alpha$ and $\angle EBC = \beta$.

$\therefore\ \angle ACB = \angle EBC = \beta$ [Alternate angles]

Let CD = H metres. Then,

CE = AB = h metres and ED = (H – h) metres.

From right ΔBED, we have

$$\frac{BE}{ED} = \cot\alpha \Rightarrow \frac{BE}{(H-h)} = \cot\alpha$$

$\Rightarrow$ BE = $\cot\alpha\ (H - h)$ ... (i)

In right ΔBEC, we have

$$\tan\beta = \frac{CE}{BE} \Rightarrow BE = h\cot\beta \qquad \text{... (ii)}$$

From (i) and (ii), we have

$(H - h)\cot\alpha = h\cot\beta$

$H\cot\alpha - h\cot\alpha = h\cot\beta$

$H\cot\alpha = h(\cot\alpha + \cot\beta)$

$$\Rightarrow \quad H = \frac{h(\cot\alpha + \cot\beta)}{\cot\alpha}$$

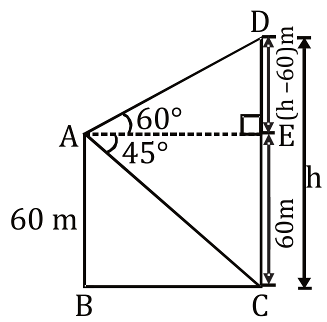
$\Rightarrow \quad H = h(1 + \tan\alpha\cot\beta)$

Hence, the height of the opposite house is $h(1 + \tan\alpha\cot\beta)$ metres.

**34.** From a window (60 m high above the ground) of a house in a street, the angles of elevation and depression of the top and the foot of another house on opposite side of street are 60° and 45° respectively. Show that the height of the opposite house is $60\left(1+\sqrt{3}\right)$ metres.

**Sol.** Let the height of the opposite house be DC = h metres.

In right ΔADE,




$$\tan 60° = \frac{DE}{AE}$$

$$\sqrt{3} = \frac{h-60}{AE}$$

$$AE = \frac{h-60}{\sqrt{3}} \quad \text{... (1)}$$

In right ΔACE,

$$\tan 45° = \frac{CE}{AE}$$

$$AE = 60 \quad \text{... (2)}$$

Comparing (1) and (2), we get

$$\frac{h-60}{\sqrt{3}} = 60$$

$$\Rightarrow \quad h - 60 = 60\sqrt{3}$$

$$\Rightarrow \quad h = 60\sqrt{3} + 60$$

$$\Rightarrow \quad h = 60(1+\sqrt{3})$$

Therefore, height of the opposite house is

60 (1 + $\sqrt{3}$ ) metres

**35.** The angles of depression of the top and bottom of a 8 m tall building from the top of a multi-storey building are 30° and 45° respectively. Find the height of the multi-storey building and the distance between the two buildings.

**Sol.** Let AB and CD be the multi-storey building and the other building respectively.

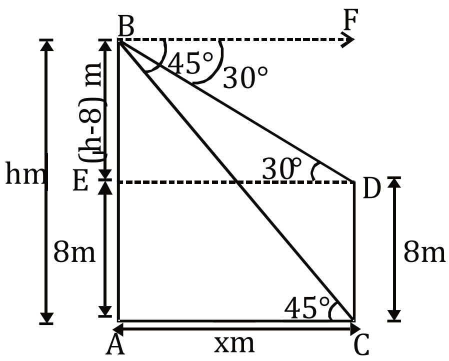
Let the height of the multi-storey building = h m

and the distance between the two buildings = x m

AE = CD = 8 m (Given)

BE = AB – AE = (h – 8) m

& AC = DE = x m




Also,

$\angle FBD = \angle BDE = 30°$ (alternate angles)

$\angle FBC = \angle BCA = 45°$ (alternate angles)

Now,

In $\Delta ACB$,

$\Rightarrow \tan 45° = \dfrac{AB}{AC}$

$\Rightarrow 1 = \dfrac{h}{x}$

$\Rightarrow x = h$ ... (1)

In $\Delta BDE$,

$\Rightarrow \tan 30° = \dfrac{BE}{ED}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h-8}{x}$

$\Rightarrow x = \sqrt{3}(h-8)$ ... (2)

From (1) and (2), we get

$\Rightarrow h = \sqrt{3}h - 8\sqrt{3}$

$\Rightarrow \sqrt{3}h - h = 8\sqrt{3}$

$\Rightarrow h(\sqrt{3}-1) = 8\sqrt{3}$

$\Rightarrow h = \dfrac{8\sqrt{3}}{\sqrt{3}-1}$

$\Rightarrow h = \dfrac{8\sqrt{3}}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1} = \dfrac{8\sqrt{3}(\sqrt{3}+1)}{2} = 4\sqrt{3}(\sqrt{3}+1)\text{m}$

Therefore, the height of multi-storey building is $4\sqrt{3}(\sqrt{3}+1)$ m

From equation (1)

x = h

$\therefore x = 4\sqrt{3}\left(\sqrt{3}+1\right)$ m

Therefore, the distance between two building is $4\sqrt{3}\left(\sqrt{3}+1\right)$ m.

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**Case Study type questions** **(4 marks)**

**1.** Mr. Suresh is an electrician. He receives a call regarding a fault on a pole from three different colonies A, B & C. He reaches one by one to each colony to repair that. He need to reach a point 1.3 m below the top of each pole to undertake the repair work. Observe the following diagrams.

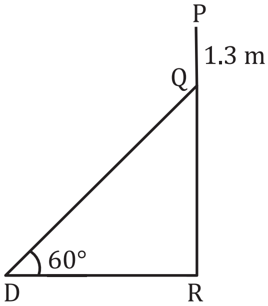


Fig A

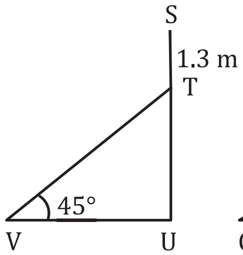


Fig B

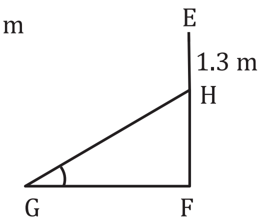


Fig C

(i) What should be the length of ladder DQ in fig. A that enable him to reach the required position if the height of the pole is 4 m?

(ii) What is the distance of the point where the ladder is placed on the ground from the foot of the pole if the height of pole is 4m?

(iii) Given that the length of ladder is $\frac{4}{\sqrt{2}}$ m in fig. B what is height of pole?

**[OR]**

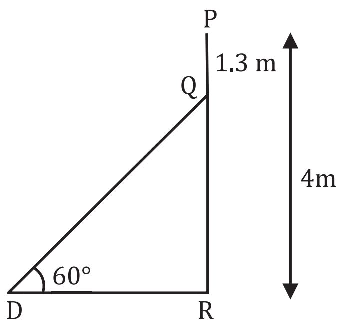
(iii) Find the angle of elevation of reaching point of ladder at pole, if the height of the pole is 8.3 m and the distance GF is $7\sqrt{3}$ m in fig C.

**Sol.** (i) QR = PR – PQ

$\Rightarrow$ QR = 4 – 1.3

QR = 2.7

In $\Delta$QDR




$$\Rightarrow \sin 60° = \frac{QR}{DQ} = \frac{\sqrt{3}}{2} = \frac{2.7}{DQ}$$

$$DQ = \frac{2.7 \times 2}{\sqrt{3}}$$

$$= \frac{5.4}{\sqrt{3}} = \frac{5.4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5.4\sqrt{3}}{3}$$

$$= 1.8\sqrt{3} \Rightarrow \frac{9\sqrt{3}}{5} \text{ m}$$

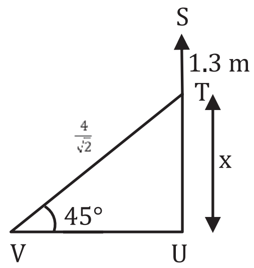
(ii) In $\Delta QDR$

$$\tan 60° = \frac{QR}{DR} \Rightarrow \sqrt{3} = \frac{2.7}{DR}$$

$$DR = \frac{2.7}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow \frac{2.7}{3} \times \sqrt{3}$$

$$= 0.9\sqrt{3}\,\text{m}$$

(iii) In $\Delta TUV$,




$$\sin 45° = \frac{TU}{TV} = \frac{x}{4/\sqrt{2}}$$

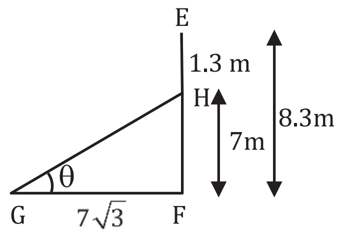
$$\frac{1}{\sqrt{2}} = \frac{x}{4/\sqrt{2}}$$

$$x = \frac{1}{\sqrt{2}} \times \frac{4}{\sqrt{2}} = \frac{4}{2} = 2\text{ m}$$

Height of pole = SU = 1.3 + 2 = 3.3 m

**[OR]**

(iii) In $\Delta HFG$



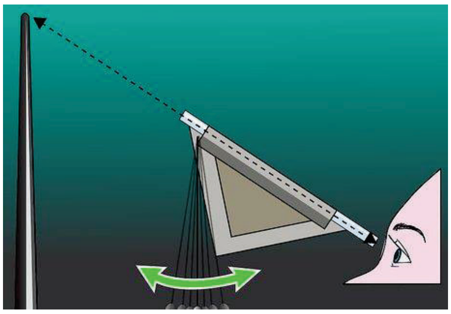

$$\tan\theta = \frac{HF}{GF}$$

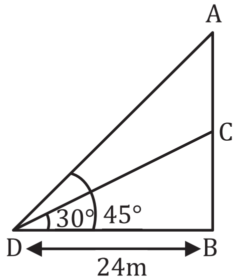
$$\tan\theta = \frac{7}{7\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$\Rightarrow \theta = 30°$

**2.** A clinometer is a tool that is used to measure the angle of elevation, or angle from the ground in a right-angled triangle. We can use a clinometer to measure the height of tall things that you can't possibly reach like flags, poles, buildings, trees etc. Ravish got a clinometer from school lab and started measuring elevation angle in surrounding. He saw a building on which society logo is painted on the wall of building.






From a point D on the ground level, the angle of elevation of the roof of the building is 45°. The angle of elevation of the centre of logo is 30° from same point. The point D is at a distance of 24 m from the base of building.

(i) What is the height of building logo from the ground?

(ii) What is the height of building from the ground?

(iii) What is the distance of point D from the top of the building?

**[OR]**

(iii) If the point of observation D is moved 9 m toward the base of the building, then find the tangent of angle of elevation ($\theta$) of logo on the building.

**Sol.** (i) In $\Delta CBD$, $\tan 30^\circ = \dfrac{CB}{DB} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{CB}{24}$

$$CB = \frac{24}{\sqrt{3}} = 8\sqrt{3} \text{ m}$$

(ii) In $\Delta ABD$, $\tan 45^\circ = \dfrac{AB}{DB} \Rightarrow 1 = \dfrac{AB}{24}$

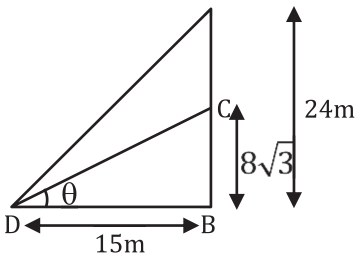
AB = 24 m

(iii) In $\Delta ADB$, $\sin 45^\circ = \dfrac{AB}{AD}$

$$\frac{1}{\sqrt{2}} = \frac{24}{AD} \qquad AD = 24\sqrt{2} \text{ m}$$

**[OR]**

(iii)

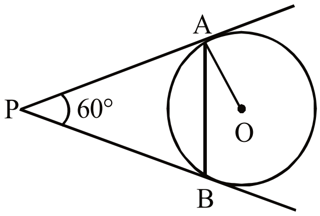



$$\tan\theta = \frac{8\sqrt{3}}{15}$$

# 10 Circles

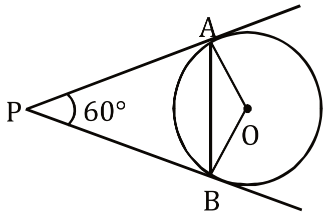
**Multiple choice questions** **(1 mark)**

**1.** In the given figure, PA and PB are two tangents to the circle with centre O. If $\angle APB = 60°$ then $\angle OAB$ is




(1) 15° (2) 30° (3) 60° (4) 90°

**Sol.** **Option (2)**




Join OB.

We know that the radius and tangent are perpendicular at their point of contact.

$\therefore \quad \angle OBP = \angle OAP = 90°$

Now, in a quadrilateral AOBP

$\Rightarrow \quad \angle AOB + \angle OBP + \angle APB + \angle OAP = 360°$ [ Sum of four angles of a quadrilateral is 360°]

$\Rightarrow \quad \angle AOB + 90° + 60° + 90° = 360°$

$\Rightarrow \quad 240° + \angle AOB = 360°$

$\Rightarrow \quad \angle AOB = 120°$

Since OA and OB are the radius of a circle then, $\Delta AOB$ is an isosceles triangle.

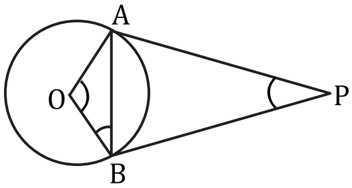
$\Rightarrow \quad \angle AOB + \angle OAB + \angle OBA = 180°$

$\Rightarrow \quad 120° + 2\angle OAB = 180°$ [Since, $\angle OAB = \angle OBA$]

$\Rightarrow \quad 2\angle OAB = 60°$

$\therefore \quad \angle OAB = 30°$

**2.** Two tangents are drawn from an external point P (as shown in fig.) such that $\angle OBA = 10°$. Then angle $\angle BPA$ is.




(1) 10° (2) 20° (3) 30° (4) 40°

**Sol. Option (2)**

Since tangent and radius are perpendicular at the point of contact, we have:

$\angle OAP = \angle OBP = 90°$.

So, $\angle AOB + \angle BPA = 180°$ ... (1)

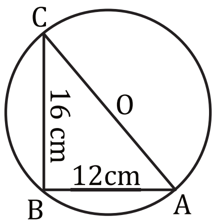
In $\Delta OAB$, OA = OB, so, $\angle OAB = \angle OBA = 10°$.

So, $\angle AOB = 180° - 10° - 10° = 160°$ ... (2)

From (1) and (2), we get,

$\angle BPA = 180° - 160° = 20°$

**3.** If AB = 12 cm, BC = 16 cm and AB is perpendicular to BC then the radius of the circle passing through the points A, B and C is :




(1) 6 cm (2) 8 cm (3) 10 cm (4) 12 cm

**Sol. Option (3)**

It is given that AB is perpendicular to BC, therefore, $\angle ABC = 90°$.

Every angle inscribed in a semicircle is a right angle.

Since the inscribed $\angle ABC = 90°$, the arc ABC is a semicircle.

Therefore, AC is the diameter of the circle passing through the centre.

Apply Pythagoras theorem in triangle ABC, we have

$AC^2 = AB^2 + BC^2$

$AC^2 = 12^2 + 16^2$

$AC^2 = 144 + 256$

$AC^2 = 400$

$AC = \sqrt{400}$

AC = 20 cm

Diameter of circle is 20 cm and hence the radius is 10 cm as radius is half of diameter.

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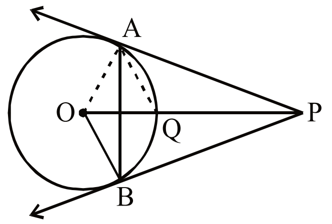
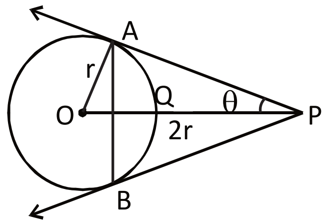
**4.** From a point P, two tangents PA and PB are drawn to a circle C (O, r). If OP = 2r, then ΔAPB is




(1) Scalene Triangle (2) Isosceles Triangle

(3) Right angled Triangle (4) Equilateral Triangle

**Sol. Option (4)**




Let ∠APO = θ and ∠OAP = 90°

$\sin\theta = \frac{OA}{OP} = \frac{1}{2}$

⇒ θ = 30°

⇒ ∠APB = 2θ = 60°

Also, ∠PAB = ∠PBA = 60° (∵ PA = PB)

⇒ ΔAPB is equilateral.

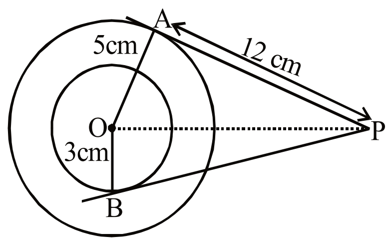
**5.** How many parallel tangents can a circle have?

(1) 1 (2) 2 (3) ∞ (4) None of these

**Sol. Option (2)**

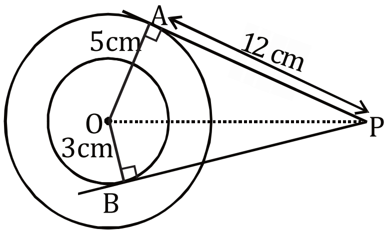
A circle can have exactly two parallel tangents and they must pass through ends of a diameter. That is their point of contacts must be diametrically opposite.

**6.** In the given figure, there are two concentric circles with centre O of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP.




(1) 13 cm (2) $5\sqrt{2}$ cm (3) $2\sqrt{3}$ cm (4) $4\sqrt{10}$ cm

**Sol. Option (4)**




PA and PB are the tangent drawn from the external point P to outer and inner circle respectively.

$\angle OAP = \angle OBP = 90°$

Given OA = 5 cm, OB = 3 cm and AP = 12 cm

In $\Delta OAP$

$OP^2 = (12)^2 + (5)^2 = 169 \text{ cm}^2$

OP = 13 cm

In $\Delta OBP$

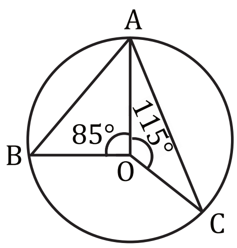
$PB^2 = OP^2 - OB^2$

$PB^2 = (13)^2 - (3)^2 = 160 \text{ cm}^2$

PB = $4\sqrt{10}$ cm

Thus, the length of PB = $4\sqrt{10}$ cm

**7.** In the figure, O is the centre of the circle with $\angle AOB = 85°$ and $\angle AOC = 115°$, then $\angle BAC$ is




(1) 115° (2) 85° (3) 80° (4) 100°

**Sol. Option (3)**

$\angle BOC = 360 - 115 - 85 = 160°$

Angle subtended at the centre is twice the angle subtended anywhere on the arc, so

$\angle BAC = \frac{1}{2}\angle BOC = 80°$

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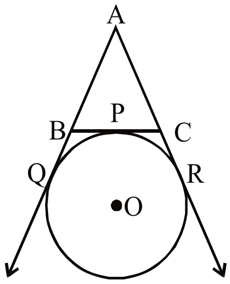
**8.** In the adjoining figure, AQ and AR are tangents from A to the circle. P is a point on the circle. Which of the following statement is not true ?




(1) AB + BQ = AC + CR (2) AB + BP = AC + CR

(3) AB + BP = AC + CP (4) AB + CP = AC + BP

**Sol.** **Option (4)**

AQ = AR [tangents from point A]

AB + BQ = AC + CR

AB + BP = AC + CR [$\because$ BQ = BP]

AB + BP = AC + CP [$\because$ CP = CR]

AB + CP $\neq$ AC + BP

## Assertion reason questions (1 mark)

**9.** **Assertion (A) :** AB and CD are two parallel chords of a circle whose diameter is AC. Then AB ≠ CD.

**Reason (R) :** Perpendicular from the centre of a circle to a chord bisect the chord.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol.** **Option (4)**

Assertion (A) is false but Reason (R) is true.

**10.** **Assertion:** Angles in the same segment of a circle are equal.

**Reason:** In cyclic quadrilateral, opposite angles are supplementary.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

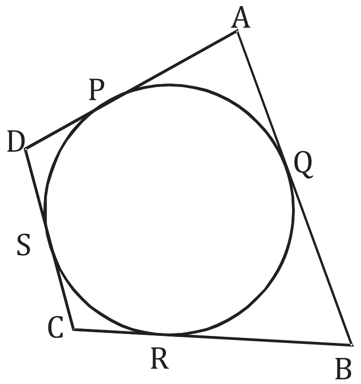
(4) Assertion (A) is false but Reason (R) is true.

**Sol.** **Option (2)**

Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

**Very short answer type questions** **(2 mark)**

**11.** Quadrilateral ABCD circumscribes a circle as shown in figure. Find the side of the quadrilateral which is equal to AP + BR.

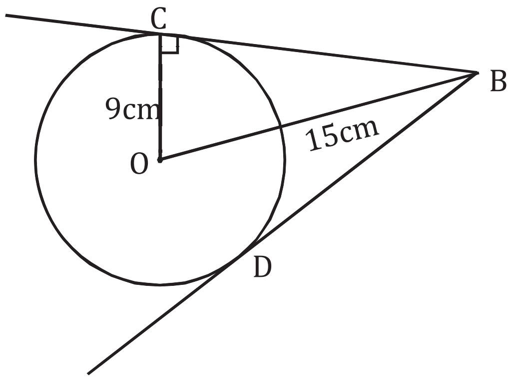



**Sol.** Since the lengths of tangents drawn from an external point to the circle are equal, then

AP = AQ and BR = QB.

$\therefore$ AP + BR = AQ + QB = AB [$\because$ AQ + QB = AB]

**12.** In figure, if OC = 9 cm and OB = 15 cm, then BC + BD is




**Sol.** In $\Delta$OCB, by Pythagoras theorem

$$CB = \sqrt{(15)^2 - (9)^2} = \sqrt{225 - 81} = \sqrt{144}$$

CB = 12 cm

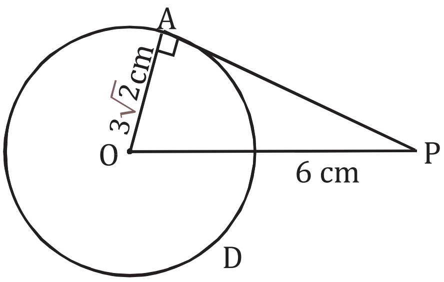
$\therefore$ BD = BC = 12 cm [$\because$ length of tangents drawn from an external point to the circle are equal]

Now, BC + BD = 12 cm +12 cm = 24 cm

**13.** A tangent PA is drawn from an external point P to a circle of radius $3\sqrt{2}$ cm such that the distance of the point P from O is 6 cm as shown in figure. Then find value of $\angle$APO.

**Sol.** Let $\angle APO = \theta$.

In right $\Delta OAP$, we have

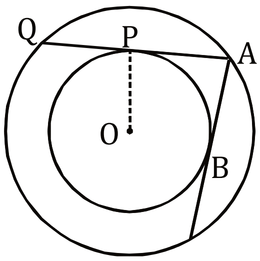



$$\sin\theta = \frac{OA}{OP} \Rightarrow \sin\theta = \frac{3\sqrt{2}}{6} = \frac{1}{\sqrt{2}}$$

$\Rightarrow$ $\sin\theta = \sin 45°$

$\Rightarrow$ $\theta = 45°$

**14.** The figure shows two concentric circles with centre O. AB and APQ are tangents to the inner circle from point A lying on the outer circle. If AB = 7.5 cm, then find AQ.



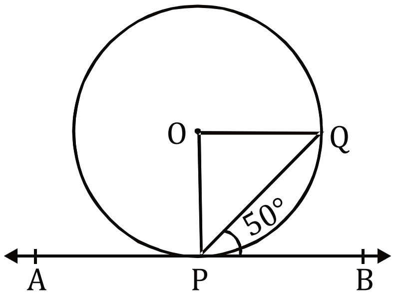

**Sol.** AP = AB = 7.5 cm

[Since the length of tangents drawn from an external point to a circle are equal]

Now, AQ = 2AP = 2 × 7.5 cm = 15 cm

[since, perpendicular from the centre bisects the chord]

**15.** In figure, APB is a tangent to a circle with centre O, at point P. If $\angle QPB = 50°$, then find the measure of $\angle POQ$.




**Sol.** $\angle OPB = 90°$

[The tangent at any point of a circle is perpendicular to the radius through the point of contact]

$\angle OPQ = 90° - \angle QPB = 90° - 50° = 40°$

Now, since OP = OQ (radii)

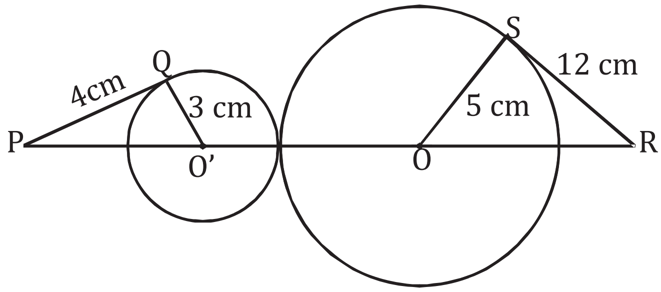
$\therefore$ $\angle OQP = \angle OPQ = 40°$

$\therefore$ $40° + 40° + \angle POQ = 180$ [Angle sum property of triangle]

$\Rightarrow$ $\angle POQ = 180° - 80° = 100°$

**Short answer type questions** **(3 marks)**

**16.** In the given figure, find the length of PR.




**Sol.** In right $\Delta OSR$, $OR^2 = 5^2 + 12^2 = 169$ [By pythagoras theorem]

$\Rightarrow \quad OR = \sqrt{169} \Rightarrow OR = 13$ cm

In right $\Delta PQO'$.

$PO'^2 = 4^2 + 3^2 = 25$

$\Rightarrow \quad PO' = \sqrt{25}$

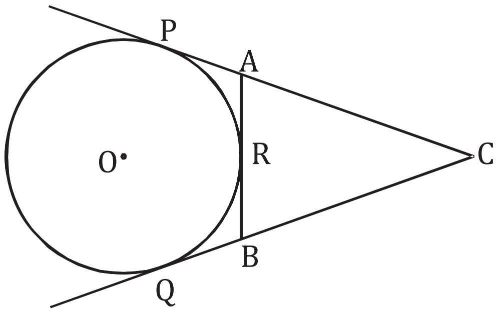
$\Rightarrow \quad PO' = 5$ cm

Now, $OO' = (3 + 5)$ cm $= 8$ cm

$\therefore \quad PR = PO' + OO' + OR$

$= 5$ cm $+ 8$ cm $+ 13$ cm $= 26$ cm

**17.** CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm, BC = 7 cm, then find the length BR is




**Sol.** CP = CQ [$\because$ length of tangents drawn from an exterior point to a circle are equal]

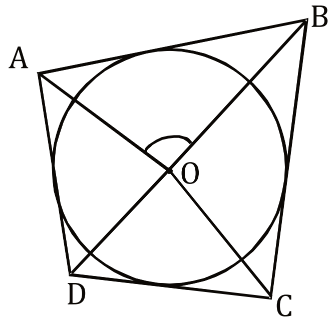
CQ = 11 cm = QB + BC = QB + 7 cm

$\Rightarrow$ QB = 11 cm – 7 cm = 4 cm

$\Rightarrow$ BR = QB = 4 cm

[ $\because$ length of tangents drawn from an exterior point to a circle are equal]

**18.** In the given figure, if $\angle AOB = 125°$, then find $\angle COD$.

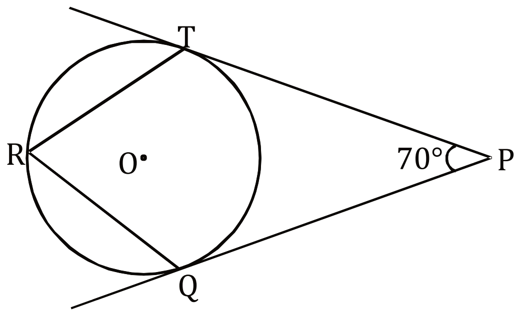



**Sol.** If a circle touches the sides of a quadrilateral, then the angles subtended at the centre by a pair of opposite sides are supplementary.

$\therefore \quad \angle AOB + \angle COD = 180°$

$\Rightarrow \quad \angle COD = 180° - 125° = 55°$

**19.** In the given figure O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If $\angle TPQ = 70°$, find $\angle TRQ$.



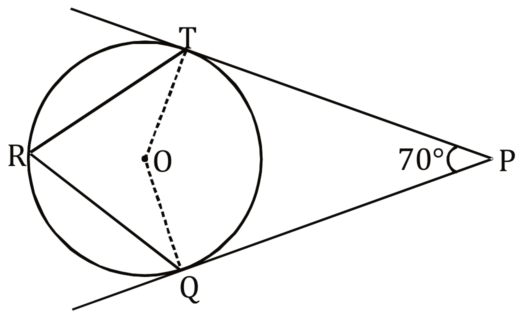

**Sol.** Given : O is the centre of a circle. PT and PQ are tangents to the circle from an external point P.

$\angle TPQ = 70°$

To find : $\angle TRQ$

Construction : Join T to O and Q to O

Solution : $\because$ OT $\perp$ PT and OQ $\perp$ QP




$\therefore \quad \angle OTP = \angle OQP = 90°$

In quadrilateral PTOQ,

$\angle OTP + \angle OQP + \angle TOQ + \angle TPQ = 360°$

$\Rightarrow \quad 90° + 90° + \angle TOQ + 70° = 360°$

$\Rightarrow \quad \angle TOQ = 110° \qquad \Rightarrow \angle TOQ = 2\angle TRQ$

[$\because$ Angle subtended by an arc at centre of the circle is double the angle subtended by the same arc at the remaining part of the circle]

$\therefore \quad 110° = 2\angle TRQ \Rightarrow \angle TRQ = \dfrac{110°}{2} = 55°$

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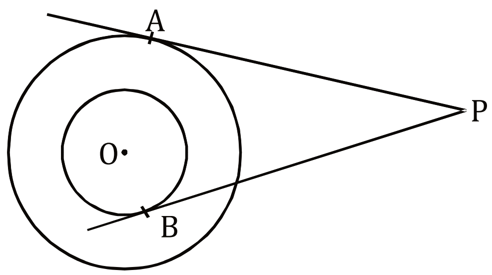
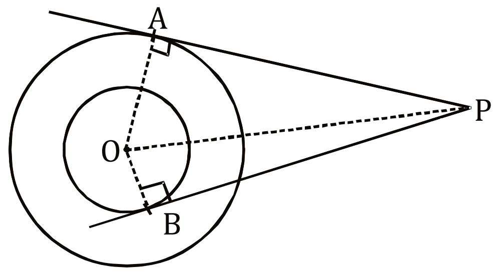
**20.** In the given figure, there are two concentric circles with centre O and of radii 7 cm and 15 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 24 cm, find the length of BP.




**Sol.** Join OA, OB and OP.

Then OA = 7 cm, OB = 3 cm and PA = 12 cm




In $\Delta OPA$, $OP^2 = OA^2 + AP^2$

$= 7^2 + 24^2$

$= 49 + 576 = 625$

$\therefore \quad OP = \sqrt{625} = 25 \text{ cm}$

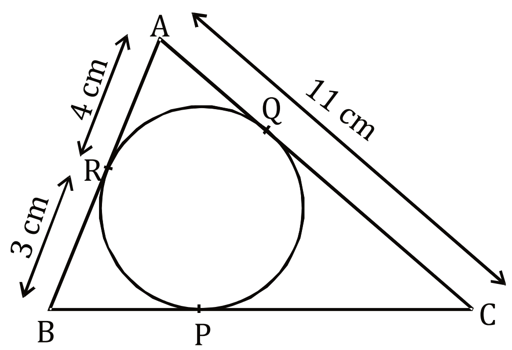
Now, in $\Delta OBP$, $OP^2 = OB^2 + BP^2$

$\Rightarrow \quad BP^2 = OP^2 - OB^2$

$= (25)^2 - (15)^2 = 625 - 225$

$\Rightarrow \quad BP = \sqrt{400} \text{ cm} = 20 \text{ cm}$

**21.** In the figure, $\Delta ABC$ is circumscribing a circle. Find the length of BC.




**Sol.** We know that the lengths of tangents drawn from an external point to a circle are equal.

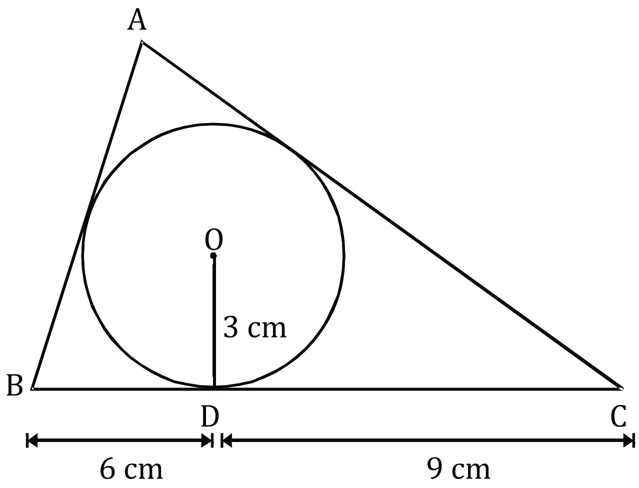
So, AR = AQ = 4 cm ... (i)

BR = BP = 3 cm ... (ii)

CP = CQ = (AC – AQ) = (11 – 4) cm = 7 cm

So, BC = BP + PC = BP + CQ = (3 + 7) cm = 10 cm

**22.** In the given figure, $\Delta ABC$ is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of $\Delta ABC$ is 54 cm$^2$, then find the lengths of sides AB and AC.




**Sol.** Let E and F be the points where the tangents AB and AC touches the circle respectively. Join OE and OF. Now, radius is perpendicular to tangent at the point of contact. So, $OD \perp BC$, $OE \perp AB$ and $OF \perp AC$. Join OA, OB and OC. Since, tangents drawn from an external point to a circle are equal.

$\therefore$ BD = BE = 6 cm, CD = CF = 9 cm

and AE = AF = x cm (say).

Now, area of $\Delta ABC$ = Area of $\Delta AOB$ + Area of $\Delta BOC$ + Area of $\Delta AOC$

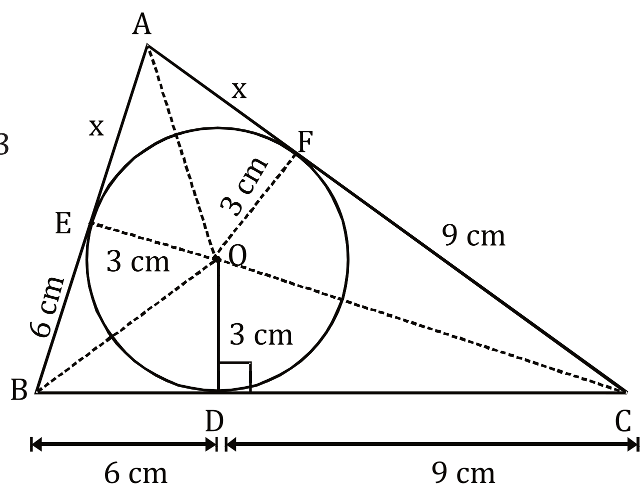
$\Rightarrow \quad 54 = \frac{1}{2} \times AB \times OE + \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AC \times OF$

$\Rightarrow \quad 54 = \frac{1}{2} \times (x+6) \times 3 + \frac{1}{2} \times 15 \times 3 + \frac{1}{2} \times (x+9) \times 3$

$\Rightarrow \quad 54 = \frac{1}{2} \times (3x + 18 + 45 + 3x + 27)$

$\Rightarrow \quad 6x + 90 = 108 \Rightarrow 6x = 108 - 90 = 18$

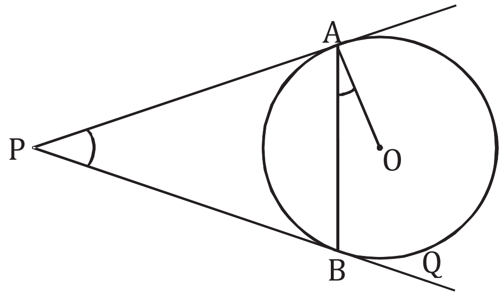
$\Rightarrow \quad x = \frac{18}{6} = 3$




$\therefore$ AB = 3 + 6 = 9 cm

and AC = 3 + 9 = 12 cm

**23.** Two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that $\angle APB = 2\angle OAB$.




**Sol.** Given : PA and PB are tangents to a circle at O.

To prove : $\angle APB = 2\angle OAB$.

Proof : $\angle OAP = 90°$ [Radius through the point of contact is perpendicular to the tangent]

$\angle PAB = 90° - \angle OAB$

In $\Delta APB$, PA = PB [Tangents drawn from an external point to a circle are equal]

$\Rightarrow \angle PBA = \angle PAB = 90° - \angle OAB$ .... (i)

Also, In $\Delta PAB$, $\angle APB = 180° - (\angle PBA + \angle PAB)$

$= 180° - 2(90° - \angle OAB)$

$\Rightarrow \angle APB = 180° - 180° + 2\angle OAB$

$\Rightarrow \angle APB = 2\angle OAB$

**24.** ABC is an isosceles triangle, in which AB = AC, circumscribed about a circle. Show that BC is bisected at the point of contact.

**Sol.** Given : An isosceles triangle ABC, in which AB = AC, circumscribed about a circle.

To Prove : BF = FC

Proof : We know that the tangents drawn from an external point to a circle are equal.

So, AD = AE ... (i)

CE = CF ... (ii)

BD = BF ... (iii)

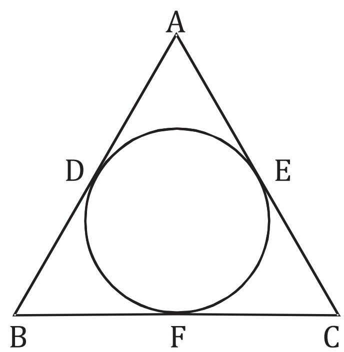
Also, AB = AC (Given)

$\Rightarrow$ AD + BD = AE + CE

Since AD = AE

$\Rightarrow$ BD = CE ... (iv)

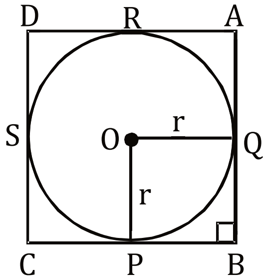
[from (i)]




From (ii), (iii) and (iv), we have BF = FC

Hence, BC is bisected at the point of contact.

**25.** In the figure, a circle is inscribed in a quadrilateral ABCD in which $\angle B = 90°$. If AD = 23 cm and AB = 29 cm, DS = 5 cm, find the radius 'r' of the circle.




**Sol.** We know that the tangents drawn from an external point to a circle are equal.

$\therefore$ AR = AQ ... (i)

DR = DS ... (ii)

BQ = BP ... (iii)

AD = AR + DR

$\Rightarrow$ AD = AR + DS [From (ii)]

$\Rightarrow$ 23 = AR + 5

$\Rightarrow$ AR = 23 – 5 = 18 cm

$\Rightarrow$ AB = AQ + BQ

$\Rightarrow$ AB = AR + BQ [From (i)]

$\Rightarrow$ 29 = 18 + BQ

$\Rightarrow$ BQ = 29 – 18 = 11 cm ... (iv)

In quadrilateral OPBQ,

$\angle B = 90°$

$\angle OQB = \angle OPB = 90°$

[Tangent is perpendicular to the radius at the point of contact]

BQ = BP [From (iii)]

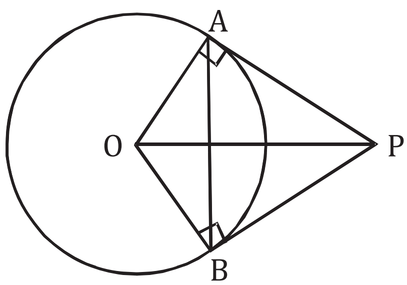
OQ = OP = r [Given]

$\Rightarrow$ OPBQ is a square.

$\Rightarrow$ OP = OQ = PB = BQ

$\Rightarrow$ r = 11 cm [From iv]

**26.** In the figure, OP is equal to diameter of the circle. Prove that ABP is an equilateral triangle.

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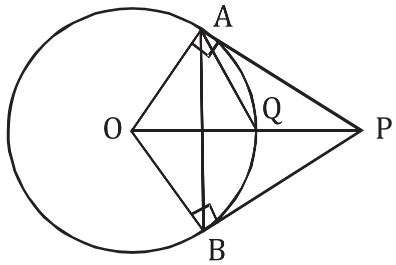
**Sol.** Given : OP is equal to diameter of circle with centre O.

To prove : $\Delta$ABP is equilateral $\Delta$.

Proof : Suppose OP meets the circle at Q. Join QA.




We have, OP = diameter

$\Rightarrow$ OQ + PQ = diameter

$\Rightarrow$ PQ = diameter – radius.

$\Rightarrow$ PQ = radius

So, OQ = PQ = radius

Thus, OP is the hypotenuse of right triangle OAP and Q is the mid-point of OP.

$\therefore$ OA = AQ = OQ

[$\because$ Mid-point of hypotenuse of a right triangle is equidistant from the vertices.]

$\Rightarrow$ $\Delta$OAQ is an equilateral triangle.

$\Rightarrow$ $\angle AOQ = 60°$

So, $\angle APO = 30°$

$\therefore$ $\angle APB = 2\angle APO = 60°$

Also, PA = PB

$\Rightarrow$ $\angle PAB = 60°$

Hence, $\Delta$APB is an equilateral triangle.

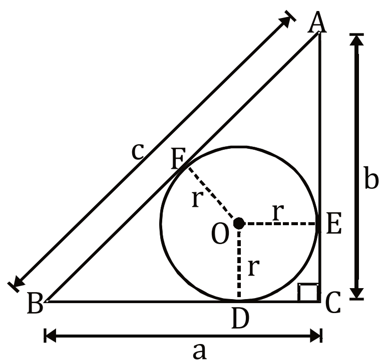
**27.** If a, b, c are the sides of a right angled triangle, where c is hypotenuse, then prove that the radius r of the circle which touches the sides of the triangle is given by $r=\dfrac{a+b-c}{2}$.

**Sol.** Let a, b and c be the sides of right angled $\Delta$ABC, such that BC = a, CA = b and AB = c.

Let the circle touches the sides BC, CA, AB at D, E and F, respectively.

Then, AE = AF and BD = BF and CD = CE

[$\because$ Tangents drawn from an external point to a circle are equal in length]




In quadrilateral OECD, each angle is 90°.

So, OECD is a square therefore, we have

OE = EC = CD = OD = r

$\therefore$ AF = AE = b – r and BF = BD = a – r ... (i)

$\Rightarrow$ AF + BF = (b – r) + (a – r) [from (i)]

$\Rightarrow$ AB = b – r + a – r

$\Rightarrow$ c = a + b – 2r [$\because$ AB = c]

$\Rightarrow$ 2r = a + b – c

$\therefore$ $r=\dfrac{a+b-c}{2}$ [Hence proved]

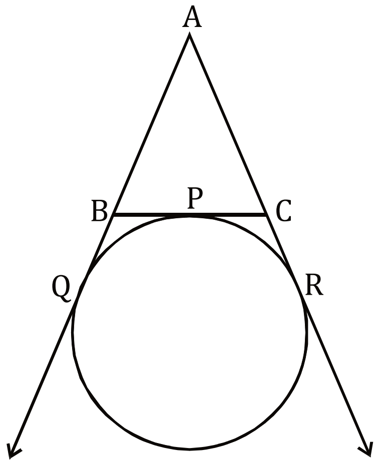
**28.** A circle touches the side BC of $\Delta ABC$ at P and touch AB and AC produced at Q and R respectively. Prove that $AQ = \frac{1}{2}\text{Perimeter}$ of $\Delta ABC$.

**Sol.** Since tangents from an exterior point to a circle are equal in length.

$\therefore$ BP = BQ ... (i)

CP = CR ... (ii)

And AQ = AR ... (iii)




Now, perimeter of $\Delta ABC$ = AB + BC + AC

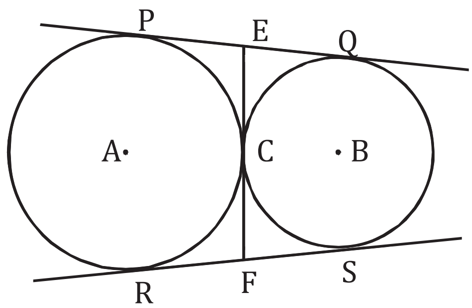
= AB + (BP + PC) + AC

= (AB + BQ) + (AC + CR) [From (i) and (ii)]

= AQ + AR = 2AQ [$\because$ AQ = AR]

So, $AQ = \frac{1}{2}(\text{Perimeter of } \Delta ABC)$

**29.** In the figure, two circles touch each other externally at C. Prove that the common tangent at C bisects the other two common tangents.




**Sol.** Given: Two circles with centres at A and B touch each other externally at C. RS and PQ are two common tangents to the circles.

To prove: (i) PE = EQ (ii) RF = FS.

Proof: We know that the tangents drawn from an exterior point to a circle are equal.

$\because$ PE = EC ... (i)

Similarly, EQ = EC ... (ii)

PE = EQ [From (i) and (ii)]

$\Rightarrow$ EC bisects PQ at E.

Similarly, we can prove that CF is the bisector of RS. So, EF bisects PQ and RS.

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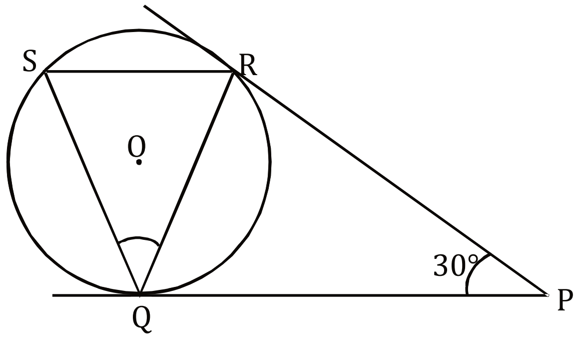
**Long answer type questions** **(5 marks)**

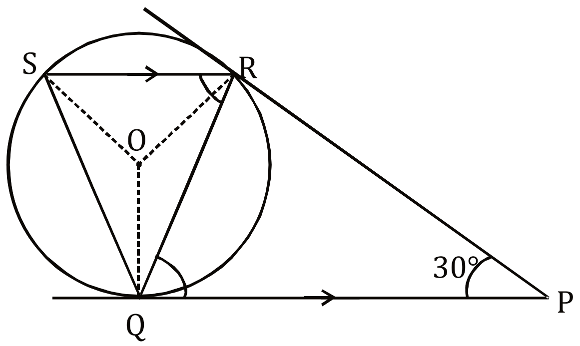
**30.** In the given figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30°$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$.




**Sol.** We know that, tangents drawn from an external point to a circle are equal in length.

So, PQ = PR

$\therefore \quad \angle PRQ = \angle PQR$




In $\Delta PQR$,

$\angle PQR + \angle PRQ + \angle QPR = 180°$ [Angle sum property]

$\Rightarrow \quad \angle PQR + \angle PQR + 30° = 180°$

$\Rightarrow \quad 2\angle PQR = 150°$

$\Rightarrow \quad \angle PQR = 75°$

Since, SR ∥ QP and QR is a transversal,

$\therefore \quad \angle SRQ = \angle PQR = 75°$ [Alternate angles]

Now, join OR, OS and OQ

We know that, angle subtended by an arc at the centre is double the angle subtended by same arc at any point on the circle.

So, $\angle SOQ = 2\angle SRQ$

$\Rightarrow \quad \angle SOQ = 2 \times 75° = 150°$

Now, in $\Delta OSQ$, OS = OQ [Radius of circle]

$\Rightarrow \quad \angle OQS = \angle OSQ$

Now, by angle sum property,

$2\angle OQS = 180^\circ - 150^\circ$

$\Rightarrow \quad \angle OQS = 15^\circ$

We know that, tangent is perpendicular to radius at the point of contact.

So, $\angle OQP = 90^\circ$

$\Rightarrow \quad \angle OQR + \angle PQR = 90^\circ$

$\Rightarrow \quad \angle OQR = 90^\circ - 75^\circ = 15^\circ$

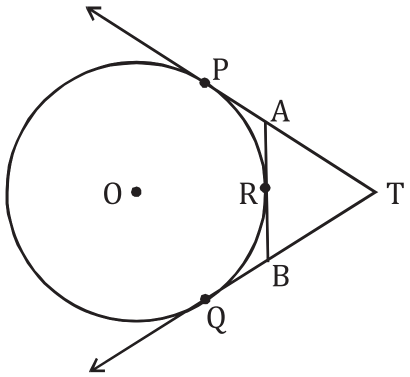
Now, $\angle RQS = \angle OQR + \angle OQS$

$\Rightarrow \quad \angle RQS = 15^\circ + 15^\circ = 30^\circ$

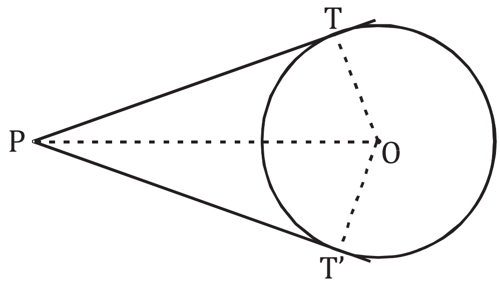
**31.** Prove that the lengths of the tangents drawn from an external point to a circle are equal.

Using the above, do the following:

In the figure, TP and TQ are tangents from T to the circle with centre O and R is any point on the circle. If AB is a tangent to the circle at R, prove that : TA + AR = TB + BR.




**Sol. Part I**




Given : PT and PT' are two tangents drawn from an external point P to a circle C (O, r).

To prove: PT = PT'

Construction: Join TO, T'O and PO.

Proof: In $\Delta POT$ and $\Delta POT'$

| | |
|---|---|
| PO = PO | (Common) |
| OT = OT' | (Radii of the same circle) |
| $\angle PTO = \angle PT'O = 90^\circ$ | [$OT \perp PT$, $OT' \perp PT'$] |
| $\therefore \quad \Delta POT \cong \Delta POT'$ | (RHS congruence) |
| $\Rightarrow \quad PT = PT'$ | (CPCT) |

**Part II**

From given figure, we have

TP = TQ ... (i)

AP = AR ... (ii)

BQ = BR ... (iii)

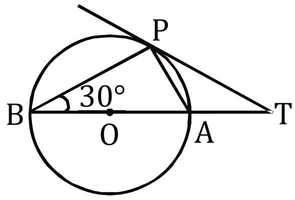
[Tangents drawn from an external point to a circle are equal in length]

From (i),

TA + AP = TB + BQ

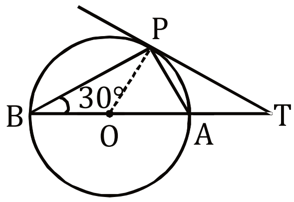
$\Rightarrow$ TA + AR = TB + BR [From (ii) and (iii)]

**32.** In the given figure, O is the centre of the circle and TP is the tangent to the circle from an external point T.




If $\angle PBT = 30^\circ$, prove that BA : AT = 2 : 1.

**Sol.** Given : O is the centre of the circle and TP is the tangent to circle and $\angle PBT = 30^\circ$




To prove : $\frac{BA}{AT} = \frac{2}{1}$

Construction : Join OP

Proof : $\angle BPA = 90^\circ$ [Angle in a semi circle]

In $\Delta BPA$, $\angle P + \angle PBA + \angle BAP = 180^\circ$ [Angle sum property]

$\Rightarrow$ $90^\circ + 30^\circ + \angle BAP = 180^\circ$

$\Rightarrow$ $\angle BAP = 60^\circ$

Also, $\angle OPT = 90^\circ$ [Tangent is perpendicular to radius at a point of contact]

And OP = OA [Radii of same circle] ... (i)

$\therefore$ $\angle OAP = \angle OPA = 60^\circ$ ... (ii)

$\Rightarrow$ $\angle APT = 90^\circ - 60^\circ = 30^\circ$

Now, $\angle OAP + \angle PAT = 180^\circ$ [Linear pair]

$\Rightarrow$ $60^\circ + \angle PAT = 180^\circ$ [Using (ii)]

$\Rightarrow \quad \angle PAT = 120°$

In $\Delta PAT$, $\angle PAT + \angle APT + \angle PTA = 180°$ [Angle sum property]

$\Rightarrow \quad 120° + 30° + \angle PTA = 180°$

$\Rightarrow \quad \angle PTA = 30°$

$\Rightarrow \quad PA = AT$ ... (iii)

Also in $\Delta OAP$,

$\angle AOP = 180° - 60° - 60° = 60°$

$\therefore \quad \angle AOP = \angle OPA$

$\Rightarrow \quad PA = OA$ ... (iv)

Hence, $PA = AT = OA = OP$ [Using (i), (iii) and (iv)]

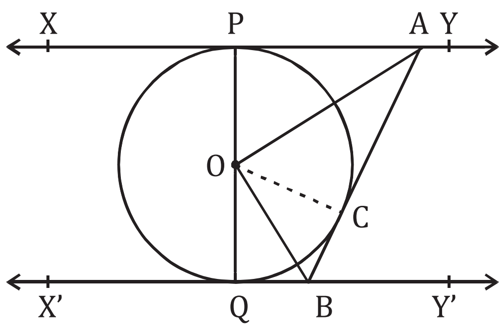
Now, $BA = BO + OA = 2OA$ [$\because$ $OA = OB$]

$\Rightarrow \quad BA = 2AT$ ($\because$ $OA = AT$)

$\Rightarrow \quad \dfrac{BA}{AT} = \dfrac{2}{1}$

Hence proved

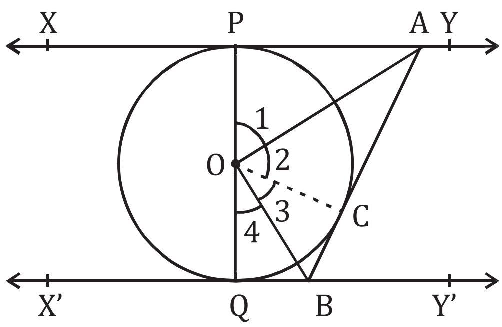
**33.** In the given figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C is intersecting XY at A and X'Y' at B. Prove that $\angle AOB = 90°$.




**Sol.** Given : Two parallel tangents XY and X'Y' to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B.

To Prove : $\angle AOB = 90°$

Proof : In $\Delta APO$ and $\Delta ACO$

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AP = AC (Tangents drawn from an external point to a circle are equal in length).

OP = OC (Radii of same circle)

OA = OA (Common)

$\therefore$ $\Delta APO \cong \Delta ACO$ (SSS congruence criterion)

$\therefore$ $\angle 1 = \angle 2$ (By CPCT) ... (i)

Similarly, $\Delta OCB \cong \Delta OQB$

$\therefore$ $\angle 3 = \angle 4$ (By CPCT) ... (ii)

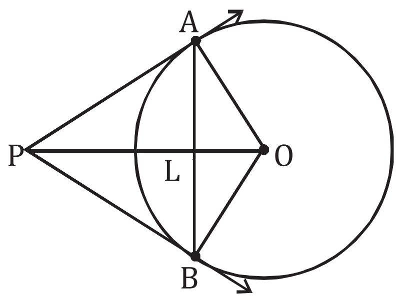
Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$ (Linear pair)

$\Rightarrow$ $2\angle 2 + 2\angle 3 = 180°$ (Using (i) and (ii))

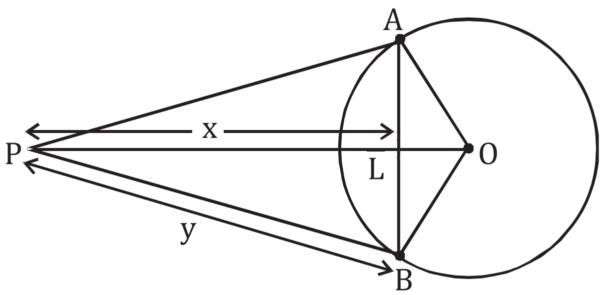
$\Rightarrow$ $\angle 2 + \angle 3 = \dfrac{180°}{2} = 90°$

$\Rightarrow$ $\angle AOB = 90°$

**34.** In the given figure, AB is a chord of a circle, with centre O, such that AB = 16 cm and radius of circle is 10 cm. Tangents at A and B intersect each other at P. Find the length of PA.




**Sol.**




We have, AB = 16 cm

$\therefore$ AL = BL = 8 cm

In $\Delta OLB$, we have

$OB^2 = OL^2 + LB^2$ [By pythagoras theorem]

$\Rightarrow$ $(10)^2 = OL^2 + (8)^2$

$\Rightarrow$ $OL^2 = 100 - 64 = 36$

$\Rightarrow$ OL = 6 cm

Let PL = x cm and PB = y cm

Then, OP = (x + 6) cm

In $\Delta PLB$, $PB^2 = PL^2 + BL^2$

$\Rightarrow$ $y^2 = x^2 + 64$ [By pythagoras theorem]

Now, $OB \perp PB$.

In $\Delta OBP$, $OP^2 = OB^2 + PB^2$

$\Rightarrow$ $(x + 6)^2 = 100 + y^2$ [By pythagoras theorem]

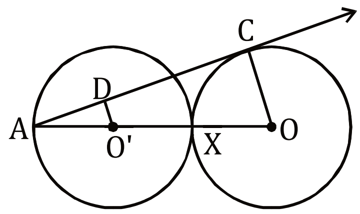
$\Rightarrow$ $x^2 + 36 + 12x = 100 + x^2 + 64$ [$\because$ $y^2 = x^2 + 64$]

$\Rightarrow$ $12x = 128 \Rightarrow x = \dfrac{32}{3}$

$\therefore$ $y^2 = \left(\dfrac{32}{3}\right)^2 + 64 = \dfrac{1600}{9} \Rightarrow y = \dfrac{40}{3}$

Hence, $PA = PB = \dfrac{40}{3}$ cm

**35.** In figure, two equal circles, with centres O and O' touch each other at X.OO' produced meets the circle with centre O' at A. AC is tangent to the circle with centre O, at the point C. O'D is perpendicular to AC. Find the value of $\dfrac{DO'}{CO}$.




**Sol.** AC is tangent to circle with centre O.

Thus, $\angle ACO = 90°$

In $\Delta AO'D$ and $\Delta AOC$

$\angle ADO'$ and $\angle ACO = 90°$

$\angle A = \angle A$ (Common)

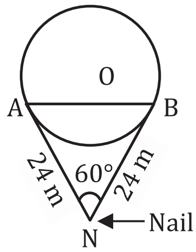
$\therefore$ $\Delta AO'D \sim \Delta AOC$ (By AA similarity)

$\Rightarrow$ $\dfrac{AO'}{AO} = \dfrac{DO'}{CO}$

$\therefore$ $\dfrac{DO'}{CO} = \dfrac{r}{3r} = \dfrac{1}{3}$ ($\because$ $AO = AO' + O'X + XO = 3r$)

**Case Study type questions** **(4 marks)**

**1.** In a Diwali mela, organisers for publicity used a huge balloon which can be seen from long distance. It was held in air by two strings 24m of length each, inclined at an angle of 60° at a point on the ground, where these were tied to a nail as shown in figure.


(i) Calculate the length of line segment AB ?

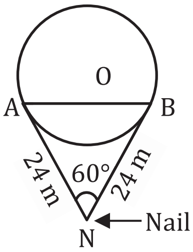
(ii) If the perpendicular distance of chord AB from centre 'O' is 5m, find the radius of balloon.

(iii) Find the area of ΔABN

**[OR]**

(iii) If the length of strings remains same but inclination angle changes to 90°, then what will be the distance between them?

**Sol.**




(i) AN = BN

$\angle NAB = \angle NBA = x$

$\angle ANB + x + x = 180°$

$60° + 2x = 180°$

$2x = 120°$

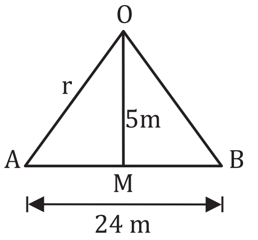
$x = 60°$

$\angle ANB = \angle NAB = \angle NBA = 60°$

Δ is equilateral

AB = 24 m

(ii)




We know that perpendicular from the centre to the chord, bisect the chord.

$AM = \frac{1}{2} \times 24 = 12$ m

In $\Delta AOM$ by Pythagoras theorem

$AO^2 = AM^2 + OM^2$

$AO^2 = (12)^2 + (5)^2$

$AO^2 = 144 + 25$

$AO^2 = 169$

$AO = 13$ m

(iii) $Ar(\Delta ABN) = \frac{\sqrt{3}}{4}(\text{Side})^2$

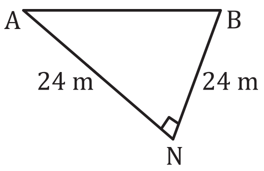
$\Rightarrow \frac{\sqrt{3}}{4}(24)^2$

$= \frac{\sqrt{3}}{4} \times 24 \times 24$

$\Rightarrow 144\sqrt{3}\ m^2$

**[OR]**

(iii)

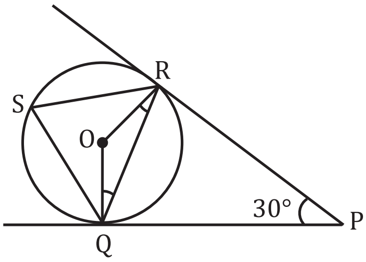



$AB^2 = 24^2 + 24^2$

$AB^2 = 2(24)^2$

$AB = 24\sqrt{2}$ m

**2.** A Ferris Wheel (or a big wheel in the United Kingdom) is an amusement side consisting of a rotating upright wheel with multiple passenger carrying components (commonly referred to as passenger cars, cabins, tubes, capsules or pods) attached to the rim in such a way that as the wheel turns, they are kept upright usually by gravity . After taking a ride in Ferris wheel, Aarti came out from the crowd and was observing her friends who were enjoying the ride. She was curious about the different angles and measure that the wheel will form. She forms the figure as given below.




(i) In the given fig, find the value of $\angle ROQ$.

**[OR]**

(i) Find the measurement of $\angle RSQ$.

(ii) What is the measurement of $\angle ORP$?

(iii) If $\angle P = 60°$ and OQ = OR = 5 m then find the length of PQ.

**Sol.** (i) In quadrilateral PQOR, by angle sum property,

$\angle ORP + \angle OQP + \angle QPR + \angle QOR = 360°$

$90° + 90° + 30° + \angle QOR = 360°$

$210° + \angle QOR = 360°$

$\angle QOR = 360° - 210° = 150°$

**[OR]**

(i) We know that angle subtended by an arc at the centre is double of the angle subtended by it at any other point in the alternate segment

$\angle QOR = 2\ \angle RSQ$

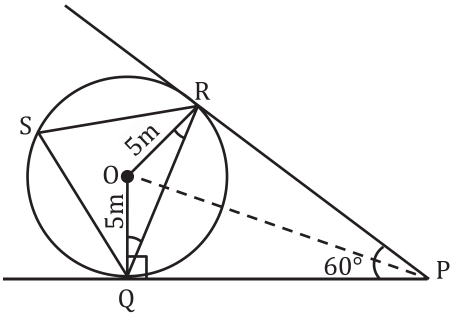
$150° = 2\angle RSQ$

$\angle RSQ = \frac{1}{2} \times 150° = 75°$

(ii) We know that radius is perpendicular to the tangent.

So, $\angle ORP = 90°$

(iii)




If we join the points O and P, OP will be angle bisector of $\angle P$

So, in $\Delta OQP$, $\angle OQP = 90°$ (tangent is perpendicular to radius)

$\angle OPQ = 30°$

$\tan 30° = \frac{OQ}{PQ}$

$\frac{1}{\sqrt{3}} = \frac{5}{PQ}$

$PQ = 5\sqrt{3}\,m$

# 11 Areas Related to Circles

**Multiple choice questions** **(1 mark)**

**1.** The perimeter of a circle having radius 5 cm is equal to:

(1) 30 cm (2) 3.14 cm (3) 31.4 cm (4) 40 cm

**Sol.** **Option (3)**

The perimeter of the circle is equal to the circumference of the circle.

Circumference = $2\pi r$

$= 2 \times 3.14 \times 5 = 31.4$ cm

**2.** Area of the circle with radius 5 cm is equal to

(1) 60 sq.cm (2) 75.5 sq.cm (3) 78.5 sq.cm (4) 10.5 sq.cm

**Sol.** **Option (3)**

Radius = 5cm

Area = $\pi r^2 = 3.14 \times 5 \times 5 = 78.5$ sq.cm

**3.** The largest triangle inscribed in a semi-circle of radius r, then the area of that triangle is

(1) $r^2$ (2) $\frac{1}{2}r^2$ (3) $2r^2$ (4) $\sqrt{2}r^2$

**Sol.** **Option (1)**

The height of the largest triangle inscribed will be equal to the radius of the semi-circle and base will be equal to the diameter of the semi-circle.

Area of triangle = $\frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 2r \times r = r^2$

**4.** If the perimeter of the circle and square are equal, then the ratio of their areas will be equal to:

(1) 14 : 11 (2) 22 : 7 (3) 7 : 22 (4) 11 : 14

**Sol.** **Option (1)**

Given, The perimeter of circle = perimeter of the square

$2\pi r = 4a$

$a = \frac{\pi r}{2}$

Area of square = $a^2 = \left(\frac{\pi r}{2}\right)^2$

$$\frac{A_{\text{Circle}}}{A_{\text{Square}}} = \frac{\pi r^2}{\left(\frac{\pi r}{2}\right)^2} = \frac{14}{11}$$

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**5.** The area of the circle that can be inscribed in a square of side 8 cm is

(1) 36 π $cm^2$ (2) 16 π $cm^2$ (3) 12 π $cm^2$ (4) 9 π $cm^2$

**Sol. Option (2)**

Given,

Side of square = 8 cm

Diameter of a circle = side of square = 8 cm

Therefore, Radius of circle = 4 cm

Area of circle = $\pi(4)^2$

$= \pi\ (4)^2$

$= 16\pi\ cm^2$

**6.** The area of the square that can be inscribed in a circle of radius 8 cm is

(1) 256 $cm^2$ (2) 128 $cm^2$ (3) 642 $cm^2$ (4) 64 $cm^2$

**Sol. Option (2)**

Radius of circle = 8 cm

Diameter of circle = 16 cm = diagonal of the square

Let "a" be the triangle side, and the hypotenuse is 16 cm.

Using Pythagoras theorem, we can write.

$16^2 = a^2 + a^2$

$256 = 2a^2$

$a^2 = \frac{256}{2}$

$a^2 = 128\ cm^2$ = area of a square.

**7.** The area of a sector of a circle with radius 6 cm if the angle of the sector is 60°.

(1) $\frac{142}{7}$ $cm^2$ (2) $\frac{152}{7}$ $cm^2$ (3) $\frac{132}{7}$ $cm^2$ (4) $\frac{122}{7}$ $cm^2$

**Sol. Option (3)**

Angle of the sector is 60°

Area of sector = $\left(\frac{\theta}{360°}\right) \times \pi r^2$

Area of the sector with angle 60° = $\left(\frac{60°}{360°}\right) \times \pi r^2\ cm^2$

$= \left(\frac{36}{6}\right) \pi\ cm^2$

$= 6 \times \left(\frac{22}{7}\right) cm^2 = \frac{132}{7}\ cm^2$

**8.** In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. The length of the arc is

(1) 20cm (2) 21cm (3) 22cm (4) 25cm

**Sol. Option (3)**

Length of an arc = $\left(\frac{\theta}{360°}\right)\times 2\pi r$

$\therefore$ Length of an arc AB = $\left(\frac{60°}{360°}\right)\times 2\times\frac{22}{7}\times 21$

= $\left(\frac{1}{6}\right)\times 2\times\left(\frac{22}{7}\right)\times 21$ or arc AB Length = 22 cm

**Assertion reason questions** **(1 mark)**

**9.** **Assertion (A):** If the circumference of a circle is 176 cm, then its radius is 28 cm.

**Reason (R):** Circumference = $2\pi$ × radius.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol. Option (1)**

Circumference = $2\pi r = 176$

r = $\frac{176}{2}\times\frac{7}{22} = 28\text{cm}$

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

**10.** **Assertion (A):** If the outer and inner diameter of a circular path is 10 m and 6 m respectively, then area of the path is $16\pi$ m$^2$.

**Reason (R):** If R and r be the radius of outer and inner circular path respectively, then area of circular path = $\pi(R^2 - r^2)$.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol. Option (1)**

Area of path = $\pi(R^2 - r^2) = \pi(5^2 - 3^2) = 16\pi$ m$^2$

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

**Very short answer type questions** **(2 marks)**

**11.** The circumference of a circle is 22 cm. Find the area of quadrant (in cm$^2$) of this circle.

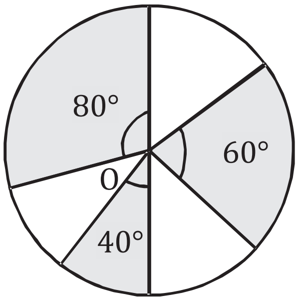
**Sol.** Let the radius of the circle be r cm. Given circumference of circle, $2\pi r = 22$ cm

$$\Rightarrow \quad r = \frac{22\times 7}{2\times 22} \Rightarrow r = \frac{7}{2}\text{cm}$$

Area of quadrant of circle $= \frac{1}{4}\pi r^2$

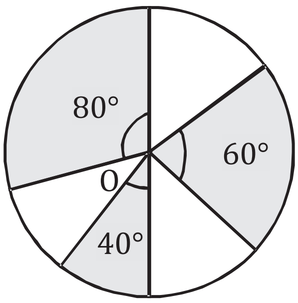
$$= \frac{1}{4}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2} = \frac{77}{8}\ \text{cm}^2$$

**12.** In the given figure, three sectors of a circle of radius 7 cm, making angles of 60°, 80°, 40° at the centre are shaded. Find the area of the shaded region (in cm$^2$) $\left[\text{Using } \pi = \frac{22}{7}\right]$.




**Sol.** Radius (r) of circle = 7 cm

Area of shaded region $= \frac{\pi(7)^2 .40°}{360°} + \frac{\pi(7)^2 \cdot 60°}{360°} + \frac{\pi(7)^2 \cdot 80°}{360°}$ $\left[\because \text{Area of sector} = \frac{\theta}{360°}\pi r^2\right]$




$$= \frac{\pi(7)^2}{9} + \frac{\pi(7)^2}{6} + \frac{\pi(7)^2\cdot 2}{9}$$

$$= \pi(7)^2\left[\frac{1}{9}+\frac{1}{6}+\frac{2}{9}\right] = \frac{22}{7}\times 7\times 7\times\frac{9}{18} = 77\,\text{cm}^2$$

**13.** In the given figure, the shape of the top of a table is like that of a sector of a circle with centre O and $\angle AOB = 90$. If AO = OB = 42 cm, then find the perimeter of the top of the table. $\left[\text{Use } \pi = \frac{22}{7}\right]$

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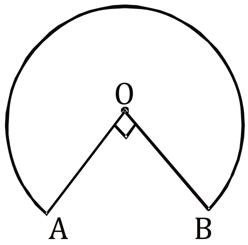


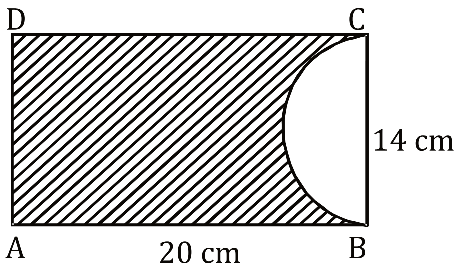

**Sol.** Given, radius (r) of the table = 42 cm and $\angle AOB = 90°$.

Perimeter of top of the table $= \frac{2\pi r\theta}{360°} + 2r$ (where $\theta = 360° - 90°) = 270°$

$$= 2 \times \frac{22}{7} \times 42 \times \frac{270°}{360°} + 2(42) = 198 + 84 = 282 \text{cm}$$

**14.** A paper is in the form of a rectangle ABCD in which AB = 20 cm, BC = 14 cm. A semi-circular portion with BC as diameter is cut off. Find the area of the remaining part. $\left(\text{Use } \pi = \frac{22}{7}\right)$

**Sol.** Area of remaining part = Area of rectangle – Area of semi-circle




$= 20 \times 14 - \frac{22 \times 7 \times 7}{7 \times 2}$

$= 280 - \frac{154}{2}$

$= 280 - 77 = 203 \text{ cm}^2$.

**15.** A cow is tied with a rope of length 14 m at one corner of rectangular field of dimensions 20 m × 15 m. find the area of the field in which the cow cannot graze. $\left[\text{Use } \pi = \frac{22}{7}\right]$

**Sol.** Let ABCD be a rectangular field of dimensions 20 m × 15 m.

Since, ABCD is a rectangular field.

$\therefore$ Each angle will be a right angle.

Length of rope = 14 m

$\therefore$ Area of field in which the cow can graze $= \frac{\theta}{360°}\pi r^2 = \frac{90°}{360°} \times \frac{22}{7} \times (14)^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = 154 \text{m}^2$$

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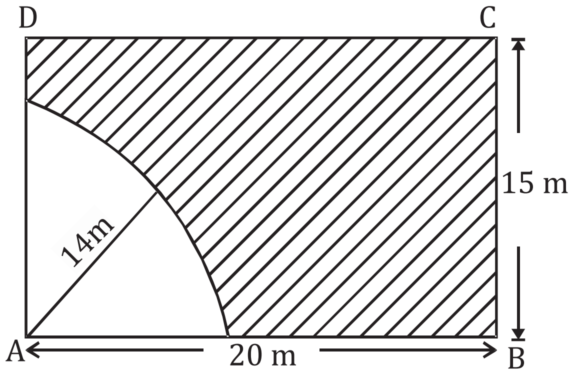
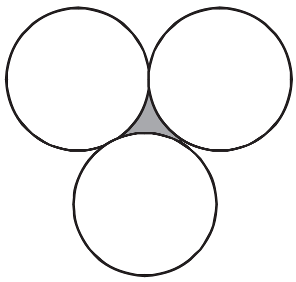
Area of rectangular field = 20 × 15 = 300 $m^2$

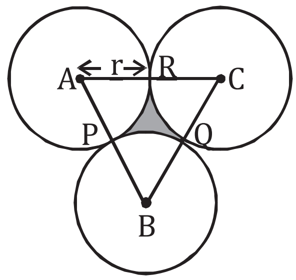
Hence, required area = Area of rectangular field – Area of field in which the cow can graze

= (300 – 154) $m^2$ = 146 $m^2$




**16.** In the given figure, three circles each of radius 3.5 cm are drawn in such a way that each of them touches the other two. Find the area enclosed between these three circles (shaded region). $\left[\text{Use } \pi = \frac{22}{7}, \sqrt{3} = 1.732\right]$






**Sol.** Let A, B and C be the centres of given circles.

Radius of each circle, (r) = 3.5 cm.

Thus, sides of ΔABC is AB = BC = CA = (3.5 + 3.5) = 7 cm

Hence, ABC is an equilateral triangle.

∴ ∠A = ∠B = ∠C = 60°

Area of the shaded region = Area of ΔABC – (Sum of areas of sectors APR, BPQ and CQR)

$$= \frac{\sqrt{3}}{4}(AB)^2 - 3 \times \frac{\theta}{360°} \times \pi r^2$$

$$= \frac{1.732}{4} \times (7)^2 - 3 \times \frac{60°}{360°} \times \frac{22}{7} \times 3.5 \times 3.5$$

= 21.217 – 19.25 = 1.967 $cm^2$

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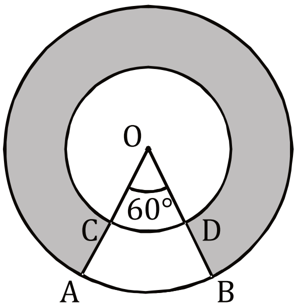
### Short answer type questions (3 marks)

**17.** In the given figure, two concentric circles with centre O have radii 21 cm and 42 cm. If $\angle AOB = 60°$, find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$




**Sol.** Area of region ABDC = (Area of sector OAB) – (Area of sector OCD)

$$= \frac{\pi(OA)^2 \times 60°}{360°} - \frac{\pi(OC)^2 \times 60°}{360°}$$

$$= \frac{\pi}{6} \times (42)^2 - \frac{\pi}{6} \times (21)^2 \quad [\because OA = 42 \text{ cm and } OC = 21 \text{ cm}]$$

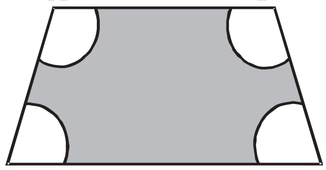
$$= \frac{\pi}{6}(1764 - 441) = \frac{1323\pi}{6}$$

Area of circular ring = $\pi[(42)^2 - (21)^2] = \pi(1764 - 441) = 1323\pi$

$\therefore$ Area of shaded region = Area of circular ring – Area of region ABDC

$$= 1323\pi - \frac{1323\pi}{6} = 1323 \times \frac{22}{7} \times \frac{5}{6} = 3465 \text{cm}^2$$

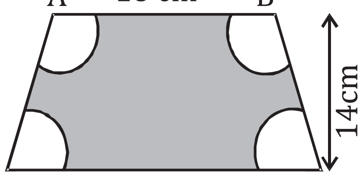
**18.** In the given figure, ABCD is a trapezium with AB ∥ DC, AB = 18 cm, DC = 32 cm and the distance between AB and DC is 14 cm. If arcs of equal radii 7 cm have been drawn, with centres A, B, C and D, then find the area of the shaded region.




**Sol.** Given, AB = 18 cm, DC = 32 cm,

Radius of each sector = 7 cm

Now, area of shaded region = Area of trapezium – Area of four sectors



$$=\frac{1}{2}h(AB+DC)-\left[\frac{\angle A}{360^\circ}\pi(7)^2+\frac{\angle B}{360^\circ}\pi(7)^2+\frac{\angle C}{360^\circ}\pi(7)^2+\frac{\angle D}{360^\circ}\pi(7)^2\right]$$

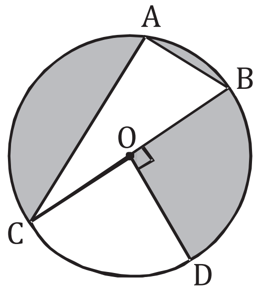
[where h is distance between AB and CD].

$$=\frac{1}{2}\times 14(18+32)-\frac{\pi}{360^\circ}(7)^2[\angle A+\angle B+\angle C+\angle D]$$

$$=7\times 50-\frac{\pi}{360^\circ}\times 49\times 360^\circ$$ (Sum of angles of quadrilateral is 360°)

$$=7\times 50-\frac{22}{7}\times 49=350-154=196\,cm^2$$

**19.** In the given figure, O is the centre of the circle with AC = 24 cm, AB = 7 cm and $\angle BOD = 90^\circ$. Find the area of the shaded region. [Use $\pi = 3.14$]




**Sol.** Given, O is the centre of circle with AC = 24 cm, AB = 7 cm and $\angle BOD = 90^\circ$

$\therefore$ $\angle BAC = 90^\circ$.

So, area of $\Delta ABC=\frac{1}{2}\times 7\times 24=84\,cm^2$

Using Pythagoras theorem in $\Delta ABC$, we have

$(BC)^2 = (24)^2 + (7)^2 = 576 + 49 = 625$

$\Rightarrow$ BC = 25 cm

$\because$ BC is diameter of circle.

$\therefore$ OC = Radius of circle $=\frac{25}{2}$ cm

Area of the sector $COD=\frac{\theta}{360^\circ}\times\pi r^2$

$$=\frac{90^\circ}{360^\circ}\times 3.14\times\frac{25}{2}\times\frac{25}{2}=\frac{1}{4}\times 3.14\times\frac{625}{4}$$

$$=\frac{1962.5}{16}=122.65\,cm^2$$

Area of circle $\pi r^2=3.14\times\frac{25}{2}\times\frac{25}{2}=\frac{1962.5}{4}=490.62\,cm^2$

Area of the shaded region = Area of circle – (Area of $\Delta ABC$ + Area of sector COD)

= 490.62 – (84 + 122.65)

= 490.62 – 206.65 = 283.97 cm$^2$



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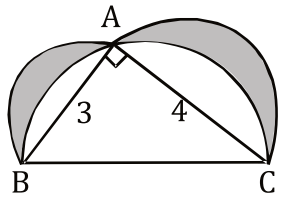
**20.** In the given figure, ABC is a right-angled triangle, right-angled at A. Semi-circles are drawn on AB, AC and BC as diameters. Find the area of shaded region.




**Sol.** Using Pythagoras theorem in $\Delta ABC$, we obtain $BC^2 = AB^2 + AC^2 = 3^2 + 4^2 = 9 + 16 = 25$

$\Rightarrow$ BC = 5 units

Area of shaded region = (Area of semi-circle of diameter AB + Area of semi-circle of diameter AC) – (Area of semi-circle of diameter BC – area of $\Delta ABC$) ... (i)

Area of semi-circle whose diameter is AB

$$=\frac{1}{2}\pi\left(\frac{3}{2}\right)^2=\frac{9\pi}{8} \text{ square units}$$

Area of semi-circle whose diameter is AC $=\frac{1}{2}\pi\left(\frac{4}{2}\right)^2=2\pi$ square units

Area of semi-circle whose diameter is BC $=\frac{1}{2}\pi\left(\frac{5}{2}\right)^2=\frac{25\pi}{8}$ square units

Area of $\Delta ABC=\frac{1}{2}\times \text{base}\times\text{height}$

$$=\frac{1}{2}\times 3\times 4=6 \text{ sq.units}$$

From (i), we obtain, Area of shaded region

$$=\left(\frac{9\pi}{8}+2\pi\right)-\left(\frac{25\pi}{8}-6\right)$$

$$=\left(\frac{9\pi+16\pi-25\pi+48}{8}\right)=\frac{48}{8}=6 \text{ square units}$$

**21.** The long and short hands of a clock are 6 cm and 4 cm long respectively. Find the sum of the distances travelled by their tips in 24 hours. (Use $\pi = 3.14$)

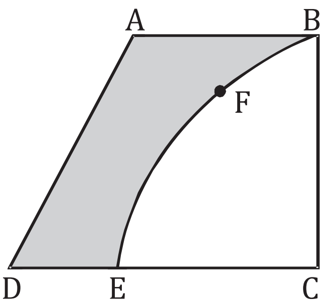
**Sol.** In 24 hours, short-hand of clock i.e., hour hand takes 2 complete revolutions and long hand i.e., minute hand takes 24 revolutions.

$\therefore$ Distance travelled by hour hand in 24 hours = $(2\pi r) \times 2 = 2\pi(4)(2) = 16\pi$ cm

Distance travelled by minute hand in 24 hours = $2\pi(6) \times 24 = 288\pi$ cm

Sum of the distance travelled = $16\pi + 288\pi = 304\pi = 304 \times 3.14 = 954.56$ cm

**22.** From a thin metallic piece, in the shape of a trapezium ABCD in which AB ∥ CD and ∠BCD=90°, a quarter circle BFEC is removed (see figure). Given AB = BC = 3.5 cm and DE = 2 cm, calculate the area of remaining (shaded) part of the metal sheet. $\left[\text{Use } \pi = \frac{22}{7}\right]$




**Sol.** We have, AB = BC = 3.5 cm, DE = 2 cm, ∠BCD = 90,

DC = DE + EC = DE + BC

= 2 + 3.5 = 5.5 cm [∵ EC = BC (radii of same quarter circle)]

Height of trapezium, BC = 3.5 cm

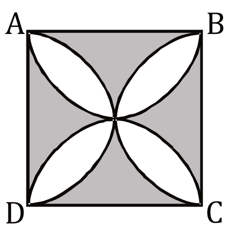
Area of the shaded part of the metal sheet = Area of trapezium ABCD – Area of quarter circle BFEC

$$= \frac{1}{2}(AB + CD)BC - \frac{1}{4}\pi(BC)^2$$

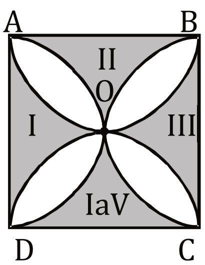
$$= \frac{1}{2}(3.5 + 5.5)3.5 - \frac{1}{4} \times \frac{22}{7} \times (3.5)^2$$

= 15.75 – 9.625 = 6.125 $cm^2$

**23.** In the given figure, ABCD is a square of side 14 cm. Semi-circles are drawn with each side of square as diameter. Find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$




**Sol.**



Given, ABCD is a square of side 14 cm.

$\therefore$ Area of square = 14 × 14 = 196 $cm^2$

Area of semicircle AOD = Area of semicircle DOC

= Area of semicircle BOC = Area of semicircle BOA

$$= \frac{\pi}{2}r^2 = \frac{22}{7\times 2}\times 7\times 7 = 77\,cm^2$$

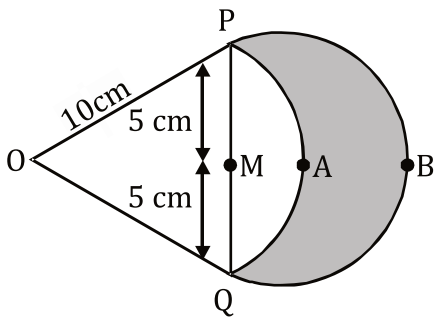
Hence, area of shaded region I and III = area of square – [area of semicircle AOB + area of semicircle DOC] = 196 – (77 + 77) $cm^2$ = 42 $cm^2$

Similarly, area of shaded region II and IV = 42 $cm^2$

$\therefore$ Required area of all four shaded regions = 42 + 42 = 84 $cm^2$

**24.** The given figure shows two arcs PAQ and PBQ. Arc PAQ is a part of circle with centre O and radius OP while arc PBQ is a semi-circle drawn on PQ as diameter with centre M.

If OP = PQ = 10 cm show that area of shaded region is $25\left(\sqrt{3}-\frac{\pi}{6}\right)cm^2$.




**Sol.** Given, OP and OQ are radii of the circle.

$\therefore$ OP = OQ = 10 cm

$\Rightarrow$ $\Delta$OPQ is an equilateral triangle.

$\therefore$ $\angle$POQ = 60°

Area of region PAQP = Area of sector OPAQO – Area of $\Delta$OPQ

$$= \frac{\theta}{360^\circ}\pi r^2 - \frac{\sqrt{3}}{4}(\text{side})^2$$

$$= \frac{60^\circ}{360^\circ}\pi(10)^2 - \frac{\sqrt{3}}{4}(10)^2 = \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right)cm^2$$

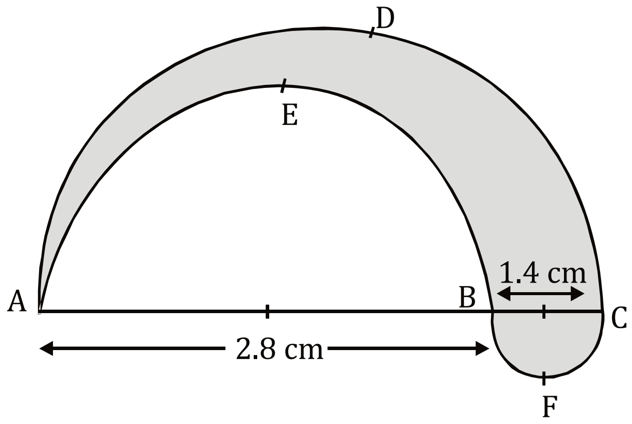
Area of semi-circle PBQ $= \frac{\pi r^2}{2} = \frac{\pi(5)^2}{2}$

$\therefore$ Area of shaded region = Area of semi-circle PBQP – Area of region PAQP

$$= \frac{\pi(25)}{2} - \frac{100\pi}{6} + \frac{100\sqrt{3}}{4}$$

$$= \frac{100\sqrt{3}}{4} - \frac{25\pi}{6} = 25\left(\sqrt{3}-\frac{\pi}{6}\right)cm^2$$

**25.** In the figure, find the perimeter of the shaded region where ADC, AEB and BFC are semi-circles on diameters AC, AB and BC respectively.




**Sol.** Diameter of semi-circle ADC = 2.8 cm + 1.4 cm = 4.2 cm

$\Rightarrow$ Radius of semi-circle $ADC = \frac{4.2}{2}\text{cm} = 2.1\text{cm}$

$\therefore$ Circumference of semi-circle ADC $= \frac{22}{7} \times 2.1\text{cm} = \frac{22}{7} \times \frac{21}{10} = \frac{66}{10}\text{cm} = 6.6\text{cm}$

Diameter of semi-circle AEB = 2.8 cm

$\Rightarrow$ Radius of semi-circle $AEB = \frac{2.8}{2} = 1.4\text{cm}$

$\therefore$ Circumference of semi-circle AEB $= \pi \times r = \frac{22}{7} \times 1.4\text{cm} = 22 \times \frac{2}{10}\text{cm} = 4.4\text{cm}.$

Diameter of semi-circle BFC = 1.4 cm

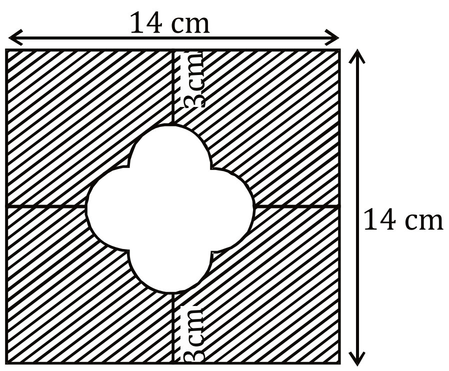
$\Rightarrow$ Radius of semi-circle BFC = 0.7 cm

$\therefore$ Circumference of semi-circle BFC $= \frac{22}{7} \times 0.7 = 2.2\text{cm}$

$\therefore$ Total perimeter of the shaded region

= 6.6 cm + 4.4 cm + 2.2 cm = 13.2 cm.

**26.** In figure, find the area of the shaded region (Use $\pi$ = 3.14)

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**Sol.** Area of square ABCD = 14 × 14 = 196 $cm^2$

Radius of the semi-circle formed inside = 2 cm

Area of 4 semi-circles = $4 \times \frac{1}{2} \pi r^2$

$= 4 \times \frac{1}{2} \times 3.14 \times 2 \times 2$

= 25.12 $cm^2$

Length of the side of square formed inside the semi-circle = 4 cm.

Area of the square = 4 × 4 = 16 $cm^2$

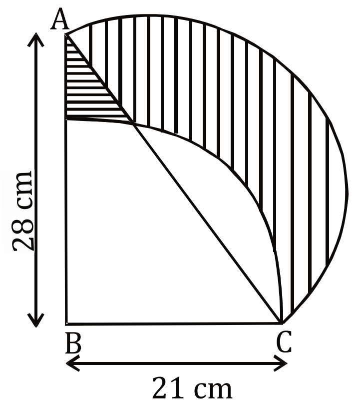
Area of the shaded region = Area of square ABCD – (Area of 4 semi-circle + Area of square)

= 196 – (25.12 + 16)

= 196 – 41.12

= 154.88 $cm^2$

**27.** In the figure, ABC is a right-angled triangle, ∠B = 90°, AB = 28 cm and BC = 21 cm. With AC as diameter, a semi-circle is drawn and with BC as radius a quarter circle is drawn. Find the area of the shaded region.




**Sol.** In right ΔABC, right angled at B,

$AC^2 = AB^2 + BC^2$

$= 28^2 + 21^2$

AC = 35 cm

Area of shaded region = area of ΔABC + area of semi-circle with diameter AC – area of quadrant with radius BC

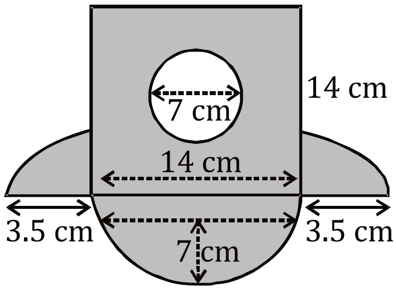
$= \frac{1}{2}(21 \times 28) + \frac{1}{2} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} - \frac{1}{4} \times \frac{22}{7} \times 21 \times 21$

= 294 + 481.25 – 346.5

= 775.25 – 346.5

= 428.75 $cm^2$

**28.** In figure find the area of the shaded region. $\left[\text{Use}\,\pi = \frac{22}{7}\right]$




**Sol.** Area of square = $(14)^2$ cm$^2$ = 196 cm$^2$

Area of internal circle = $\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\ \text{cm}^2 = \frac{77}{2} = 38.5\ \text{cm}^2$

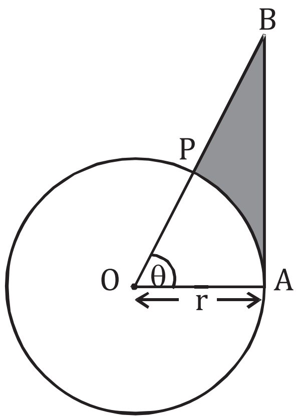
Area of semi-circle with 14 cm diameter $= \frac{1}{2} \times \frac{22}{7} \times 7^2\,\text{cm}^2$ = 77 cm$^2$

Area of two quarter circles of radius $\frac{7}{2}$ cm $= 2 \times \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = \frac{77}{4} = 19.25\,\text{cm}^2$

$\therefore$ Shaded area = 196 – 38.5 + 77 + 19.25 = 292.25 – 38.5 = 253.75 cm$^2$

**Long answer type questions** **(5 marks)**

**29.** In the given figure, a sector OAP of a circle with centre O, containing $\angle\theta$ is shown. AB is perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is $r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180°} - 1\right]$




**Sol.** (i) Length of sector $PA = 2\pi r \frac{\theta}{360°} = \frac{\pi r\theta}{180°}$

In $\Delta OAB,\ \tan\theta = \frac{AB}{OA} \Rightarrow AB = r\tan\theta$

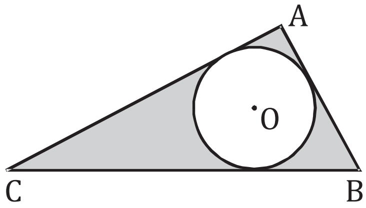
$\text{Now}, \sec\theta = \frac{BO}{r}$ So, BO = r sec θ

Length of BP = OB – OP = r sec θ – r

So, perimeter = AP + AB + BP $= \pi r \dfrac{\theta}{180^\circ} + r\tan\theta + r\sec\theta - r$

$= r\left[\tan\theta + \sec\theta + \dfrac{\pi\theta}{180^\circ} - 1\right]$

**30.** In the given figure, ABC is a right triangle right angled at A. Find the area of shaded region if AB = 6 cm, BC =10 cm and O is the centre of the incircle of ΔABC. (Take π = 3.14)




**Sol.** Given, AB = 6 cm and BC = 10 cm.

By Pythagoras theorem, in ΔABC, we get $AC^2 = BC^2 - AB^2 = (10)^2 - (6)^2 = 64$

⇒ AC = 8 cm

Let the radius of the incircle be r.

Let the circle touch side AB at P, side AC at Q and side BC at R.

Join OP, OQ and OR.

We know that the radius from the centre of the circle is perpendicular to the tangent through the point of contact.

∴ OP ⊥ AB, OQ ⊥ AC and OR ⊥ BC

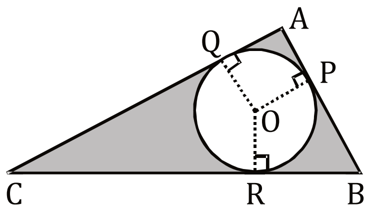
Also, the tangents drawn from an external point to the circle are equal.

∴ AP = AQ, BP = BR, CR = CQ

Now, in quadrilateral OPAQ, AQ = AP and

∠AQO = ∠APO = ∠PAQ = 90°

Therefore, OPAQ is a square.




∴ OP = AQ = AP = OQ = r

∴ PB = 6 – r ⇒ BR = 6 – r

CQ = 8 – r ⇒ CR = 8 – r

Now, BC = BR + CR

⇒ 10 = 6 – r + 8 – r

⇒ 10 = 14 – 2r

⇒ r = 2 cm

Now, area of shaded region

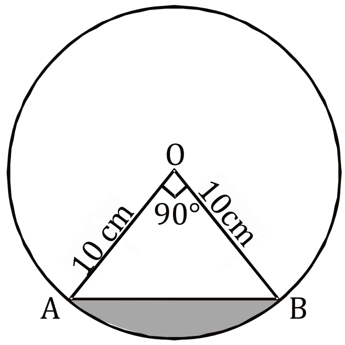
= Area of ΔABC – Area of circle $= \dfrac{1}{2} \times AB \times AC - \pi r^2 = \dfrac{1}{2} \times (8) \times (6) - 3.14(2)^2$

$= 24 - 12.56 = 11.44\ cm^2$

**31.** A chord AB of a circle of radius 10 cm makes a right angle at the centre of the circle. Find the area of the major and minor segments (Take $\pi = 3.14$ )

**Sol.** We know that the area of a minor segment of angle θ° in a circle of radius r is given by

$$A = \left\{\frac{\pi\theta}{360°} - \frac{1}{2}\sin\theta\right\} r^2$$




Here, $r = 10$ and $\theta = 90°$

$$\therefore \quad A = \left\{\frac{3.14 \times 90°}{360°} - \frac{1}{2}\sin 90°\right\}(10)^2 \text{ cm}^2$$

$$\Rightarrow \quad A = \left\{\frac{3.14}{4} - \frac{1}{2}\right\}(10)^2 \text{ cm}^2$$

$\Rightarrow$ $A = \{3.14 \times 25 - 50\}$ cm$^2$ = $(78.5 - 50)$ cm$^2$

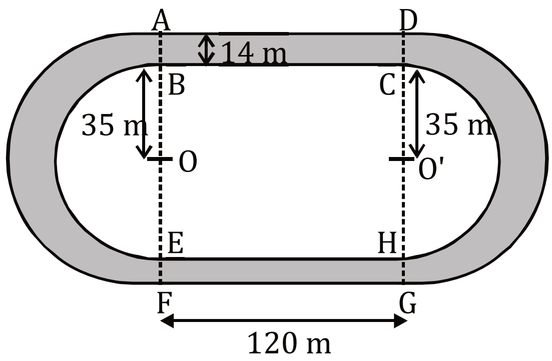
= 28.5 cm$^2$

Area of the major segment = Area of the circle – Area of the minor segment

= $(3.14 \times 10^2 - 28.5)$ cm$^2$ = $(314 - 28.5)$ cm$^2$

= 285.5 cm$^2$

**32.** An athletics track 14 m wide consists of two straight sections 120 m long joining semi-circular ends whose inner radius is 35 m. Calculate the area of the shaded region.




**Sol.** We have,

OB = O'C = 35 m and AB = CD = 14 m

OA = O'D = (35 + 14) m = 49 m

Now,

Area of the shaded region = Area of rectangle ABCD + Area of rectangle EFGH

+ 2 {Area of the semi-circle with radius 49 m} – 2 {Area of the semi-circle with radius 35 m}

$$= (14 \times 120) + (14 \times 120) + 2\left\{\frac{1}{2} \times \frac{22}{7} \times (49)^2\right\} - 2\left\{\frac{1}{2} \times \frac{22}{7} \times (35)^2\right\}$$

$= \{1680 + 1680 + \frac{22}{7}\{(49)^2 - (35)^2\}\ m^2$

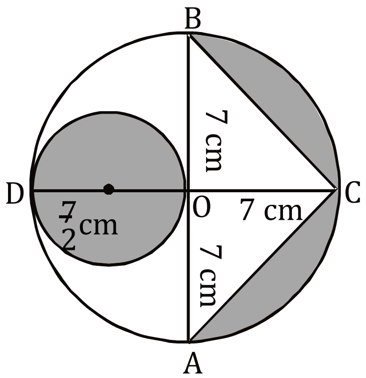
$= \{3360 + \frac{22}{7}(49 + 35)(49 - 35)\}\ m^2$

$= \{3360 + \frac{22}{7} \times 84 \times 14\}\ m^2$

$= \{3360 + 44 \times 84\}\ m^2 = 7056\ m^2$

Hence, the area of the shaded region is $7056\ m^2$

**33.** In figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.

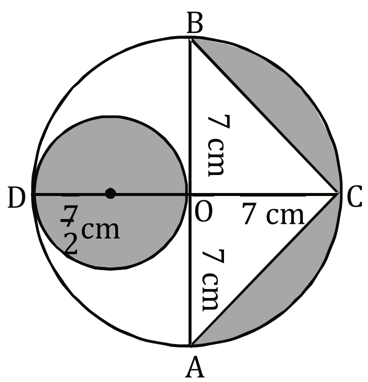



**Sol.** We have,

Area of the shaded region

= (Area of circle with OD (= 7 cm) as diameter)

+ Area of semi-circle with AB as diameter – Area of ΔABC



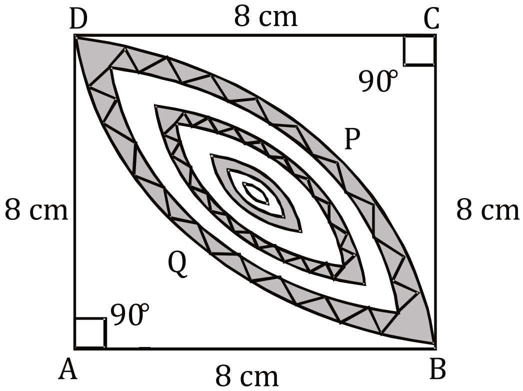

$$= \pi \times \left(\frac{7}{2}\right)^2 + \frac{1}{2} \times \pi \times (7)^2 - \frac{1}{2} \times AB \times OC$$

$$= \left\{\frac{\pi}{4} \times 49 + \frac{\pi}{2} \times 49 - \frac{1}{2} \times 14 \times 7\right\} cm^2$$

$$= \left(\frac{3\pi}{4} \times 49 - 49\right) cm^2 = \left(\frac{3}{4} \times \frac{22}{7} \times 49 - 49\right) cm^2$$

$$= \frac{231 - 98}{2} cm^2 = 66.5\ cm^2$$

**34.** Calculate the area of the designed region in figure common between two quadrants of circles of radius 8 cm each.




**Sol.** We have,

Area of the designed region

= 2 (Area of quadrant ABPD – Area of ΔABD)

$$= 2\left\{\frac{\pi}{4}\times(8)^2 - \frac{1}{2}\times 8\times 8\right\}\text{cm}^2$$

$$= 2\left\{\frac{22}{7}\times\frac{1}{4}\times 64 - 32\right\}\text{cm}^2$$

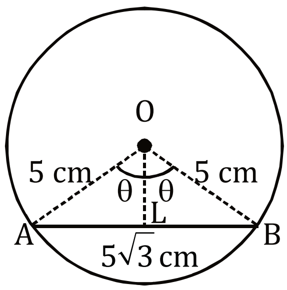
$$= 2\left\{\frac{22\times 16}{7} - 32\right\}\text{cm}^2$$

$$= 2\left\{\frac{352-224}{7}\right\}\text{cm}^2 = \frac{256}{7}\ \text{cm}^2$$

**35.** In a circle with centre O and radius 5 cm, AB is a chord of length. Find the area of sector AOB.

**Sol.** We have,

$$AB = 5\sqrt{3}\,\text{cm} \Rightarrow AL = BL = \frac{5\sqrt{3}}{2}\ \text{cm}$$




Let ∠AOB = 2θ. Then, ∠AOL = ∠BOL = θ

In ΔOLA, we have

$$\sin\theta = \frac{AL}{OA} = \frac{\frac{5\sqrt{3}}{2}}{5} = \frac{\sqrt{3}}{2}$$

$\Rightarrow$ θ = 60°

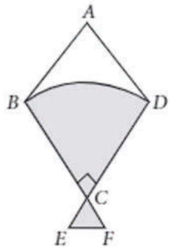
$\Rightarrow$ ∠AOB = 120°

$\Rightarrow$ Area of sector AOB = $\frac{120°}{360°}\times\pi\times 5^2\,\text{cm}^2 = \frac{25\pi}{3}\text{cm}^2$

## Case Study type questions (4 marks)

**1.** Kite Flying

Makar Sankranti is a fun and delightful occasion. Like many other festivals, the kite flying competition also has a historical and cultural significance attached to it. The following figure shows a kite in which BCD is the shape of quadrant of a circle of radius 42 cm, ABCD is a square and CEF is an isosceles right-angled triangle, whose equal sides are 7 cm long.


(i) Find the area of square.

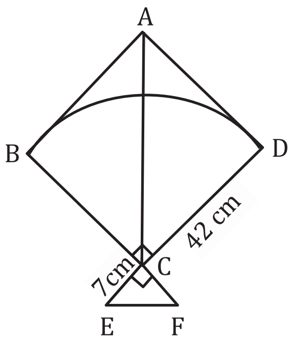
**[OR]**

(i) Find the area of $\Delta CEF$.

(ii) Find the area of shaded portion.

(iii) Find the area of unshaded portion.

**Sol.**




(i) Area of square = $(\text{side})^2 = 42 \times 42 = 1764 \text{ cm}^2$

**[OR]**

(i) CEF is an isosceles right-angled triangle with base & height 7 cm

$$Ar(\Delta CEF) = \frac{1}{2} \times CE \times CF = \frac{1}{2} \times 7 \times 7 = \frac{49}{2} = 24.5 \text{ cm}^2$$

(ii) $Ar(\text{Quadrant } BCD) = \frac{\theta}{360^\circ}\pi(42)^2$

$\Rightarrow \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times 42 \times 42$

$\Rightarrow 22 \times 3 \times 21$

$\Rightarrow 1386 \text{ cm}^2$

Ar(shaded portion) = Ar(Quadrant) + Ar($\Delta$CEF)

= 1386 + 24.5

= $1410.5 \text{ cm}^2$

(iii) Ar(unshaded portion) = Ar(Square) – Ar(Quadrant)

= 1764 – 1386

= $378 \text{ cm}^2$

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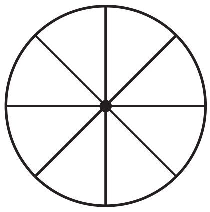
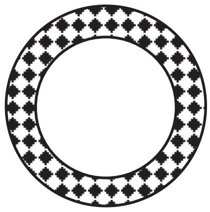
**2.** A brooch is a small piece of jewellery which has a pin at the back so it can be fastened on a dress, blouse or coat. Designs of some brooches are shown below. Observe them carefully.


A


B

**Design A :** Brooch A is made with silver wire in the form of a circle with diameter 28 mm. A wire is used for making 4 diameters which divide the circle into 8 equal parts.

**Design B :** Brooch B is made of two colours, gold and silver. Outer part is made with gold. The circumference of silver part is 44 mm and the golden part is 3 mm wide everywhere.

Refer to design A

(i) Determine the total length of silver wire required for design A.

(ii) Calculate the area of each sector of the brooch.

Refer to design A.

(iii) Calculate the difference of area of golden and silver part in terms of $\pi$.

**[OR]**

(iii) A boy is playing with brooch B. He makes revolution with it along its edge. how many complete revolutions must it take to cover 80 $\pi$ mm?

**Sol.** (i) Total Silver wire required = Circumference + 4 (diameter)

$= 2\pi r + 4d = 2 \times \frac{22}{7} \times 14 + 4 \times 28 \Rightarrow 88 + 112 = 200$ mm

(ii) $\theta = \frac{360°}{8} = 45°$

r = 14 mm

Area of sector $= \frac{\theta}{360°}\pi r^2 = \frac{45°}{360°} \times \frac{22}{7} \times 14 \times 14 = 11 \times 7 = 77 \text{ mm}^2$

(iii) Circumference of Silver part = 44 mm

$2\pi r_{silver} = 44$

$2 \times \frac{22}{7} \times r_{silver} = 44$

$r_{silver} = 7$ mm

Area of silver part $= \pi(7)^2 = 49\pi \text{ mm}^2$

Outer radius = 7 + 3 = 10 mm.

Area of golden part $= \pi(10)^2 = 100\pi \text{ mm}^2$

Difference in areas $= 100\pi - 49\pi = 51\pi \text{ mm}^2$

**[OR]**

(iii) Distance covered in 1 revolution = Circumference $= 2\pi(10) = 20\pi$ mm

Distance to be covered $= 80\pi$

Number of revolutions $= \frac{80\pi}{20\pi} = 4$

# 12 Surface Areas and Volumes

**Multiple choice questions** **(1 marks)**

**1.** The shape of an ice-cream cone is a combination of:

(1) Sphere + cylinder (2) Sphere + cone

(3) Hemisphere + cylinder (4) Hemisphere + cone

**Sol. Option (4)**

The shape of an ice-cream cone is a combination of a hemisphere and a cone.

**2.** If a cone is cut parallel to the base of it by a plane in two parts, then the shape of the top of the cone will be a:

(1) Sphere (2) Cube (3) Cone itself (4) Cylinder

**Sol. Option (3)**

If we cut a cone into two parts parallel to the base, then the shape of the upper part remains the same.

**3.** If r is the radius of the sphere, then the surface area of the sphere is given by.

(1) $4\pi r^2$ (2) $2\pi r^2$ (3) $\pi r^2$ (4) $\frac{4}{3}\pi r^2$

**Sol. Option (1)**

If r is the radius of the sphere, then the surface area of the sphere is given by $4\pi r^2$.

**4.** If we change the shape of an object from a sphere to a cylinder, then the volume of cylinder will

(1) Increase (2) Decrease

(3) Remains unchanged (4) Doubles

**Sol. Option (3)**

If we change the shape of a three-dimensional object, the volume of the new shape will be same.

**5.** Fifteen solid spheres are made by melting a solid metallic cone of base diameter 2cm and height 15cm. The radius of each sphere is:

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{1}{\sqrt[3]{2}}$ (4) $\frac{1}{\sqrt[3]{4}}$

**Sol. Option (4)**

Volume of 15 spheres = Volume of a cone

$$15 \times \left(\frac{4}{3}\right)\pi r^3 = \frac{1}{3}\pi r^2 h$$

$$5 \times 4\pi r^3 = \frac{1}{3}\pi\, 1^2(15)$$

$$20r^3 = 5$$

$$r^3 = \frac{5}{20} = \frac{1}{4}$$

$$r = \frac{1}{\sqrt[3]{4}}\ \text{cm}$$

**6.** The radius of the top and bottom of a bucket of slant height 35 cm are 25 cm and 8 cm. The curved surface of the bucket is:

(1) 4000 sq.cm (2) 3500 sq.cm (3) 3630 sq.cm (4) 3750 sq.cm

**Sol. Option (3)**

Curved surface of bucket = $\pi(R_1 + R_2) \times$ slant height ($\ell$)

$$\text{Curved Surface} = \left(\frac{22}{7}\right) \times (25 + 8) \times 35$$

CSA = 22 × 33 × 5 = 3630 sq.cm.

**7.** If a cylinder is covered by two hemispheres shaped lid of equal shape, then the total curved surface area of the new object will be

(1) $4\pi rh + 2\pi r^2$ (2) $4\pi rh - 2\pi r^2$ (3) $2\pi rh + 4\pi r^2$ (4) $2\pi rh + 4\pi r$

**Sol. Option (3)**

Curved surface area of cylinder = $2\pi rh$

The curved surface area of hemisphere = $2\pi r^2$

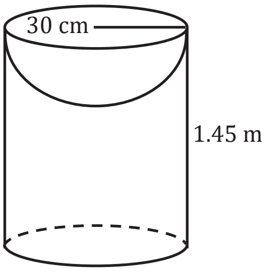
Here, we have two hemispheres.

So, total curved surface area = $2\pi rh + 2(2\pi r^2) = 2\pi rh + 4\pi r^2$

**8.** A tank is made of the shape of a cylinder with a hemispherical depression at one end and open at other end. The height of the cylinder is 1.45 m and radius is 30 cm. The total surface area of the tank is:

(1) 30 m (2) 3.3 m (3) 30.3 m (4) 3300 m

**Sol.** **Option (2)**




Total surface area of tank = CSA of cylinder + CSA of hemisphere

$= 2\pi rh + 2\pi r^2 = 2\pi\, r(h + r)$

$= 2 \times \dfrac{22}{7} \times 30(145 + 30)\ cm^2$

$= 33000\ cm^2$

$= 3.3\ m^2$

**Assertion Reason questions** **(1 marks)**

**9.** **Assertion:** If we join two hemispheres of same radius along their bases, then we get a sphere.

**Reason:** Ratio of C.S.A of hemisphere to sphere is 1 : 2.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol.** **Option (2)**

Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

**10.** **Assertion (A)** : The volume and surface Area of a sphere are related to each other by radius.

**Reason (R) :** Relation between Surface Area S and Volume V is $S^3 = 36\pi V^2$.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol.** **Option (1)**

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

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**Very short answer type questions** **(2 mark)**

**11.** A solid metallic cuboid of dimensions 9m, 8m, 2m is melted and recast into solid cubes of edge 2m. Find the number of cubes so formed.

**Sol.** Number of cubes = $\dfrac{\text{Volume of cuboid}}{\text{Volume of one cube}} = \dfrac{9\times8\times2}{2\times2\times2}$

Number of cubes = 18

**12.** Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere?

**Sol.** Let r be radius of hemisphere then

According to question

$\dfrac{2}{3}\pi r^3 = 3\pi r^2$; $r = \dfrac{3\times3}{2} = \dfrac{9}{2}$ units, diameter = 9 units

**13.** A cylinder, a cone and a hemisphere have same base and same height. Find the ratio of their volumes.

**Sol.** Let base radius and height of each be 'r' and 'h' respectively.

Volume of cylinder : Volume of cone : Volume of hemisphere.

$= \pi r^2 h : \dfrac{1}{3}\pi r^2 h : \dfrac{2}{3}\pi r^3$

$= \pi r^3 : \dfrac{1}{3}\pi r^3 : \dfrac{2}{3}\pi r^3 \qquad (\because h = r)$

$= 1 : \dfrac{1}{3} : \dfrac{2}{3}$

= 3 : 1 : 2

**14.** The curved surface area of a cylinder is 264 $m^2$ and its volume is 924 $m^3$. Find the ratio of its height to its diameter.

**Sol.** Let r and h be radius and height of cylinder respectively.

Curved surface of cylinder = $2\pi rh$

Volume of cylinder = $\pi r^2 h$

$\dfrac{\pi r^2 h}{2\pi rh} = \dfrac{924}{264} \Rightarrow \dfrac{r}{2} = \dfrac{7}{2}$

r = 7 m

$\therefore \quad 2\pi rh = 264$

$\Rightarrow \quad 2\times\dfrac{22}{7}\times7\times h = 264$

$\Rightarrow \quad h = 6$ m

$\therefore \quad \dfrac{h}{2r} = \dfrac{6}{14} = \dfrac{3}{7}$

$\therefore \quad h : d = 3 : 7$

**Short answer type questions** **(3 marks)**

**15.** The radius and height of a solid right circular cone are in the ratio of 5 : 12. If its volume is 314 $cm^3$, find its total surface area. [Take $\pi = 3{\cdot}14$]

**Sol.** Let r and h be radius and height of cone respectively.

Given r : h = 5 : 12

Let r = 5x and h = 12x

Volume of cone = 314 $cm^3$

i.e., $\left(\frac{1}{3}\right)\pi r^2 h = 314$

$\left(\frac{1}{3}\right) \times (3.14) \times (5x)^2 \times (12x) = 314$

$(25x^2)(12x) = 100 \times 3$

$x^3 = 1$

Hence x = 1

Therefore, r = 5 cm and h = 12 cm

Slant height, $\ell^2 = r^2 + h^2$

$= 5^2 + 12^2 \Rightarrow \ell^2 = 169$

Therefore, slant height = 13 cm

Total surface area = $\pi r\ell + \pi r^2$

$= \pi(5)(13) + \pi(5)^2$

$= 282.6\ cm^2$

**16.** A wire of diameter 3 mm is wound about a cylinder whose height is 12 cm and radius 5 cm so as to cover the curved surface of the cylinder completely. Find the length of the wire.

**Sol.** Let r and h be radius and height of cylinder respectively.

Diameter of the wire = 3 mm = 0.3 cm

Height covered by the wire in one round of wounding = 0.3 cm

Number of rounds

= Height of the cylinder / Diameter of the wire

$= \frac{12}{0.3} = 40$

Length of the wire in one round = Circumference of the base of cylinder

$= 2\pi r = 2 \times \frac{22}{7} \times 5 = \frac{220}{7}$

Length of the wire in 40 rounds

$= \left(\frac{220}{7}\right)40 = \frac{8800}{7}$ cm = 12.57 m

**17.** A solid sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel?

**Sol.** Let R be radius of sphere.

Given, the diameter of sphere = 6 cm

Radius of sphere = R = $\frac{6}{2}$ = 3 cm

Let r and h be radius and height of cylinder respectively.

And diameter of cylindrical vessel = 12 cm.

Radius of cylindrical vessel = r = $\frac{12}{2}$ = 6 cm.

Now, volume of sphere = $\frac{4}{3}\pi R^3 = \left(\frac{4}{3}\right)\pi(3)^3$ and volume of cylindrical vessel = $\pi r^2 h = \pi(6)^2 h$

Here, the volume of sphere is equal to the volume increased in the cylindrical vessel.

$$\frac{4}{3}\pi(3)^3 = \pi(6)^2 h$$

$$h = \frac{(4 \times 3 \times 3 \times 3)}{(3 \times 6 \times 6)} = 1 \text{ cm.}$$

Therefore, 1 cm level of water rise in the cylindrical vessel.

**18.** A conical vessel of internal radius 12 cm and height 50 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cm. Find the height of water in cylindrical vessel.

**Sol.** Let R and H be radius and height of cone respectively.

Volume of water in conical vessel $= \frac{1}{3}\pi R^2 H$

$$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 12 \times 12 \times 50 \text{ cm}^3$$

$$= \left(\frac{22 \times 4 \times 12 \times 50}{7}\right) \text{cm}^3$$

Let the height of the liquid in the vessel be h.

Volume of the liquid in the cylindrical vessel = Volume of the conical vessel

Then, $\left(\frac{22}{7}\right) \times 10 \times 10 \times h = \left(\frac{22 \times 4 \times 12 \times 50}{7}\right)$ or $\Rightarrow h = \left(\frac{4 \times 12 \times 50}{100}\right) = 24$ cm

**19.** A metallic solid sphere of radius 10.5 cm is melted and recasted into smaller solid cones, each of radius 3.5 cm and height 3 cm. How many cones will be made?

**Sol.** Let R be radius of sphere.

Volume of sphere = $\frac{4}{3}\pi R^3$

$= \frac{4}{3} \times \frac{22}{7} \times 10.5 \times 10.5 \times 10.5$

Volume of metallic sphere = 4851 $cm^3$

Now,

Let r and h be radius and height of cone respectively.

Volume of one cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3$

Volume of one cone = 38.5 $cm^3$

Total number of cones

= Volume of metallic sphere/volume of one cone

$= \frac{4851}{38.5}$ = 126 cones

Total number of cones can be formed is 126.

**20.** A 5 m wide cloth is used to make a conical tent of base diameter 14 m and height 24 m. Find the cost of cloth used at the rate of Rs. 25 per metre.

**Sol.** Let r a nd h be radius and height of cone respectively.

Given, radius and height = 7 m and 24 m.

Slant height($\ell$) = $\sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2}$

$= \sqrt{625} = 25\text{m}$

C.S.A. = $\pi r \ell$

$= \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$

Let x m of cloth is required C.S.A. = area of cloth.

$\Rightarrow \quad 5x = 550 \Rightarrow x = \frac{550}{5} = 110 \text{ m}$

$\therefore$ Cost of cloth = 110 × 25 = Rs. 2750

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**21.** A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of each bottle, if 10% liquid is wasted in this transfer.

**Sol.** Let r be radius of sphere.

Volume of bowl = $\frac{2}{3}\pi r^3$

Volume of liquid in bowl $= \frac{2}{3}\pi \times (18)^3 \text{cm}^3$

Volume of liquid after wastage $= \frac{2}{3}\pi \times (18)^3 \times \frac{90}{100} \text{ cm}^3$

Let R and H be radius and height of cylinder respectively.

Volume of bottle = $\pi R^2 H$

Volume of liquid in 72 bottles = $\pi \times (3)^2 \times H \times 72 \text{ cm}^3$

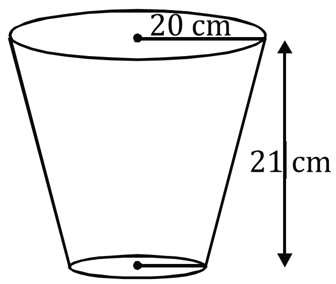
Volume of liquid in bottles = volume after wastage

$\pi \times (3)^2 \times H \times 72 = \frac{2}{3}\pi \times (18)^3 \times \frac{90}{100}$

$$\Rightarrow \quad H = \frac{\frac{2}{3}\pi \times (18)^3 \times \frac{90}{100}}{\pi \times (3)^2 \times 72}$$

= 5.4 cm

**22.** A metal container, open from the top, is in the shape of a frustum of cone of height 21 cm with radii of its lower and upper circular ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at the rate of Rs.35 per litre. $\left(\text{Use } \pi = \frac{22}{7}\right)$




**Sol.** If $r_1$ and $r_2$ be the radii of two circular ends and h be the height of frustum, then volume

$= \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$

Given that : $r_1 = 8$ cm; $r_2 = 20$ cm and h = 21 cm

$\therefore$ Volume = $\frac{1}{3}\times\frac{22}{7}\times 21[(8)^2+(20)^2+8\times 20]$

= 22[64 + 400 + 160]

= 22 × 624

= 13728 $cm^3$

V = 13.728 litre

$\therefore$ Total cost = Rs. 13.728 × 35

= Rs. 480.48

**23.** In given figure, is a decorative block, made up of two solids a cube and a hemisphere. The base of the block is a cube of side 6 cm and the hemisphere fixed on the top has a diameter of 3.5 cm. Find the total surface area of the block. $\left(\text{Use}\pi=\frac{22}{7}\right)$.

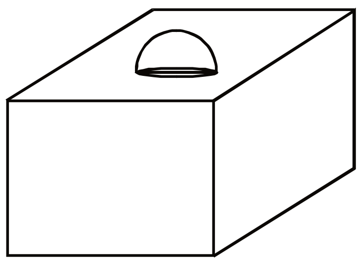
**Sol.** Area of six faces of cube = $6a^2$

= $6(6)^2$

= 216 $cm^2$

Area of circular part covered by hemisphere = $\pi r^2$

$=\frac{22}{7}\times\left(\frac{3.5}{2}\right)^2$ = 9.625 $cm^2$



Area of curved surface of hemisphere = $2\pi r^2$

$=2\times\frac{22}{7}\times\left(\frac{3.5}{2}\right)^2$ = 19.25

Required area = 216 – 9.625 + 19.25 $cm^2$

= 225.625 $cm^2$

**24.** A right triangle whose sides are 15 cm and 20 cm is made to revolve about its hypotenuse. Find the volume and the surface area of the double cone so formed. (Use $\pi$ = 3.14)

**Sol.** (i) $AC^2 = 20^2 + 15^2 = 625$

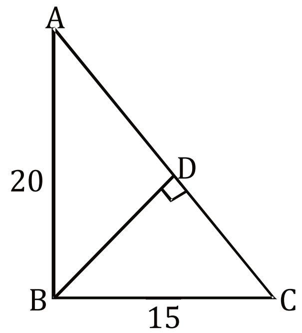
$\Rightarrow$ AC = 25 cm

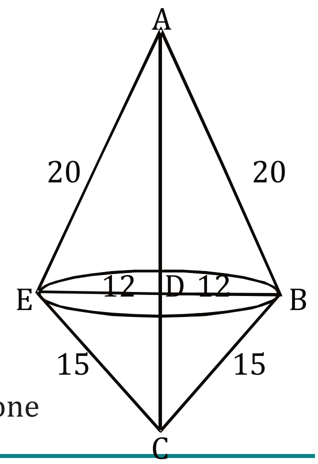
(ii) ar($\Delta$ABC) = ar($\Delta$ABC)

$\frac{1}{2}$ × BC × AB = $\frac{1}{2}$ × AC × BD

$\Rightarrow$ 15 × 20 = 25 × BD

$\Rightarrow$ BD = 12 cm




Volume of double cone = Volume of upper cone + Volume of lower cone

$$=\frac{1}{3}\pi(BD)^2\times AD+\frac{1}{3}\pi(BD)^2(DC)$$

$$=\frac{1}{3}\times 3.14\times 144\times 25=3768\ cm^3$$

Surface area = C.S.A. of upper cone + C.S.A of lower cone

= $\pi(12)(20) + \pi(12)(15) = 12\pi\{20 + 15\}$

= 12 × 3.14 × 35

= 1318.8 $cm^2$

**25.** A hemispherical tank of diameter 3 m is full of water. It is being emptied by a pipe at the rate of $3\frac{4}{7}$ litre per second. How much time will it take to make the tank half empty? $\left[\text{Use}\ \pi=\frac{22}{7}\right]$

**Sol.** Diameter of tank = 3m

$\therefore$ Radius = $r=\frac{3}{2}$ m

Volume of hemispherical tank = $\frac{2}{3}\pi r^3$

$$\Rightarrow V=\frac{2}{3}\pi\left(\frac{3}{2}\right)^3 m^3$$

$$=\frac{2}{3}\times\frac{22}{7}\times\frac{27}{8}\ m^3$$

$$=\frac{11}{7}\times\frac{9}{2}=\frac{99}{14}\ m^3$$

$V=\frac{99}{14}\times 1000$ litre [1 $m^3$ = 1000 litre]

$\therefore$ Half the volume of hemisphere i.e., $\frac{V}{2}=\frac{1}{2}\times\frac{99}{14}\times 1000$ litres

Let the time taken for this volume to flow out be 't' sec.

Then according to question,

$$t\times 3\frac{4}{7}=\frac{1}{2}\times\frac{99}{14}\times 1000$$

$$t\times\frac{25}{7}=\frac{1}{2}\times\frac{99}{14}\times 1000$$

$$\Rightarrow t=\frac{7}{25}\times\frac{1}{2}\times\frac{99}{14}\times 1000$$

= 990 sec = 16 minutes 30 sec.

## Long answer type questions (5 marks)

**26.** Water in a canal, 6 m wide and 1·5 m deep, is flowing with a speed of 4 km/hour. How much area can it irrigate in 10 minutes, if 8 cm of standing water is required for irrigation?

**Sol.** Width of the canal = 6 m, Depth of the canal = 1.5 m

It is given that water is flowing at a speed of $\frac{4\text{km}}{\text{hr}} = \frac{4000\text{m}}{\text{hr}}$

Thus, length of water column formed in 10 minutes = $\left(\frac{1}{6}\right)$ hour = $\left(\frac{1}{6}\right) \times 4000 = \frac{4000}{6}$ m

Hence, Volume of the water flowing in $\left(\frac{1}{6}\right)$ hour = Volume of the cuboid of length $\frac{4000}{6}$ m width 6 m and depth 1.5 m

Volume of the water flowing in $\left(\frac{1}{6}\right)$ hour = $\frac{4000}{6} \times 6 \times 1.5$

= 6000 $m^3$

Suppose x $m^2$ area is irrigated in $\left(\frac{1}{6}\right)$ hour, Then $x \times \frac{8}{100} = 6000$

$\Rightarrow x = \frac{600000}{8} m^2 = 75000 m^2$

**27.** The slant height of a frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

**Sol.** Given slant height($\ell$) = 4 cm

Circumference of the end ($C_1$) = 18 cm

$2\pi r_1 = 18$ cm

$2 \times \frac{22}{7} \times r_1 = 18$ cm

Therefore, $r_1 = \frac{63}{22}$ cm

Similarly, $C_2 = 6$ cm

Circumference of other end $\Rightarrow 2\pi r_2 = 6$

$= 2\left(\frac{22}{7}\right) r_2 = 6$

So, $r_2 = \frac{21}{22}$ cm

Now, CSA of frustum of cone = $\pi \ell (r_1 + r_2)$

$= \frac{22}{7} \times 4\left(\frac{63}{22} + \frac{21}{22}\right)$

$= \frac{84}{22} \times 4 \times \frac{22}{7}$

= 48 sq. cm

**28.** The dimensions of a solid iron cuboid are 4.4 m, 2.6 m and 1.0 m. It is melted and recast into a hollow cylindrical pipe of 30 cm inner radius and thickness 5 cm. Find the length of the pipe.

**Sol.** Internal radius of hollow cylinder, r = 30 cm

Thickness of hollow cylinder, w = 5 cm

External radius of hollow cylinder, R = 30 + 5 = 35 cm

Here, volume of cuboid = volume of hollow cylinder

$440 \times 260 \times 100 = \pi(R^2 - r^2)\, h$

$440 \times 260 \times 100 = \pi(35^2 - 30^2)\, h$

$440 \times 260 \times 100 = \pi(325)\, h$

$$\frac{440 \times 260 \times 100 \times 7}{325 \times 22} = h$$

$\Rightarrow$ h = 11200 cm

$\Rightarrow$ h = 112 m

Thus, length of pipe is 112 m.

**29.** A sphere of diameter 12cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by 32/9 cm. Find the diameter of cylindrical vessel.

**Sol.** Diameter of the sphere = 12 cm

Radius of the sphere = 6 cm

R, r be the radii of the cylinder and sphere respectively.

Let height to which water level rise, $H = \frac{32}{9}$ cm

Volume of water raised in the cylindrical vessel = Volume of sphere

$$\pi R^2 H = \left(\frac{4}{3}\right)\pi r^3$$

$$\pi R^2\left(\frac{32}{9}\right) = \left(\frac{4}{3}\right)\pi 6^3$$

$$R^2 = \left(\frac{4}{3}\right) \times \left(\frac{9}{32}\right) \times 6^3$$

$R^2 = 81$

$\Rightarrow$ R = 9 cm

Diameter of cylindrical vessel = 2R =18 cm.

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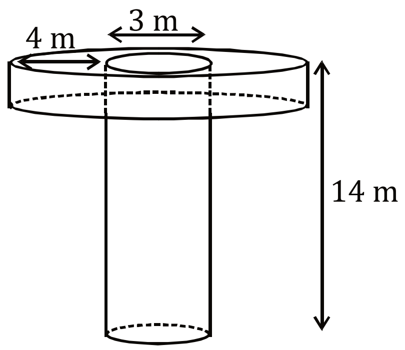
**30.** A well of diameter 3 m is dug 14 m deep. The soil taken out of it is spread evenly all around it to a width of 4 m to form an embankment. Find the height of the embankment.

**Sol.** Height of the well = 14 m

Diameter of the well = 3 m




So, Radius of the well = $\frac{3}{2}$ m

Volume of the earth taken out of the well = $\pi r^2 h$

$= \frac{22}{7} \times \left(\frac{3}{2}\right)^2 \times 14$

= 99 m

Outer radius of the embankment

$R = \left(\frac{3}{2} + 4\right) m = \frac{11}{2}$ m

Area of embankment = outer area – inner area

$= \pi R^2 - \pi r^2$

$= \frac{22}{7} \times \left[\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right]$

$= \frac{22}{7} \times \left[\left(\frac{121}{4}\right) - \left(\frac{9}{4}\right)\right]$

$= \frac{22}{7} \times \frac{112}{4}$ = 88 m$^2$

Height of the embankment = $\frac{\text{Volume}}{\text{Area}} = \frac{99}{88}$ m

Height of the embankment = 1.125 m

**31.** A hollow cone is cut by a plane parallel to the base at some height and the upper portion is removed. If the curved surface area of the remainder is $\frac{8}{9}$ of the curved surface of the whole cone, find the ratio of the two parts into which the cone's altitude is divided.

**Sol.** Let for the whole cone,

radius = R

height = H

slant height = L

for the removed portion of the cone, (which is also a cone) radius = r

height = h

slant height = $\ell$

Since they are from same cone, $\frac{r}{R} = \frac{\ell}{L} = \frac{h}{H}$ ...(1)

CSA of remainder = $\frac{8}{9}$ of C.S.A of whole cone

So, CSA of removed portion $= 1 - \frac{8}{9} = \frac{1}{9}$ of C.S.A of whole cone

CSA of original cone = $\pi RL$

CSA of removed cone = $\pi r\ell$

$$\Rightarrow \pi r\ell = \frac{1}{9}\pi RL$$

$$\Rightarrow \frac{r\ell}{RL} = \frac{1}{9} \Rightarrow \left(\frac{r}{R}\right)^2 = \frac{1}{9} \quad \text{[By equation (1)]}$$

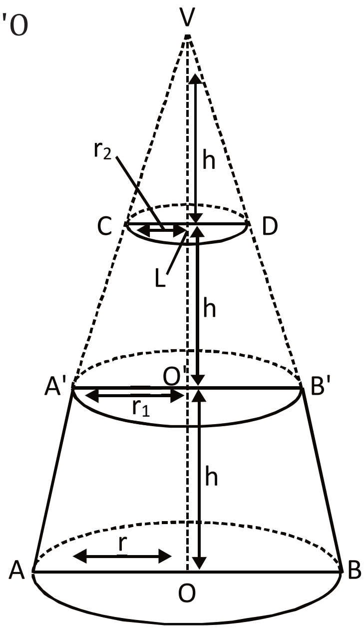
$$\frac{r}{R} = \frac{1}{3}$$

$$\frac{h}{H} = \frac{1}{3}$$

So, the cone's altitude is divided in the ratio 1 : 3.

**32.** A right circular cone is divided into three parts by trisecting its height by two planes drawn parallel to the base. Show that the volumes of the three portions starting from the top are in the ratio 1 : 7 : 19.

**Sol.** Let VAB be a right circular cone of height 3h and base radius r. This cone is cut by planes parallel to its base at points O' and L such that VL = LO' = h = O'O




Since triangles VOA and VO'A' are similar.

$$\therefore \frac{VO}{VO'} = \frac{OA}{O'A'} \Rightarrow \frac{r}{r_1} = \frac{3h}{2h} \Rightarrow r_1 = \frac{2r}{3}$$

Also, $\Delta VOA \sim \Delta VLC$

$$\therefore \frac{VO}{VL} = \frac{OA}{LC} \Rightarrow \frac{3h}{h} = \frac{r}{r_2} \Rightarrow r_2 = \frac{r}{3}$$

Let $V_1$ be the volume of cone VCD. Then,

$$V_1 = \frac{1}{3}\pi r_2^2 h = \frac{1}{3}\pi\left(\frac{r}{3}\right)^2 h = \frac{1}{27}\pi r^2 h$$

Let $V_2$ be the volume of the frustum A'B'DC. Then,

$$V_2 = \frac{1}{3}\pi(r_1^2 + r_2^2 + r_1 r_2)h$$

$$\Rightarrow \quad V_2 = \frac{1}{3}\pi\left(\frac{4r^2}{9}+\frac{r^2}{9}+\frac{2r^2}{9}\right)h \left[\therefore r_1 = \frac{2r}{3} \text{ and } r_2 = \frac{r}{3}\right]$$

$$\Rightarrow \quad V_3 = \frac{1}{3}\pi(r^2 + r_1^2 + r_1 r)h$$

Let $V_3$ be the volume of the frustum ABB'A'. Then,

$$\Rightarrow \quad V_3 = \frac{1}{3}\pi\left(r^2 + \frac{4r^2}{9} + \frac{2r^2}{3}\right)h$$

$$\Rightarrow \quad V_3 = \frac{19\pi}{27}r^2h$$

$\therefore$ Required ratio = $V_1 : V_2 : V_3$

$$= \frac{1}{27}\pi r^2h : \frac{7}{27}\pi r^2h : \frac{19\pi}{27}r^2h = 1:7:19$$

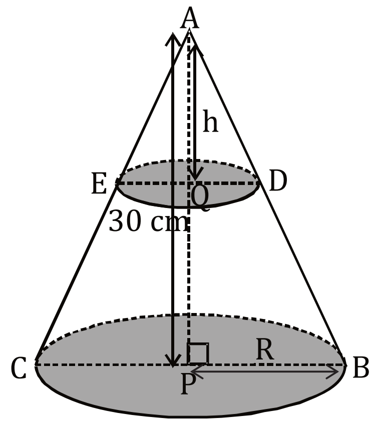
**33.** The height of a cone is 30 cm. From its topside a small cone is cut by a plane parallel to its base. If volume of smaller cone is 1/27 of the given cone, then at what height it is cut from its base?

**Sol.** Assume that big cone of radius R and H.

Assume that small cone of height h is cut off from the top of this cone whose base is parallel to the big cone.

And the radius of the cone that cut off from the original cone be r.

Given : H = 30 cm.




In $\Delta APC$ and $\Delta AQE$

$PC \parallel QE$

$\Delta APC \sim \Delta AQE$

$$\frac{AP}{AQ} = \frac{PC}{QE}$$

$$\frac{h}{H} = \frac{r}{R} \qquad \text{....... (1)}$$

Given that Volume of cone ADE

$$= \left(\frac{1}{27}\right) \text{Volume of cone ABC}$$

$$\frac{\text{Volume of cone ADE}}{\text{Volume of cone ABC}} = \frac{1}{27}$$

$$\Rightarrow \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi R^2 H} = \frac{1}{27}$$

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$$\left(\frac{r}{R}\right)^2 \times \left(\frac{h}{H}\right) = \frac{1}{27}$$

$$\left(\frac{h}{H}\right)^2 \times \left(\frac{h}{H}\right) = \left(\frac{1}{3}\right)^3 \quad \text{[ from (1)]}$$

$$\left(\frac{h}{H}\right)^3 = \left(\frac{1}{3}\right)^3$$

$$\left(\frac{h}{H}\right) = \left(\frac{1}{3}\right)$$

$$h = \left(\frac{1}{3}\right) \times H$$

$$h = \left(\frac{1}{3}\right) \times 30 = 10\text{cm}$$

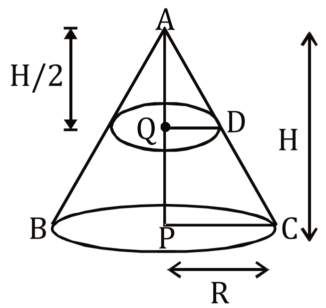
From the figure QP = H – h = 30 – 10 = 20 cm.

Hence, the section of the cone is made at a height of 20 cm above the base.

**34.** A cone of radius 10 cm is divided into two parts by a plane parallel to its base through the mid-point of its height. Compare the volumes of the two parts.

**Sol.** As the cone is divided into two equal parts by the axis so, $AQ = \frac{AP}{2}$ with the help of similarity

theory, $\frac{QD}{PC} = \frac{AQ}{AP}$




So, $\frac{QD}{PC} = \frac{1}{2}$

So, radius $QD = \frac{PC}{2} = \frac{R}{2}$

Now, the volume of frustum

$$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi\left(\frac{R}{2}\right)^2\left(\frac{H}{2}\right)$$

$$= \frac{1}{3}\pi R^2 H \times \frac{7}{8}$$

Compare the two parts in cone, 1st is the volume of small cone and 2nd the volume of frustum i.e.

$$\frac{\text{Volume of small cone}}{\text{Volume of frustum}} = \frac{\left(\frac{1}{3}\right)\pi\left(\frac{R}{2}\right)^2\left(\frac{H}{2}\right)}{\left(\frac{1}{3}\right)\pi R^2 H \times \frac{7}{8}} = \frac{\left(\frac{1}{8}\right)}{\left(\frac{7}{8}\right)} = \frac{1}{7}$$

**35.** In a rain-water harvesting system, the rain-water from a roof of 22 m × 20 m drains into a cylindrical tank having diameter of base 2 m and height 3.5 m. If the tank is full, find the rainfall in cm.

**Sol.** Volume of roof = $\ell \times b \times h = 22 \times 20 \times h$

Height of roof = Height of rainfall

Volume of cylinder = $\frac{22}{7} \times 1 \times 1 \times \frac{35}{10}$

and volume of water on roof

= volume of water in cylindrical tank

$22 \times 20 \times h = \frac{22}{7} \times 1 \times 1 \times \frac{35}{10}$

$22 \times 20 \times h = 11$

$h = \frac{11}{440}$

$h = \frac{1}{40}$ m

h = 0.025 m = 2.5 cm

So, the rainfall = 2.5 cm

**36.** Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make tents cost Rs.120 per sq m, find the amount shared by each school to set up the tents. (Use = 22/7)

**Sol.** CSA of Cylinder = $2 \times \frac{22}{7} \times 2.8 \times 3.5 = 61.6$ sq. m

Slant Height of Cone = $\sqrt{(2.8)^2 + (2.1)^2} = \sqrt{12.25} = 3.5$ m

CSA of conical part = $\pi r \ell = \frac{22}{7} \times 2.8 \times 3.5 = 30.8$ sq. m

Total Area of one Tent = 61.6 + 30.8 = 92.4 sq. m

Rate of Canvas = Rs. 120 per sq. metre

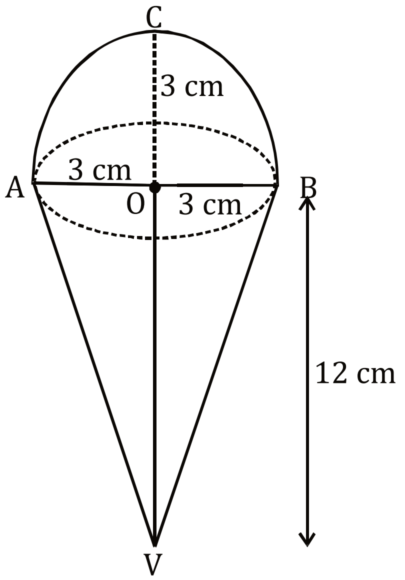
Number of Tents = 1500

Number of School to share the cost = 50

Hence, the amount contributed by each school = $\left[\frac{1500}{50}\right] \times [120 \times 92.4]$ = Rs.332640

**37.** A right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. The ice-cream is to be filled in cones of height 12 cm and diameter 6 cm having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream.

**Sol.**




We have,

Radius of the cylinder = 6 cm

Height of the cylinder = 15 cm

$\therefore$ Volume of the cylinder = $\pi r^2 h$

$\Rightarrow$ Volume of the cylinder = $\pi \times 6^2 \times 15$ cm$^3$

$\Rightarrow$ Volume of the cylinder = $540\ \pi$ cm$^3$

Radius of the ice-cream cone = 3 cm

Height of the ice-cream cone = 12 cm

Volume of the conical part of ice-cream cone = $\frac{1}{3}\pi r^2 h$

$\Rightarrow$ Volume of the conical part of ice-cream cone = $\frac{1}{3}\pi \times 3^2 \times 12 \text{cm}^3$

$\Rightarrow$ Volume of the conical part of ice-cream cone = $36\ \pi$ cm$^3$

Volume of the hemispherical top of the ice-cream cone = $\frac{2}{3}\pi r^3 = \frac{2}{3} \times \pi \times 3^3 = 18\ \pi \text{ cm}^3$

Total volume of the ice-cream cone = $(36\pi + 18\pi)$ cm$^3$ = $54\pi$ cm$^3$

$\therefore$ Number of ice-cream cones $= \dfrac{\text{Volume of the cylinder}}{\text{Total volume of ice-cream cone}} = \dfrac{540\pi}{54\pi} = 10$

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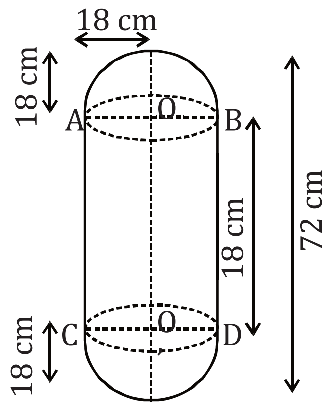
**38.** A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 108 cm and the diameter of the hemispherical ends is 36 cm, find the cost of polishing the surface of the solid at the rate of 7 paise per sq. cm. (Use $\pi = 22/7$)

**Sol.** We have,

r = radius of the cylinder = radius of hemispherical ends = 18 cm

h = height of the cylinder = 72 cm




$\therefore$ Total surface area = Curved surface area of the cylinder + Surface areas of hemispherical ends

$= (2\pi rh + 2 \times 2\pi r^2)\ cm^2$

$= (2\pi rh + 4\pi r^2)\ cm^2$

$= 2\pi r(h + 2r)\ cm^2$

$= 2\times\frac{22}{7}\times 18\times(72+36)cm^2$ $\quad [\because\ r = 18\ cm,\ h = 72\ cm]$

$= 2\times\frac{22}{7}\times 18\times 108 cm^2$

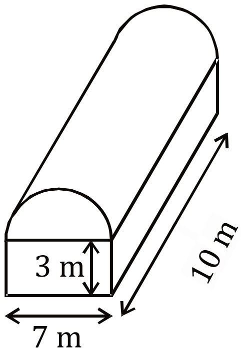
$= 12219.42\ cm^2$

Rate of polishing = 7 paise per sq. cm.

$\therefore$ Cost of polishing $= Rs.\left(12219.42\times\frac{7}{100}\right)$

= Rs. 855.36

**39.** A godown building is in the form as shown in figure. The vertical cross section parallel to the width side of the building is a rectangle of dimensions 7 m × 3 m, mounted by a semi-circle of radius 3.5 m. The inner measurments of the cuboidal portion of the building are 10 m × 7 m × 3 m. Find the volume of the godown and the total interior surface area excluding the floor (base).




**Sol.** Since the top of the building is in the form of half of the cylinder of radius 3.5 m, and length 10 m, split along the diameter.

$\therefore$ V = Volume of the godown

$\Rightarrow$ V = Volume of the cuboid + $\frac{1}{2}$ (Volume of the cylinder of radius 3.5 m and length 10 m)

$$= \left\{10\times7\times3+\frac{1}{2}\left(\frac{22}{7}\times3.5\times3.5\times10\right)\right\}m^3$$

$= (210 + 192.5)m^3 = 402.5\ m^3$

Total interior surface area excluding the base floor

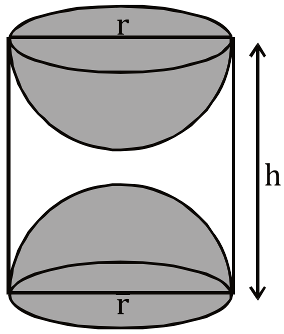
= Area of four walls + $\frac{1}{2}$ (Curved surface area of the cylinder) + 2 (Area of the semi-circles)

$$\left[2(10+7)\times3+\frac{1}{2}\left(2\times\frac{22}{7}\times3.5\times10\right)+2\left(\frac{1}{2}\times\frac{22}{7}\times(3.5)^2\right)\right]m^2$$

$= (102 + 110 + 38.5)\ m^2 = 250.5\ m^2$

**40.** A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in figure. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.

**Sol.** Let r be the radius of the base of the cylinder and h be its height Then,




Total surface area of the article

= Curved surface area of the cylinder + 2 (Curved surface area of a hemisphere)

$= 2\pi rh + 2\ (2\pi r^2)$

$= 2\pi r\ (h + 2r)$

$= 2 \times \frac{22}{7} \times 3.5\ (10 + 2 \times 3.5)\ cm^2$

$= 22 \times 17\ cm^2 = 374\ cm^2$

**Case Study Questions** **(4 marks)**

**1.** In an army base, for accommodation soldiers prepared conical tents of base diameter 14m. Area of canvas required for one tent was 550 $m^2$. Four soldiers were to be accommodated in each tent. Answer the following questions.

(i) Calculate the slant height of each tent.

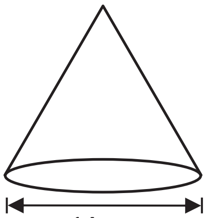
(ii) Calculate the vertical height of each tent

(iii) Calculate the volume of air available for each soldier in the tent.

**[OR]**

(iii) If the width of canvas is 5m, then find the cost of canvas at the rate of Rs. 20 per meter for one tent.

**Sol.**




(i) CSA of tent = 550

$\pi r \ell = 550$

$\frac{22}{7} \times 7 \times \ell = 550$

$\ell = \frac{550}{22}$

$\ell = 25$ m

(ii) $h^2 = \ell^2 - r^2$

$h = \sqrt{25^2 - 7^2}$

$h = \sqrt{576}$

$h = 24$ m

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(iii) Volume of air inside tent = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7}\times 7\times 7\times 24$

$= 22 \times 7 \times 8$

$= 1232 \text{ m}^3$

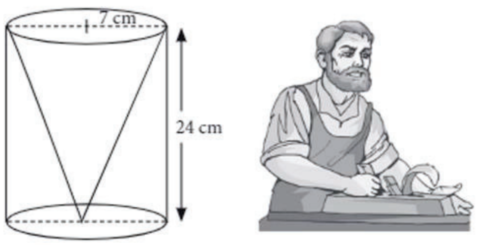
Volume of air available for each soldier = $\frac{1232}{4} = 308 \text{ m}^3$

**[OR]**

(iii) Length of canvas = $\frac{\text{Total area}}{\text{width}} = \frac{550}{5} = 110$ m

Cost of canvas = 110 × 20 = ₹ 2200

**2.** One day Rinku was going home from school, saw carpenter working on wood. He found that he is carving out a cone of same height and same diameter from a cylinder. The height of cylinder is 24 cm and base radius is 7 cm. While watching this, some question came into Rinku's mind. Help Rinku to find the answer of the following questions.




(i) Find the slant height of the conical cavity so formed.

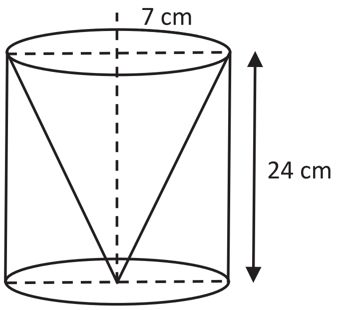
(ii) Find the CSA of conical cavity so formed.

(iii) Find the external curved surface area of the cylinder.

**[OR]**

(iii) Find the volume of conical cavity.

**Sol.**




(i) Slant height $(\ell) = \sqrt{r^2 + h^2}$

$= \sqrt{(7)^2 + (24)^2} = \sqrt{49 + 576}$

$= \sqrt{625} = 25$ cm

(ii) CSA = $\pi r\ell = \frac{22}{7}\times 7\times 25 = 550\ cm^2$

(iii) External curved surface area = $2\pi rh = 2\times\frac{22}{7}\times 7\times 24 = 1056\ cm^2$

**[OR]**

(iii) Volume of conical cavity = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7}\times 7\times 7\times 24 = 1232\ cm^3$

**3.** Adventure camps are the perfect place for the children to practice decision making for themselves without parents and teachers guiding their every move. Some students of a school reached for adventure at Sakleshpur. At the camp, the waiters served some students with a welcome drink in a cylindrical glass while some students in a hemispherical cup whose dimensions are shown below. After that, they went for a jungle trek. The jungle trek was enjoyable but tiring. As dusk fell, it was time to take shelter. Each group of four students was given a canvas tent of area 551 $m^2$. Each group had to make a conical tent to accommodate all the four students. Assuming that all the stitching and wasting incurred while cutting would amount to $1m^2$, the students put the tents. The radius of the tent is 7m.

d = 7 cm
h = 10.5 cm

d = 7 cm

(i) Calculate the volume of hemispherical cup.
(ii) Calculate the volume of cylindrical cup.
(iii) Calculate the height of the conical tent prepared to accommodate four students.

**[OR]**

(iii) How much space on the ground is occupied by each student in the conical tent?

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**Sol.** (i) $d = 7cm, r = \frac{7}{2}$ cm

Volume of hemispherical cup $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times\frac{7}{2}$

$= \frac{539}{6}$ cm$^3$ or 89.83cm$^3$

(ii) $d = 7$ cm, $r = \frac{7}{2}$ cm, $h = 10.5$ cm

Volume of cylindrical cup $= \pi r^2 h$

$= \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times 10.5$

$= 404.25$ cm$^3$

(iii) Radius of tent = 7 m,

Total areas of canvas = 551 m$^2$

wastage canvas (cutting and stitching) = 1 m$^2$

Area of canvas used = 551 – 1 = 550 m$^2$

Area of canvas used = CSA of tent

$\pi rl$ = CSA of tent

$\frac{22}{7}\times 7\times \ell = 550$ m$^2$

$\ell = \frac{550}{22} = 25$ m

$\ell^2 = h^2 + r^2 \Rightarrow h^2 = \ell^2 - r^2 \Rightarrow h^2 = 25^2 - (7)^2$

$h^2 = 625 - 49$

$h^2 = 576$

$h = 24$ m

**[OR]**

(iii) Area of base of the tent $= \pi r^2$

$\frac{22}{7}\times 7\times 7 = 154$ m$^2$

Area occupied by 1 student $= \frac{\text{Total area of base}}{4}$

Area occupied by each student $= \frac{154}{4} = 38.5$ m$^2$

# 13 Statistics

## Multiple choice questions **(1 mark)**

**1.** Find the mean of given data.

| **Class Interval** | 16-18 | 19-21 | 22-24 | 25-27 | 28-30 |
|---|---|---|---|---|---|
| **Frequency** | 1 | 3 | 4 | 9 | 13 |

(1) 23 (2) 24.8 (3) 25.5 (4) 26

**Sol. Option (4)**

| Class Interval | $f_i$ | $x_i$ | $d_i = x_i - a$ | $f_i d_i$ |
|---|---|---|---|---|
| 16-18 | 1 | 17 | –6 | –6 |
| 19-21 | 3 | 20 | –3 | –9 |
| 22-24 | 4 | 23 = a | 0 | 0 |
| 25-27 | 9 | 26 | 3 | 27 |
| 28-30 | 13 | 29 | 6 | 78 |
| | $\Sigma f_i = 30$ | | | $\Sigma f_i d_i = 90$ |

Let a = 23

mean $\overline{x} = a + \dfrac{\Sigma f_i d_i}{\Sigma f_i}$

$= 23 + \dfrac{90}{30}$

$= 23 + 3 = 26$

**2.** The height of 70 plants are shown in the following table

| Height (in cm) | 200-201 | 201-202 | 202-203 | 203-204 | 204-205 | 205-206 |
|---|---|---|---|---|---|---|
| No. of Plants | 12 | 26 | 20 | 9 | 2 | 1 |

Find the modal height.

(1) 201.9 (2) 201.7 (3) 201.5 (4) 201.2

**Sol. Option (2)**

| Height (in cm) | 200-201 | 201-202 | 202-203 | 203-204 | 204-205 | 205-206 |
|---|---|---|---|---|---|---|
| No. of Plants | 12 | 26 | 20 | 9 | 2 | 1 |

Modal class is 201-202 because it has maximum frequency 26

$\ell = 201$, $h = 1$, $f_1 = 26$, $f_0 = 12$, $f_2 = 20$

Mode = $\ell + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 201 + \left[\dfrac{26-12}{52-12-20}\right] \times 1$

$= 201 + \dfrac{14}{20} = 201.7$ cm

**3.** If the mean of the following distribution is 2.6, then the value of y is

| Variable | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Frequency | 4 | 5 | y | 1 | 2 |

(1) 3 (2) 8 (3) 13 (4) 24

**Sol. Option (2)**

| $x_i$ | $f_i$ | $f_ix_i$ |
|---|---|---|
| 1 | 4 | 4 |
| 2 | 5 | 10 |
| 3 | y | 3y |
| 4 | 1 | 4 |
| 5 | 2 | 10 |
| | $\Sigma f_i = 12 + y$ | $\Sigma f_ix_i = 28 + 3y$ |

Mean $(\overline{x}) = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$2.6 = \dfrac{28+3y}{12+y}$

$31.2 + 2.6y = 28 + 3y$

$3.2 = 0.4y$

$y = \dfrac{3.2}{0.4}$

$y = 8$

**4.** If three sets of data had means of 15, 22.5 and 24 based on 6, 4 and 5 observations respectively, then the mean of these three sets combined is

(1) 20.0 (2) 20.5 (3) 22.5 (4) 24.0

**Sol. Option (1)**

Combined mean of three set = $\dfrac{15\times6+22.5\times4+24\times5}{6+4+5} = \dfrac{90+90+120}{15}$

$= \dfrac{300}{15} = 20$

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**5.** If the mean and median of a set of numbers are 8.9 and 9 respectively, then the mode will be

(1) 7.2 (2) 8.2 (3) 9.2 (4) 10.2

**Sol. Option (3)**

We know that

mode = 3 median – 2 mean = 3 × 9 – 2 × 8.9

= 27 – 17.8

mode = 9.2

**6.** For a given data with 120 observations, the 'less than ogive' and more than ogive' intersect at (42.5, 60). The median of the data is

(1) 60 (2) 42.5 (3) 120 (4) 21.25

**Sol. Option (2)**

If intersection point of 'less than ogive' and 'more than ogive' is (42.5, 60) then median = x-coordinate = 42.5

**7.** In the formula $\bar{x} = a + h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$, for finding the mean of grouped frequency distribution, $u_i$ is equal:

(1) $\frac{x_i + a}{h}$ (2) $h(x_i - a)$ (3) $\frac{x_i - a}{h}$ (4) $\frac{a - x_i}{h}$

**Sol. Option (3)**

mean $(\bar{x})$ = a + h$\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$; where $u_i = \frac{x_i - a}{h}$

**8.** If the median of the distribution given below is 28.5, find the values of y–x.

| C.I. | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | Total |
|---|---|---|---|---|---|---|---|
| Fre. | 5 | x | 20 | y | 7 | 5 | 60 |

(1) 9 (2) 8 (3) 7 (4) 6

**Sol. Option (3)**

| Class interval | $f_i$ | CF |
|---|---|---|
| 0-10 | 5 | 5 |
| 10-20 | x | 5 + x |
| 20-30 | 20 | 25 + x |
| 30-40 | y | 25 + x + y |
| 40-50 | 7 | 32 + x + y |
| 50-60 | 5 | 37 + x + y |
| Total | 60 | |

N = 60

37 + x + y = 60

x + y = 23 ... (i)

median = 28.5 (Given)

So, median class = 20 – 30

$\ell = 20, h = 10, \frac{N}{2} = \frac{60}{2} = 30, f = 20, cf = 5 + x$

$$\text{median} = \ell + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$28.5 = 20 + \left(\frac{30 - 5 - x}{20}\right) \times 10$$

$$8.5 = \frac{25 - x}{2}$$

17 = 25 – x

x = 25 – 17 = 8

Put in equation (i)

$\Rightarrow$ 8 + y = 23

y = 15

So, y – x = 15 – 8 = 7

## Assertion reason questions (1 mark)

**9.** **Assertion (A):** The mode of the call received on 7 consecutive day 11,13,13,17,19,23,25 is 13.

**Reason (R):** Mode is the value that appears most frequent;

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol. Option (1)**

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

**10.** **Assertion (A):** Frequency is the number of times a particular observation occurs in data.

**Reason (R):** Data can be grouped into class intervals such that all observations in that range belong to that class.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(3) Assertion (A) is true but Reason (R) is false.

(4) Assertion (A) is false but Reason (R) is true.

**Sol.** **Option (2)**

Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

### Very short answer type questions **(2 mark)**

**11.** Find the lower limit of the modal class of the following data

| C.I | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 |
|---|---|---|---|---|---|
| **Frequency** | 5 | 8 | 13 | 7 | 6 |

**Sol.** Since, class interval 20 – 30 has highest frequency 13. Hence, it is modal class and its lower limit is 20.

**12.** Find the class mark of the class 29.5 – 30.5.

**Sol.** Class mark = $\frac{29.5+30.5}{2}=30$

**13.** Find the sum of lower limit of modal class and median class of the following data is

| Class | 30 – 40 | 40 – 50 | 50 – 60 | 60 – 70 | 70 – 80 | 80 – 90 |
|---|---|---|---|---|---|---|
| **Frequency** | 25 | 30 | 16 | 19 | 17 | 13 |

**Sol.** Since, class interval 40 – 50 has highest frequency 30. Hence, it is modal class and its lower limit is 40. Now for median class.

| Class | Frequency | Cumulative frequency |
|---|---|---|
| 30 – 40 | 25 | 25 |
| 40 – 50 | 30 | 55 |
| 50 – 60 | 16 | 71 |
| 60 – 70 | 19 | 90 |
| 70 – 80 | 17 | 107 |
| 80 – 90 | 13 | 120 |

N = 120

$\frac{N}{2} = \frac{120}{2} = 60$

Thus, median class 50 – 60.

Its lower limit is 50.

Sum of lower limit of median class and modal class = 40 + 50 = 90

**14.** The mean and median of some data are 24 and 26 respectively. Find the value of mode.

**Sol.** Mode = 3 Median – 2 Mean

= 3 × 26 – 2 × 24

= 78 – 48

= 30

**15.** If the median of a series exceeds the mean by 3, find by what number the mode exceeds its mean?

**Sol.** Given, Median = Mean + 3

Also, we know that

Mode = 3 Median – 2 Mean

= 3 (Mean + 3) – 2 Mean

$\Rightarrow$ Mode = Mean + 9

Hence, Mode exceeds Mean by 9.

**16.** Following distribution gives cumulative frequencies of 'more than type' :

| **Marks obtained** | More than or equal to 5 | More than or equal to 10 | More than or equal to 15 | More than or equal to 20 |
|---|---|---|---|---|
| **Number of students (cumulative frequency)** | 30 | 23 | 8 | 2 |

Change the above data to a continuous grouped frequency distribution.

**Sol.**

| **C.I.** | 5 – 10 | 10 – 15 | 15 – 20 | 20 – 25 |
|---|---|---|---|---|
| **f** | 7 | 15 | 6 | 2 |

**17.** Convert the following data into 'more than type' distribution

| **Class** | 50 – 55 | 55 – 60 | 60 – 65 | 65 – 70 | 70 – 75 | 75 – 80 |
|---|---|---|---|---|---|---|
| **Frequency** | 2 | 8 | 12 | 24 | 38 | 16 |

**Sol.**

| Class | Frequency |
|---|---|
| More than 50 | 100 |
| More than 55 | 98 |
| More than 60 | 90 |
| More than 65 | 78 |
| More than 70 | 54 |
| More than 75 | 16 |

## Short answer type questions (3 marks)

**18.** The mean of the following frequency distribution is 25. Find the value of p.

| Class interval | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 |
|---|---|---|---|---|---|
| **Frequency** | 4 | 6 | 10 | 6 | p |

**Sol.**

| Class interval | Mid value $x_i$ | $f_i$ | $f_i x_i$ |
|---|---|---|---|
| 0 – 10 | 5 | 4 | 20 |
| 10 – 20 | 15 | 6 | 90 |
| 20 – 30 | 25 | 10 | 250 |
| 30 – 40 | 35 | 6 | 210 |
| 40 – 50 | 45 | p | 45 p |
| | | **26 + p** | **570 + 45 p** |

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} \Rightarrow 25 = \frac{570+45p}{26+p}$$

$\Rightarrow$ 650 + 25p = 570 + 45p

$\Rightarrow$ 650 – 570 = 45p – 25p

$\Rightarrow$ 80 = 20p

$\Rightarrow$ p = 4

**19.** If the mean of the following distribution is 6, find the value of p.

**Sol.**

| x | 2 | 4 | 6 | 10 | p + 5 |
|---|---|---|---|---|---|
| f | 3 | 2 | 3 | 1 | 2 |

Calculation of mean

| $x_i$ | $f_i$ | $f_ix_i$ |
|---|---|---|
| 2 | 3 | 6 |
| 4 | 2 | 8 |
| 6 | 3 | 18 |
| 10 | 1 | 10 |
| p + 5 | 2 | 2p + 10 |
| **Total** | $\sum f_i = 11$ | $\sum f_ix_i = 2p + 52$ |

We have, $\sum f_i = 11$, $\sum f_ix_i = 2p + 52$, $\overline{x} = 6$

$\therefore$ Mean $(\overline{x}) = \dfrac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow$ $6 = \dfrac{2p+52}{11} \Rightarrow 66 = 2p + 52$

$\Rightarrow$ $2p = 14$

$\Rightarrow$ $p = 7$

**20.** Find the mean of the following distribution :

| x | 4 | 6 | 9 | 10 | 15 |
|---|---|---|---|---|---|
| f | 5 | 10 | 10 | 7 | 8 |

**Sol.** Calculation of arithmetic mean

| $x_i$ | $f_i$ | $f_ix_i$ |
|---|---|---|
| 4 | 5 | 20 |
| 6 | 10 | 60 |
| 9 | 10 | 90 |
| 10 | 7 | 70 |
| 15 | 8 | 120 |
| **Total** | $\sum f_i = 40$ | $\sum f_ix_i = 360$ |

$\therefore$ $\text{Mean}(\overline{x}) = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{360}{40} = 9$

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**21.** Find the mean of the following frequency distribution by assumed mean method.

| **Class** | 25 – 30 | 30 – 35 | 35 – 40 | 40 – 45 | 45 – 50 | 50 – 55 | 55 – 60 |
|---|---|---|---|---|---|---|---|
| **Frequency** | 14 | 22 | 16 | 6 | 5 | 3 | 4 |

**Sol.**

| **Class** | **Frequency** | $x_i$ | $d_i = x_i - a$ | $f_i d_i$ |
|---|---|---|---|---|
| 25 – 30 | 14 | 27.5 | –15 | –210 |
| 30 – 35 | 22 | 32.5 | – 10 | –220 |
| 35 – 40 | 16 | 37.5 | –5 | –80 |
| 40 – 45 | 6 | 42.5 | 0 | 0 |
| 45 – 50 | 5 | 47.5 | 5 | 25 |
| 50 – 55 | 3 | 52.5 | 10 | 30 |
| 55 – 60 | 4 | 57.5 | 15 | 60 |
| | N = 70 | | | $\sum f_i d_i = -395$ |

a = 42.5; Mean $(\overline{x}) = a + \frac{\sum f_i d_i}{N} = 42.5 - \frac{395}{70} = 36.85$

**22.** Calculate the median from the following distribution.

| **Class** | 5 – 10 | 10 – 15 | 15 – 20 | 20 – 25 | 25 – 30 | 30 – 35 | 35 – 40 | 40 – 45 |
|---|---|---|---|---|---|---|---|---|
| **Frequency** | 5 | 6 | 15 | 16 | 5 | 4 | 2 | 2 |

**Sol.**

| **Class** | **Frequency** | **Cf** |
|---|---|---|
| 5 – 10 | 5 | 5 |
| 10 – 15 | 6 | 11 |
| 15 – 20 | 15 | 26 |
| 20 – 25 | 16 | 42 |
| 25 – 30 | 5 | 47 |
| 30 – 35 | 4 | 51 |
| 35 – 40 | 2 | 53 |
| 40 – 45 | 2 | 55 |

N = 55

$\frac{N}{2} = \frac{55}{2} = 27.5$

Median class $\Rightarrow$ 20 – 25

$$\text{Median} = \ell + \frac{\frac{N}{2} - cf}{f} \times h = 20 + \frac{27.5 - 26}{16} \times 5$$

$$= 20 + \frac{15}{16 \times 10} \times 5 \quad = 20 + \frac{15}{32} = 20.46$$

**23.** The mode of the following data is 36. Find the missing frequency x in it.

| **Class** | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 | 60 – 70 |
|---|---|---|---|---|---|---|---|
| **Frequency** | 8 | 10 | x | 16 | 12 | 6 | 7 |

**Sol.**

| **Class** | **Frequency** |
|---|---|
| 0 – 10 | 8 |
| 10 – 20 | 10 |
| 20 – 30 | x |
| 30 – 40 | 16 |
| 40 – 50 | 12 |
| 50 – 60 | 6 |
| 60 – 70 | 7 |

Since mode is 36. Hence, modal class is 30 – 40

$$\text{Mode} = \ell + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$36 = 30 + \frac{16 - x}{32 - x - 12} \times 10$$

$$36 = 30 + \frac{16 - x}{20 - x} \times 10$$

$$6 = \frac{16 - x}{20 - x} \times 10$$

$6(20 - x) = (16 - x)10$

$120 - 6x = 160 - 10x \quad \Rightarrow 4x = 40$

$x = 10$

**24.** Find the mean of the data using an empirical formula when it is given that mode is 50.5 and median is 45.5.

**Sol.** Given, Mode = 50.5

Median = 45.5

Mode = 3 Median – 2 Mean

2 Mean = 3 Median – Mode

2 Mean = 3 × 45.5 – 50.5

$$\Rightarrow \quad \text{Mean} = \frac{136.5 - 50.5}{2} = 43$$

**25.** The mean and median of 100 observation are 50 and 52 respectively. The value of the largest observation is 100. It was later found that it is 110 not 100. Find the true mean and median.

**Sol.** Mean = $\frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow \quad 50 = \frac{\sum f_i x_i}{100}$

$\Rightarrow \quad \sum f_i x_i = 5000$

New $\sum f_i x_i = 5000 - 100 + 110$

$\Rightarrow$ Correct Mean = $\frac{5010}{100} = 50.1$

Median will remain same i.e. median = 52

**26.** The following distribution shows the marks scored by 140 students in an examination. Calculate the mode of the distribution :

| **Marks** | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 |
|---|---|---|---|---|---|
| **Number of students** | 20 | 24 | 40 | 36 | 20 |

**Sol.** Here, Modal class = 20 – 30

$\ell = 20, f_1 = 40, f_0 = 24, f_2 = 36, h = 10$

$$\text{Mode} = \ell + \frac{(f_1 - f_0)}{2f_1 - f_0 - f_2} \times h$$

$$= 20 + \frac{(40-24)}{80-24-36} \times 10$$

$$= 20 + \frac{16 \times 10}{20} = 28$$

**27.** The mean of the following frequency distribution is 62.8. Find the missing frequency x.

| **Class** | 0 – 20 | 20 – 40 | 40 – 60 | 60 – 80 | 80 – 100 | 100 – 120 |
|---|---|---|---|---|---|---|
| **Frequency** | 5 | 8 | x | 12 | 7 | 8 |

**Sol.** We have

| **Class interval** | **Frequency ($f_i$)** | **Class mark ($x_i$)** | **$f_i x_i$** |
|---|---|---|---|
| 0 – 20 | 5 | 10 | 50 |
| 20 – 40 | 8 | 30 | 240 |
| 40 – 60 | x | 50 | 50x |
| 60 – 80 | 12 | 70 | 840 |
| 80 – 100 | 7 | 90 | 630 |
| 100 – 120 | 8 | 110 | 880 |
| **Total** | $\sum f_i = 40 + x$ | | $\sum f_i x_i = 2640 + 50x$ |

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Here, $\sum f_i x_i = 2640 + 50x$,

$\sum f_i = 40 + x$, $\bar{x} = 62.8$

$\therefore$ Mean $(\bar{x}) = \dfrac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow$ $62.8 = \dfrac{2640 + 50x}{40 + x}$

$\Rightarrow$ $2512 + 62.8x = 2640 + 50x$

$\Rightarrow$ $62.8x - 50x = 2640 - 2512$

$\Rightarrow$ $12.8x = 128$

$\therefore$ $x = \dfrac{128}{12.8} = 10$

Hence, the missing frequency is 10.

**28.** Monthly pocket money of students of a class is given in the following frequency distribution

| **Pocket money (in Rs.)** | 100 – 125 | 125 – 150 | 150 – 175 | 175 – 200 | 200 – 225 |
|---|---|---|---|---|---|
| **Number of students** | 14 | 8 | 12 | 5 | 11 |

Find the mean of pocket money using step deviation method.

**Sol.**

| Class | Frequency | Class mark $x_i$ | $u_i = \dfrac{x_i - a}{h}$ | $f_i u_i$ |
|---|---|---|---|---|
| 100 – 125 | 14 | 112.5 | –2 | –28 |
| 125 – 150 | 8 | 137.5 | –1 | –8 |
| 150 – 175 | 12 | 162.5 | 0 | 0 |
| 175 – 200 | 5 | 187.5 | 1 | 5 |
| 200 – 225 | 11 | 212.5 | 2 | 22 |
| | $N = 50$ | | | $\sum f_i u_i = -9$ |

a = 162.5

$$\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$\bar{x} = 162.5 + \left(\frac{-9}{50}\right) \times 25$$

$$\bar{x} = 158$$

**29.** Compute the median marks for the following data.

| Marks | Number of students |
|---|---|
| 0 and above | 50 |
| 10 and above | 46 |
| 20 and above | 40 |
| 30 and above | 20 |
| 40 and above | 10 |
| 50 and above | 3 |
| 60 and above | 0 |

**Sol.** To find median marks, we convert the given data into continuous grouped frequency distribution

| Marks | Number of students | Cumulative frequency |
|---|---|---|
| 0-10 | 50 – 46 = 4 | 4 |
| 10-20 | 46 – 40 = 6 | 10 |
| 20-30 | 40 – 20 = 20 | 30 |
| 30-40 | 20 – 10 = 10 | 40 |
| 40-50 | 10 – 3 = 7 | 47 |
| 50-60 | 3 – 0 = 3 | 50 |

Here, $\frac{N}{2}=\frac{50}{2}=25$

$\therefore$ Median class = 20 – 30

$$\text{Median} = \ell + \frac{\frac{N}{2}-cf}{f} \times h = 20 + \frac{25-10}{20} \times 10$$

= 20 + 7.5 = 27.5

**30.** If the median for the following frequency distribution is 28.5, find the value of x and y

| Marks | Frequency |
|---|---|
| 0-10 | 5 |
| 10-20 | x |
| 20-30 | 20 |
| 30-40 | 15 |
| 40-50 | y |
| 50-60 | 5 |
| **Total** | **60** |

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**Sol.**

| C.I. | f | c.f. |
|---|---|---|
| 0 – 10 | 5 | 5 |
| 10 – 20 | x | x + 5 |
| 20 – 30 | 20 | x + 25 |
| 30 – 40 | 15 | x + 40 |
| 40 – 50 | y | x + y + 40 |
| 50 – 60 | 5 | x + y + 45 |
| | $N = \sum f = 60$ | |

From table N = 60 = x + y + 45

x + y = 60 – 45 = 15

$\Rightarrow$ Median = 28.5

Since, Median class = 20 – 30

$$\text{Median} = \ell + \frac{\left(\frac{N}{2} - \text{c.f.}\right)}{f} \times h$$

$\Rightarrow$ $28.5 = 20 + \dfrac{[30-(x+5)]}{20} \times 10$

$\Rightarrow$ 25 – x = 17 $\Rightarrow$ x = 25 – 17 = 8 $\Rightarrow$ x = 8

From (1), y = 15 – 8 = 7

$\therefore$ x = 8 and y = 7

**31.** The mean of the following distribution is 48 and sum of all the frequency is 50. Find the missing frequencies x and y.

| **Class** | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 | 60 – 70 |
|---|---|---|---|---|---|
| **Frequency** | 8 | 6 | x | 11 | y |

**Sol.**

| C.I | $f_i$ | $x_i$ | $u_i = \frac{x_i - a}{h}$ | $f_iu_i$ |
|---|---|---|---|---|
| 20 – 30 | 8 | 25 | –2 | –16 |
| 30 – 40 | 6 | 35 | –1 | –6 |
| 40 – 50 | x | 45 | 0 | 0 |
| 50 – 60 | 11 | 55 | 1 | 11 |
| 60 – 70 | y | 65 | 2 | 2y |
| | N = 50 | | | $\sum f_iu_i = -11 + 2y$ |

$a = 45, h = 10$

$$\text{Mean} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$\sum f_i = 25 + x + y = 50$

$\Rightarrow$ $x + y = 25$

$\Rightarrow$ $48 = 45 + \frac{2y - 11}{50} \times 10$

$\Rightarrow$ $15 = 2y - 11$

$\Rightarrow$ $y = 13$

$\Rightarrow$ $x = 25 - 13 = 12$

$\therefore$ $x = 12$ and $y = 13$

## Long answer type questions (5 marks)

**32.** Draw a 'more than type' ogive from the following distribution.

| **Marks obtained** | 10 – 19 | 20 – 29 | 30 – 39 | 40 – 49 | 50 – 59 |
|---|---|---|---|---|---|
| **Number of candidates** | 6 | 7 | 5 | 10 | 3 |

**Sol.** Here, the data is not in continuous from, so we convert it into continuous class for finding the 'more than type' ogive. For this subtract 0.5 from lower limit of each class interval and add 0.5 to upper limit of each class interval.

| **Marks obtained** | 9.5 – 19.5 | 19.5 – 29.5 | 29.5–39.5 | 39.5 – 49.5 | 49.5 – 59.5 |
|---|---|---|---|---|---|
| **Number of candidates** | 6 | 7 | 5 | 10 | 3 |

For 'more than type' ogive, we form cumulative frequency distribution of more than type as given below.

| **Marks obtained** | **Number of candidates** |
|---|---|
| 9.5 or more than 9.5 | 31 |
| 19.5 or more than 19.5 | 31 – 6 = 25 |
| 29.5 or more than 29.5 | 25 – 7 = 18 |
| 39.5 or more than 39.5 | 18 – 5 = 13 |
| 49.5 or more than 49.5 | 13 – 10 = 3 |

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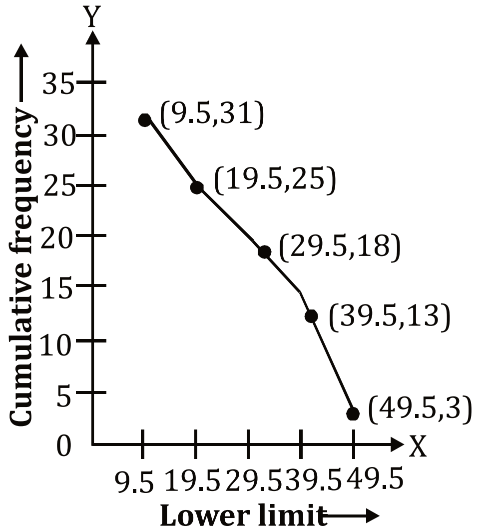
Now, take lower limits along the x-axis and cumulative frequencies along y-axis, on the graph paper. Then, plot the points (9.5, 31), (19.5, 25), (29.5, 18), (39.5, 13) and (49.5, 3) on a graph paper and join them by a freehand smooth curve.

Thus, we get 'more than type' ogive as shown in the figure




**33.** Draw a 'less than type' ogive for the following frequency distribution.

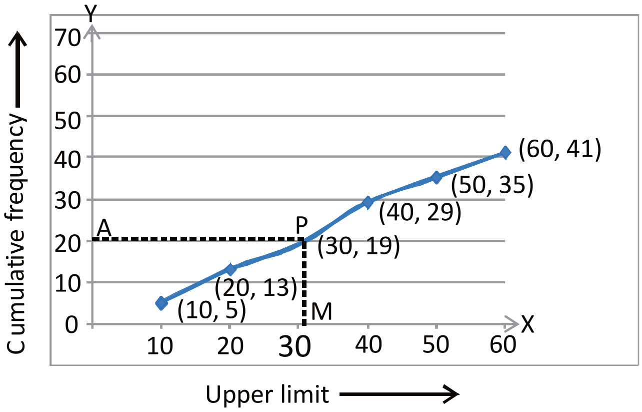
| **Marks** | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 |
|---|---|---|---|---|---|---|
| **Number of students** | 5 | 8 | 6 | 10 | 6 | 6 |

Find the median from the graph and also verify the result.

**Sol.** For the given distribution 'less than type' cumulative frequency distribution is as follows :

| Marks | Number of students | Marks | Cumulative frequency |
|---|---|---|---|
| 0 – 10 | 5 | Less than 10 | 5 |
| 10 – 20 | 8 | Less than 20 | 5 + 8 = 13 |
| 20 – 30 | 6 | Less than 30 | 13 + 6 = 19 |
| 30 – 40 | 10 | Less than 40 | 19 + 10 = 29 |
| 40 – 50 | 6 | Less than 50 | 29 + 6 = 35 |
| 50 – 60 | 6 | Less than 60 | 35 + 6 = 41 |

Now, mark the upper limits on the x-axis and cumulative frequencies along y-axis and then plot the points (10, 5), (20, 13), (30 , 19), (40, 29), (50, 35) and (60, 41). Join these points by a freehand smooth curve to obtain the 'less than type' ogive as shown in the figure.




Now, $\frac{N}{2}=\frac{41}{2}=20.5$

Take a point A on Y-axis representing 20.5. Through A, draw a line AP parallel to X-axis which meet the ogive at P. Draw PM, perpendicular to X-axis, meeting X-axis at point M. The abscissa of M is 31.5 which is the required median.

Verification

Here, N = 41

$\therefore \quad \frac{N}{2}=\frac{41}{2}=20.5$

Since the cumulative frequency just greater than 20.5 is 29 and the corresponding class interval is 30 – 40.

$\therefore \quad \ell = 30$, h = 10, cf = 19 and f = 10

Now, median

$$= \quad \ell+\left\{\frac{\frac{n}{2}-cf}{f}\right\}\times h=30+\left\{\frac{20.5-19}{10}\right\}\times 10$$

= 30 + 1.5 = 31.5 Hence proved

**34.** The following table gives the height of tree

| **Height (Less than)** | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 |
|---|---|---|---|---|---|---|---|---|
| **Number of trees** | 26 | 57 | 92 | 134 | 216 | 287 | 341 | 360 |

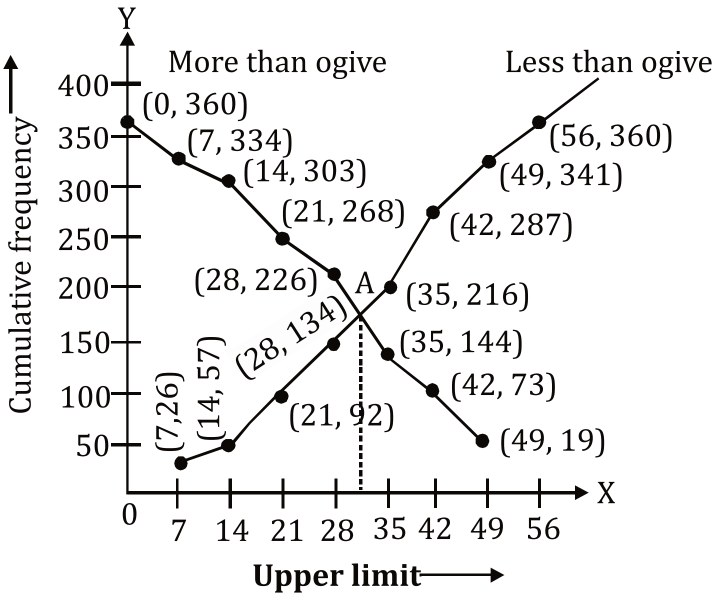
Draw 'less than ogive' and 'more than ogive'. Also, find the median.

**Sol.** Given distribution is cumulative frequency distribution of less than type.

Now, we mark the upper limits along x-axis and cumulative frequencies along y-axis, on the graph paper. Then, plot the points (7, 26), (14, 57), (21,92), (28, 134), (35, 216), (42, 287), (49,341) and (56, 360). Join all these points by a freehand smooth curve to obtain an ogive of less than type. Now, let us form the cumulative frequency distribution of more than type, as shown below.

| Height | Frequency | Height (more than equal to) | Cumulative frequency |
|---|---|---|---|
| 0 – 7 | 26 | 0 | 360 |
| 7 – 14 | 57 – 26 = 31 | 7 | 360 – 26 = 334 |
| 14 – 21 | 92 – 57 = 35 | 14 | 334 – 31 = 303 |
| 21 – 28 | 134 – 92 = 42 | 21 | 303 – 35 = 268 |
| 28 – 35 | 216 – 134 = 82 | 28 | 268 – 42 = 226 |
| 35 – 42 | 287 – 216 = 71 | 35 | 226 – 82 = 144 |
| 42 – 49 | 341 – 287 = 54 | 42 | 144 – 71 = 73 |
| 49 – 56 | 360 – 341 = 19 | 49 | 73 – 54 = 19 |

Now, we plot the points (0, 360), (7, 334), (14, 303), (21, 268), (28,226), (35, 144), (42,73) and (49, 19) on the same graph paper. Join all these points by a free hand smooth curve to obtain an ogive of more than type.




The two ogives intersect at point A. Now, we draw a perpendicular line from A to the x-axis, the intersection point Median = 31.9 ≈ 32 (approx)

**35.** Some surnames were picked up from a local telephone directory and the frequency distribution of the number of letters of the English alphabets was obtained as follows :

| Number of letters | 1 – 4 | 4 – 7 | 7 – 10 | 10 – 13 | 13 – 16 | 16 – 19 |
|---|---|---|---|---|---|---|
| Number of students | 10 | 25 | 35 | x | 12 | 8 |

If it is given that mode of the distribution is 8, then find the missing frequency (x).

**Sol.** Mode = 8

$\Rightarrow$ Modal class = 7 – 10

$$\text{Mode} = \ell + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$8 = 7 + \left(\frac{35-25}{2\times 35 - 25 - x}\right)\times 3$$

$$\Rightarrow\ 1 = \left(\frac{10}{45-x}\right)\times 3$$

$\Rightarrow$ 45 – x = 30 $\Rightarrow$ x = 15

**Case Study type questions** **(4 marks)**

**1.** The given distribution shows the number of runs scored by some top batsmen of the world in the ODI cricket matches.

| Runs Scored | Number of batsmen | CF |
|---|---|---|
| 3000-4000 | 4 | 4 |
| 4000-5000 | 18 | 22 |
| 5000-6000 | 9 | 31 |
| 6000-7000 | 7 | 38 |
| 7000-8000 | 6 | 44 |
| 8000-9000 | 3 | 47 |
| 9000-10,000 | 1 | 48 |
| 10,000-11,000 | 1 | 49 |

(i) What will be the upper limit of modal class?

(ii) How many batsmen scored more than 7000 ?

(iii) Find the modal runs scored by batsman.

**[OR]**

(iii) What will be the lower limit of median class?

**Sol.** (i) Modal class is 4000 – 5000

So, upper limit = 5000

(ii) Number of batsmen scored runs more than 7000

= 6 + 3 + 1 + 1 = 11

(iii) $\ell$ = 4000, $f_1$ = 8, $f_0$ = 4, $f_2$ = 9, h = 1000

$$\text{mode} = 4000 + \frac{18-4}{36-4-9}\times 1000$$

$$= 4000 + \frac{14}{23}\times 1000$$

= 4608.7

**[OR]**

| **Runs Scored** | **Number of batsmen** | **CF** |
|---|---|---|
| 3000-4000 | 4 | 4 |
| 4000-5000 | 18 | 22 |
| 5000-6000 | 9 | 31 |
| 6000-7000 | 7 | 38 |
| 7000-8000 | 6 | 44 |
| 8000-9000 | 3 | 47 |
| 9000-10,000 | 1 | 48 |
| 10,000-11,000 | 1 | 49 |

$\frac{N}{2} = \frac{49}{2} = 24.5$

median class = 5000 – 6000

Lower limit of median class = 5000

**2.** A fruit seller was selling mangoes in packed boxes containing varying amount of mangoes.

| **No. of Mangoes** | 5-15 | 15-25 | 25-35 | 35-45 | 45-55 | 55-65 |
|---|---|---|---|---|---|---|
| **No. of boxes** | 6 | 11 | 22 | 23 | 14 | 5 |

(i) What will be the lower limit of median class?

(ii) How many boxes contain less than 45 mangoes?

(iii) Find the modal number of mangoes kept in a packed box.

**[OR]**

(iii) Find the sum of upper limit of median class and modal class.

**Sol.** (i)

| No. of Mangoes | Number of boxes | CF |
|---|---|---|
| 5-15 | 6 | 6 |
| 15-25 | 11 | 17 |
| 25-35 | 22 | 39 |
| 35-45 | 23 | 62 |
| 45-55 | 14 | 76 |
| 55-65 | 5 | 81 |

$\frac{N}{2} = \frac{81}{2} = 40.5$

median class = 35 – 45

Lower limit = 35

(ii) Number of boxes = 6 + 11 + 22 + 23 = 62

(iii) Mode = $\ell + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$= 35 + \frac{23-22}{2(23)-22-14} \times 10 = 35 + \frac{10}{46-36}$

$= 35 + \frac{10}{10} = 36$

**[OR]**

(iii) Modal class = 35 – 45

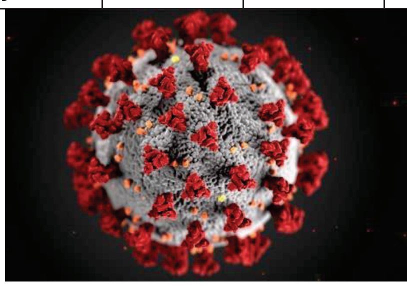
Median class = 35 – 45

Sum = 45 + 45 = 90

**3.** Direct income in India was drastically impacted due to the covid-19 lockdown. Most of the companies decided to bring down the salaries of the employees upto 50%.

The following table shows the salaries (in percent) received by 50 employees during lockdown.

| Salaries Received (in%) | 50-60 | 60-70 | 70-80 | 80-90 |
|---|---|---|---|---|
| Number of employees | 18 | 12 | 16 | 4 |



(i) Find the total number of persons whose salary is reduced by more than 20%.

(ii) Write the empirical relationship among mean, median and mode.

(iii) Find median class of given data.

**[OR]**

(iii) Find mode class of the given data.

**Sol.** (i) People whose salary has been deducted by more than 20 % will receive salary less than 80%.

Number of persons received less than 80% of the salary = 18 + 12 + 16 = 46

(ii) Mode = 3 Median – 2 Mean

(iii)

| Salaries Received (in%) | 50-60 | 60-70 | 70-80 | 80-90 |
|---|---|---|---|---|
| **Number of employees** | 18 | 12 | 16 | 4 |
| **CF** | 18 | 30 | 46 | 50 |

$\frac{N}{2} = \frac{50}{2} = 25$

So, median class will be 60-70.

**[OR]**

(iii) Modal Class = 50 – 60 (Highest Frequency)

**4.** The education of women helps to remove the social stigma that surrounds it. It is the key to eliminate social evils such as female infanticide, dowry, child marriage, harassment etc. This will not just help the women of today but of the future generations who can live in a world where gender equality exists which ultimately raises the literacy rate. The following distribution shows the number of literate females in the age group 0 to 60 years of a particular area.

| Age (in years) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 |
|---|---|---|---|---|---|---|
| **No. of literate females** | 350 | 1100 | 900 | 750 | 600 | 500 |

(i) Determine the class-mark of the class 40-50.

(ii) Find the number of literate females whose ages are between 20 years and 50 years

(iii) Find number of literate females whose ages are less than 40 years

**[OR]**

(iii) Find the upper limit of the modal class.

**Sol.** (i) Class mark = $\frac{40+50}{2} = \frac{90}{2} = 45$

Required Class Mark = 45

(ii) Literate females when ages are between 20years and 50 years = 900 + 750 + 600 = 2250

(iii) Literate females whose age is less than 40 years will be = 350 +1100 + 900 + 750 = 3100.

**[OR]**

(iii) Modal class $\Rightarrow$ Class having highest frequency = 10 – 20

Upper limit of modal class = 20

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# 14 Probability

**Multiple choice questions** **(1 mark)**

**1.** The probability of event equal to zero is called:

(1) Unsure event (2) Sure Event

(3) Impossible event (4) Independent event

**Sol. Option (3)**

The probability of an event that cannot happen or which is impossible, is equal to zero.

**2.** The probability that cannot exist among the following:

(1) $\frac{2}{3}$ (2) –1.5 (3) 15% (4) 0.7

**Sol. Option (2)**

The probability lies between 0 and 1. Hence, it cannot be negative.

**3.** If P(E) = 0.07, then what is the probability of 'not E'?

(1) 0.93 (2) 0.95 (3) 0.89 (4) 0.90

**Sol. Option (1)**

P(E) + P(not E) = 1

Since, P(E) = 0.07

So, P(not E) = 1 – P(E)

Or, P(not E) = 1 – 0.07

∴ P(not E) = 0.93

**4.** A bag has 3 red balls and 5 green balls. If we take a ball from the bag, then what is the probability of getting red balls only?

(1) 3 (2) 8 (3) $\frac{3}{8}$ (4) $\frac{8}{3}$

**Sol. Option (3)**

Number of red balls = 3

Number of green balls = 5

Total balls in bag = 3 + 5 = 8

Probability of getting red balls = number of red balls/total number of balls = $\frac{3}{8}$

**5.** A bag has 5 white marbles, 8 red marbles and 4 purple marbles. If we take a marble randomly, then what is the probability of not getting purple marble?

(1) 0.544 (2) 0.664 (3) 0.084 (4) 0.764

**Sol. Option (4)**

Total number of purple marbles = 4

Total number of marbles in bag = 5 + 8 + 4 = 17

Probability of getting purple marbles = $\frac{4}{17}$

Hence, the probability of not getting purple marbles = $1-\frac{4}{17}$ = $\frac{13}{17}=0.764$

**6.** A dice is thrown in the air. The probability of getting odd number is

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) 3 (4) 4

**Sol. Option (1)**

A dice has six faces having values 1, 2, 3, 4, 5 and 6.

There are three odd numbers and three even numbers.

Therefore, the probability of getting only odd numbers is = $\frac{3}{6}$ = $\frac{1}{2}$

**7.** If we throw two coins in the air, then the probability of getting both tails will be:

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) 2 (4) 4

**Sol. Option (2)**

When two coins are tossed, the total outcomes will be = 2 × 2 = 4

Hence, the probability of getting both tails = $\frac{1}{4}$

**8.** If two dice are thrown in the air, the probability of getting sum as 3 will be

(1) $\frac{2}{18}$ (2) $\frac{3}{18}$ (3) $\frac{1}{18}$ (4) $\frac{1}{36}$

**Sol. Option (3)**

When two dice are thrown in the air:

Total number of outcome = 6 × 6 = 36

Sum 3 is possible if we get (1, 2) or (2, 1) in the dices.

Hence, the probability will be = $\frac{2}{36}$ = $\frac{1}{18}$

## Assertion Reason questions (1 marks)

**9.** **Assertion:** The probability of winning a game is 0.4, then the probability of losing it, is 0.6.
**Reason:** P(E) + P (not E) = 1

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(3) Assertion (A) is true but Reason (R) is false.
(4) Assertion (A) is false but Reason (R) is true.

**Sol.** **Option (1)**

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

**10.** **Assertion:** The probability of getting a prime number when a die is thrown once is $\frac{2}{3}$.

**Reason:** Prime numbers on a die are 2, 3, 5.

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(2) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(3) Assertion (A) is true but Reason (R) is false.
(4) Assertion (A) is false but Reason (R) is true.

**Sol.** **Option (4)**

Assertion (A) is false but Reason (R) is true.

## Very short answer type questions (2 mark)

**11.** What is the probability of getting an even number, when a die is thrown once ?

**Sol.** Total number of outcomes = 6

Favourable number of outcomes {2, 4, 6} = 3

$\therefore$ Required probability $P(E) = \frac{3}{6} = \frac{1}{2}$

**12.** Cards each marked with one of the numbers 6, 7, 8, ....., 15 are placed in a box and mixed thoroughly. One card is drawn at random from the box. What is the probability of getting a card with number less than 10 ?

**Sol.** $\because$ There are 10 cards.

$\therefore$ Total number of possible outcomes = 10

Cards marked with a number less than 10 are = 6, 7, 8 and 9

$\therefore$ The number of favourable outcomes = 4

$\therefore$ Required probability $P(E) = \frac{4}{10}$ or $\frac{2}{5}$

**13.** A card is drawn from a well-shuffled deck of 52 playing cards. Find the probability that the card will not be an ace.

**Sol.** Total number of outcomes = 52

Number of ace cards = 4

$\therefore$ Number of non-ace cards = 52 – 4 = 48

$$P(\text{getting a non-ace card}) = \frac{48}{52} = \frac{12}{13}$$

**14.** Find the probability of a leap year having 53 Mondays.

**Sol.** Number of days in leap year = 366

So, $\frac{366}{7}$ = 52 weeks + 2 days.

These 2 days can be

(Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday)

So Favourable outcomes are (Sunday, Monday), (Monday, Tuesday).

Required probability = $\frac{2}{7}$

**15.** A letter of English alphabet is chosen at random. Determine the probability that the chosen letter is a consonant.

**Sol.** Total number of English alphabets = 26

Number of consonants = 26 – 5 = 21

$\therefore$ Number of favourable outcomes = 21

$$P(\text{chosen letter is a consonant}) = \frac{21}{26}.$$

**Short answer type questions** **(3 marks)**

**16.** Find the probability that a number selected from the numbers 1 to 25 which is not a prime number when each of the given number is equally likely to be selected.

**Sol.** Total number of given numbers = 25

Since the numbers 2, 3, 5, 7, 11, 13, 17, 19 and 23 are prime numbers.

There are 9 prime numbers.

$\therefore$ Numbers that are not prime numbers.

= 25 – 9 = 16

$\therefore$ Number of favourable outcomes = 16

$\Rightarrow$ Required probability $P(E) = \frac{16}{25}$.

**17.** The probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18. What is the number of rotten apples in the heap ?

**Sol.** P(selecting rotten apple)

$$= \frac{\text{Number of rotten apples}}{\text{Total number of apples}}$$

$$\Rightarrow \quad 0.18 = \frac{\text{Number of rotten apples}}{900}$$

$\Rightarrow$ Number of rotten apples = 0.18 × 900 = 162

Thus, there are 162 rotten apples in the heap.

**18.** A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability that the drawn card is neither a king nor a queen.

**Sol.** Total number of possible outcomes = 52

Cards that are neither a king nor a queen

= 52 – (4 + 4) = 44

$\therefore$ Total number of favourable outcomes = 44

$\therefore$ Required probability $= \frac{44}{52} = \frac{11}{13}$

**19.** A bag contains 4 red, 5 black and 3 yellow balls. A ball is taken out of the bag at random. Find the probability that the ball drawn is

(i) yellow colour

(ii) not of red colour.

**Sol.** Number of red balls = 4

Number of black balls = 5

Number of yellow balls = 3

$\therefore$ Total number of outcomes = 4 + 5 + 3 = 12

(i) Number of favourable outcomes = 3

$\therefore$ P(getting a yellow ball) $= \frac{3}{12} = \frac{1}{4}$

(ii) Total number of non-red balls = 5 + 3 = 8

$\therefore$ Number of favourable outcomes = 8.

$\therefore$ P(getting a non-red ball) $= \frac{8}{12} = \frac{2}{3}$

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**20.** From a group of 2 boys and 3 girls, two children are selected at random. Find the probability such that at least one boy is selected.

**Sol.** Let $B_1$ and $B_2$ be two boys and $G_1$, $G_2$ and $G_3$ be the three girls.

Since two children are selected at random,

$\therefore$ Following are the possible groups:

$B_1B_2$, $B_1G_1$, $B_1G_2$, $B_1G_3$, $B_2G_1$, $B_2G_2$, $B_2G_3$, $G_1G_2$, $G_1G_3$, $G_2G_3$

$\therefore$ Total number of possible outcomes = 10

Since, atleast one boy is to be selected,

$\therefore$ Favourable outcomes are :

$B_1B_2$, $B_1G_1$, $B_1G_2$, $B_1G_3$, $B_2G_1$, $B_2G_2$ and $B_2G_3$.

$\therefore$ Number of favourable outcomes = 7

$\therefore$ Required probability $P(E) = \frac{7}{10}$.

**21.** The king, queen and jack of diamonds are removed from a pack of 52 cards and then the pack is well-shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of

(i) diamonds

(ii) a Jack

**Sol.** $\because$ There are 52 cards in the pack.

And number of cards removed = 1 king + 1 queen + 1 jack = 3 cards

$\therefore$ Remaining cards = 52 – 3 = 49

(i) $P(\text{a diamond}) = \frac{13-3}{49} = \frac{10}{49}$ (Total diamonds are 13)

(ii) Total jacks are 4

$$P(\text{a jack}) = \frac{4-1}{49} = \frac{3}{49}$$

**22.** 20 tickets, on which numbers 1 to 20 are written, are mixed thoroughly and then a ticket is drawn at random out of them. Find the probability that the number on the drawn ticket is a multiple of 3 or 7.

**Sol.** Total number of possible outcomes = 20

Multiples of 3 from 1 to 20 are {3, 6, 9, 12, 15, 18} i.e. 6 in number.

Multiples of 7 from 1 to 20 are {7, 14} i.e. 2 in number.

So, number of favourable outcomes = 6 + 2 = 8

$\therefore$ Required probability $P(E) = \frac{8}{20} = \frac{2}{5}$

**23.** Three distinct coins are tossed together. Find the probability of getting

(i) at least 2 heads

(ii) at most 2 heads

**Sol.** Possible outcomes when three distinct coins tossed together are {(HHH), (HHT), (HTH), (THH), (HTT), (THT), (TTH), (TTT)}.

$\therefore$ Total number of possible outcomes = 8

(i) Favourable outcomes for at least two heads are {(HHH), (HHT), (HTH), (THH)}.

$\therefore$ Number of favourable outcomes = 4

$$P(\text{getting at least 2 heads}) = \frac{4}{8} = \frac{1}{2}$$

(ii) Favourable outcomes for atmost two heads are {(HHT), (HTH), (THH), (HTT) (THT), (TTH), (TTT)}.

$\therefore$ Number of favourable outcomes = 7

$\therefore$ $P(\text{getting atmost 2 heads}) = \dfrac{7}{8}$

**24.** Two different dices are thrown together. Find the probability that the numbers obtained

(i) have a sum less than 7

(ii) have a product less than 16

(iii) is a doublet of odd numbers

**Sol.** When two different dice are rolled then possible outcomes are :

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

$\therefore$ Number of total outcomes = 36

(i) Favourable outcomes = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3,), (4, 1), (4, 2), (5, 1), i.e., 15 in number.

$\therefore$ Required probability $= \dfrac{\text{Favourable no. of outcomes}}{\text{Total number of outcomes}} = \dfrac{15}{36} = \dfrac{5}{12}$

(ii) Favourable outcomes = (1, 1), (1, 2), (1, 3), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (1, 4), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2)

i.e. 25 in number.

$\therefore$ Required probability $= \dfrac{\text{Favourable no. of outcomes}}{\text{Total number of outcomes}} = \dfrac{25}{36}$

(iii) Favourable outcomes are {(1, 1), (3, 3), (5, 5)} i.e., 3 in number.

$\therefore$ Required probability $= \dfrac{\text{Favourable no. of outcomes}}{\text{Total number of outcomes}} = \dfrac{3}{36} = \dfrac{1}{12}$

**25.** From a well-shuffled pack of playing cards, black jacks, black kings and black aces are removed. A card is then drawn at random from the pack. Find the probability of getting

(i) a red card

(ii) not a diamond card

**Sol.** Total number of cards, after removing black jacks, black kings and black aces

= 52 – (2 + 2 + 2) = 46

(i) Number of red cards = 26

∴ Required probability

(ii) Number of diamond cards = 13

∴ Required probability $=\frac{26}{46}=\frac{13}{23}$

= 1 – P(getting a diamond card)

$=1-\frac{13}{46}=\frac{33}{46}$

**26.** A box has cards numbered 14 to 99. Cards are mixed thoroughly and a card is drawn from the bag at random. Find the probability that the number on the card drawn from the box is

(i) an odd number

(ii) a perfect square number

(iii) a number divisible by 7

**Sol.** Total number of cards in the box = (99 – 14) + 1 = 86

(i) Probability of getting an odd number $=\frac{43}{86}=\frac{1}{2}$

(ii) Number of perfect square number = 16, 25, 36, 49, 64, 81 i.e. 6 numbers

∴ Probability of getting a perfect square $=\frac{6}{86}=\frac{3}{43}$

(iii) Numbers divisible by 7 are {14, 21, 28, ....,98} i.e., 13 numbers

∴ Probability of getting a number divisible by $7=\frac{13}{86}$

**27.** Cards, marked with numbers 5 to 50, are placed in a box and shuffled thoroughly. A card is drawn from the box at random. Find the probability that the number on the taken out card is

(i) a prime number less than 10

(ii) a number which is a perfect square

**Sol.** Total number of cards in the box = 50 – 5 + 1 = 46

(i) Prime numbers less than 10 are {5, 7} i.e., 2 in number.

∴ Required probability $=\frac{2}{46}=\frac{1}{23}$

(ii) From the numbers 5 to 50, the perfect square numbers are {9, 16, 25, 36, 49} i.e., 5 in number.

$\therefore$ Required probability = $\frac{5}{46}$

**28.** A child has a die whose six faces show the letters as given below :

$\boxed{A}$ $\boxed{B}$ $\boxed{C}$ $\boxed{A}$ $\boxed{D}$ $\boxed{A}$

The die is thrown once. Find the probability of getting.

(i) A (ii) D

**Sol.** Total number of faces in die = 6

(i) Number of favourable outcomes = 3

$\therefore$ $P(\text{getting A}) = \frac{3}{6} = \frac{1}{2}$

(ii) Number of favourable outcomes = 1

$\therefore$ $P(\text{getting D}) = \frac{1}{6}$

## Long answer type questions (5 marks)

**29.** The probability of selecting a red ball at random from a jar that contains only red, blue and orange balls is 1/4. The probability of selecting a blue ball at random from the same jar is 1/3. If the jar contains 10 orange balls, find the total number of balls in the jar.

**Sol.** Given, probability of selecting a red ball, $P(R) = \frac{1}{4}$.

Probability of selecting a blue ball, $P(B) = \frac{1}{3}$

Since, P(R) + P(B) + P(O) = (where P(O) is probability of selecting an orange ball)

$$\Rightarrow \frac{1}{4} + \frac{1}{3} + P(O) = 1$$

$$\Rightarrow P(O) = 1 - \frac{1}{4} - \frac{1}{3} = \frac{12-3-4}{12} = \frac{12-7}{12} = \frac{5}{12}$$

Also, $P(O) = \frac{\text{Number of orange balls}}{\text{Total number of balls}}$

$$\Rightarrow \frac{5}{12} = \frac{10}{\text{Total number of balls}}$$

$$\Rightarrow \text{Total number of balls} = \frac{12 \times 10}{5} = 24$$

**30.** A game consists of tossing a coin 3 times and noting its outcome each time. Hanif wins if he gets three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

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**Sol.** When a coin is tossed 3 times, total possible outcomes are {HHH, HTH, HHT, THH, HTT, THT, TTH, TTT}

$\therefore$ Number of possible outcomes = 8

Possible outcomes for Hanif to lose the game are [HHT, HTH, THH, HTT, THT, TTH]

$\therefore$ Number of favourable outcomes = 6

$\therefore$ Required probability $=\frac{6}{8}=\frac{3}{4}$

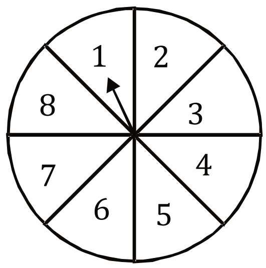
**31.** In a game of chance there is spinning of an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 and there are equally likely outcomes. What is the probability that it will point at

(i) 7 (ii) an odd number (iii) a number less than 9

**Sol.** Since, following numbers are marked on the disc:

1, 2, 3, 4, 5, 6, 7, 8

$\Rightarrow$ Possible outcomes in each case are 8.




(i) Possible outcomes = 8

Favourable outcome = 1

$\therefore$ P(The number 7) = $\frac{1}{8}$

(ii) Odd numbers on the disc are 1, 3, 5 and 7

$\therefore$ Favourable outcomes = 4

$\Rightarrow$ P(Odd number) $=\frac{4}{8}=\frac{1}{2}$

(iii) No. less than 9 are (1, 2, 3, 4, 5, 6, 7, 8)

$\therefore$ Favourable outcomes = 8

$\Rightarrow$ P(no. less than 9) = $\frac{8}{8}=1$

**32.** A piggy bank contains hundred 50 p coins, seventy Rs. 1 coins, fifty Rs. 2 coins and thirty Rs. 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin

(i) will be a Rs. 1 coin?

(ii) will not be a Rs. 5 coin?

(iii) will be a 50 p or Rs. 2 coin?

**Sol.** Total coins in the piggy bank = 100 + 70 + 50 + 30 = 250

$\therefore$ Total number of possible outcomes = 250

(i) Number of Rs. 1 coins = 70

$\therefore$ Number of favourable outcomes = 70

$\therefore$ P(getting a Rs. 1 coin) = $\frac{70}{250} = \frac{7}{25}$

(ii) Number of Rs. 5 coins = 30

$\therefore$ P(getting a Rs. 5 coin) = $\frac{30}{250} = \frac{3}{25}$

Now, required probability = $1 - \frac{3}{25} = \frac{22}{25}$

(iii) Number of 50 p or Rs. 2 coin = 100 + 50 = 150

$\therefore$ P(getting a 50 p or Rs. 2 coin) = $\frac{150}{250} = \frac{3}{5}$

**33.** Two dice are thrown simultaneously. What is the probability that

(i) 5 will not come up on either of them?

(ii) 5 will come up on at least one?

(iii) 5 will come up at both dice?

**Sol.** The two dice are thrown simultaneously.

$\therefore$ Possible outcomes are = 6 × 6 = 36

(i) When 5 will not come up on either of them:

Outcomes on which 5 will come up atleast one times of them = 11

Favourable outcomes are: 36 – 11 = 25

$\therefore$ P(5 will not come up on either dice) $= \frac{25}{36}$

(ii) When 5 will come on at least one dice

Favourable outcomes are 11

$\therefore$ P(5 will come on at least one dice) $= \frac{11}{36}$

(iii) When 5 will come up on both dice:

Favourable outcome is only one i.e. (5, 5)

$\therefore$ P(5 on both dice) $= \frac{1}{36}$

**34.** Two different dice are rolled simultaneously. Find the probability that the sum of numbers appearing on the two dice is 10.

**Sol.** When two different dice are rolled then possible outcomes are :

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)

(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)

(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

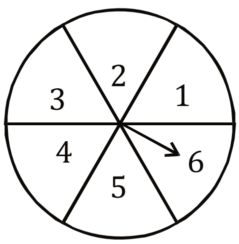
$\therefore$ Number of total outcomes = 36

$\therefore$ Sum of (5, 5), (6, 4) and (4, 6) is 10.

$\therefore$ No of favourable outcomes = 3

$\Rightarrow$ Required Probability $=\frac{3}{36}$ or $\frac{1}{12}$

**35.** The given figure shows a disc on which a player spins an arrow twice. The fraction $\frac{a}{b}$ is formed, where 'a' is the number on sector on which arrow stops on the first spin and 'b' is the number on the sector in which the arrow stops on second spin. On each spin, each sector has equal chance of selection by the arrow. Find the probability that the fraction $\frac{a}{b}>1$.




**Sol.** Total possible outcomes in spinning of an arrow twice = 6 × 6 = 36

So, favourable outcomes (a, b) for which $\frac{a}{b}>1$ are

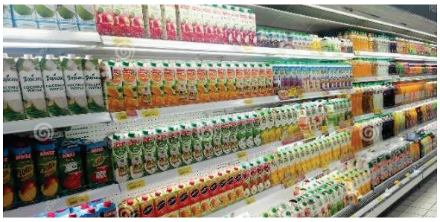
{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5)}

$\therefore$ Number of favourable outcomes = 15

$\therefore$ Required probability $=\frac{15}{36}=\frac{5}{12}$

**Case Study type questions** **(4 marks)**

**1.** Vishal goes to a store to purchase juice cartons for his shop. The store has 80 cartons of orange juice, 90 cartons of apple juice, 38 cartons of mango juice and 42 cartons of guava juice. If Vishal chooses a carton at random, then answer the following questions.



(i) Find the probability of selecting a carton of apple juice.

(ii) Find the probability that selected carton doesn't contain carton of orange juice.

(iii) Vishal buys 4 cartons of apple juice, 3 cartons of orange juice and 3 cartons of guava juice. A customer comes to Vishal's shop and picks a litre pack of juice at random. Find the probability that a customer pick a guava juice, if each carton has 10 tetra packs of juice.

**[OR]**

(iii) If the storekeeper bought 14 more cartons of apple juice, then find the probability of selecting a tetra pack of apple fruit juice, if each cartoon has 10 tetra packs of juice.

**Sol.** (i) Total cartons = 80 + 90 + 38 + 42 = 250

P(apple juice carton) = $\frac{90}{250} = \frac{9}{25}$

(ii) P(orange juice carton) = $\frac{80}{250} = \frac{8}{25}$

P(not a carton of orange juice) = $1 - \frac{8}{25} = \frac{17}{25}$

(iii) Total cartons Vishal has bought = 4 + 3 + 3 = 10 cartons

Total tetra packs = 10 × 10 = 100

Number of tetra packs having guava juice = 3 × 10 = 30

P(customer picking up a guava juice) = $\frac{30}{100} = \frac{3}{10}$

**[OR]**

(iii) Total cartons = 250 + 14 = 264

Total number of tetra packs = 264 × 10 = 2640

Number of tetra packs of apple juice = (90 +14) × 10 = 104 × 10 = 1040

P(selecting a tetra packs of apple juice) = $\frac{1040}{2640} = \frac{13}{33}$

Jot Down

Jot Down

Jot Down